# Fundamentals of Logic Design 

$6^{\text {th }}$ Edition

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## I: INTRODUCTION

The text, Fundamentals of Logic Design, 6th edition, has been designed so that it can be used either for a standard lecture course or for a self-paced course The text is divided into 20 study units in such a way that the average study time for each unit is about the same The units have undergone extensive class testing in a self-paced environment and have been revised based on student feedback The study guides and text material are sufficient to allow almost all students to achieve mastery of all of the objectives For example, the material on Boolean algebra and algebraic simplification is $21 / 2$ units because students found this topic difficult There is a separate unit on going from problem statements to state graphs because this topic is difficult for many students

The textbook contains answers for all of the problems that are assigned in the study guides This Instructor's Manual contains complete solutions to these problems Solutions to the remaining homework problems as well as all design and simulation exercises are also included in this manual In the solutions section of this manual, the abbreviation FLD stands for Fundamentals of Logic Design (6th ed )

Information on the self-paced course taught at the University of Texas using the textbook is available at www.ece. utexas edu/projects/ee316 This website also links to an updated errata list for the text In addition to the textbook and study guides, teaching a self-paced course requires that a set of tests be prepared for each study unit This manual contains a sample test for each unit

### 1.1 Using the Text in a Lecture Course

Even though the text was developed in a self-paced environment, the text is well suited for use in a standard lecture course Since the format of the text differs somewhat from a conventional text, a few suggestions for using the text in a lecture course may be appropriate Except for the inclusion of objectives and study guides, the units in the text differ very little from chapters in a standard textbook The study guides contain very basic questions, while the problems at the end of each unit are of a more comprehensive nature For this reason, we suggest that specific study guide questions be assigned for students to work through on their own before working out homework problems selected from those at the end of the unit. The unit tests given in Part IV of this manual provide a convenient source of additional homework assignments or a source of quiz problems. The text contains many examples that are completely worked out with detailed step-by-step explanations Discussion of these detailed examples in lecture may not be necessary if the students study them on their own The lecture time is probably better spent discussing general principles and applications as well as providing help with some of the more difficult topics Since all of the units have study guides, it would be possible to assign some of the easier topics for self-study and devote the lectures to the more difficult topics

At the University of Texas a class composed largely of Electrical Engineering and Computer Science sophomores and juniors covers 18 units (all units except 6 and 19) of the text in one semester Units $8,10,12,16,17$, and 20 contain design problems that are suitable for simulation and lab exercises The design problems help tie together and review the material from a number of preceding units Units 10,17 , and 20 introduce the VHDL
hardware description language These units may be omitted if desired since no other units depend on them

### 1.2 Some Remarks About the Text

In this text, students are taught how to use Boolean algebra effectively, in contrast with many texts that present Boolean algebra and a few examples of its application and then leave it to the student to figure out how to use it effectively For example, use of the theorem $x+y z=(x+y)$ $(x+z)$ in factoring and multiplying out expressions is taught explicitly, and detailed guidelines are given for algebraic simplification

Sequential circuits are given proper emphasis, with over half of the text devoted to this subject. The pedagogical strategy the text uses in teaching sequential circuits has proven to be very effective The concepts of state, next state, etc are first introduced for individual flip-flops, next for counters, then for sequential circuits with inputs, and finally for more abstract sequential circuit models The use of timing charts, a subject neglected by many texts, is taught both because it is a practical tool widely used by logic design engineers and because it aids in the understanding of sequential circuit behavior

The most important and often most difficult part of sequential circuit design is formulating the state table or graph fiom the problem statement, but most texts devote only a few paragraphs to this subject because there is no algorithm This text devotes a full unit to the subject, presents guidelines for deriving state tables and graphs, and provides programmed exercises that help the student learn this material. Most of the material in the text is treated in a faitly conventional manner with the following exceptions:
(1) The diagonal form of the 5-variable Karnaugh map is introduced in Unit 5 (We find that students make fewer mistakes when using the diagonal form of 5 -variable map in comparison with the side-by-side form ) Unit 5 also presents a simple algorithm for finding all essential prime implicants from a Karnaugh map
(2) Both the state graph approach (Unit 18) and the SM chart approach (Unit 19) for designing sequential control circuits are presented
(3) The introduction to the VHDL hardware description language in Units 10,17 , and 20 emphasizes the relation between the VHDL code and the actual hardware

### 1.3 Using the Text in a Self-Paced Course

This section introduces the personalized system of self-paced instruction (PSI) and offers suggestions for using the text in a self-paced course. PSI (Personalized System of Instruction) is one of the most popular and successful systems used for self-paced instruction The essential features of the PSI method are
(a) Students are permitted to pace themselves through the course at a rate commensurate with their ability and available time.
(b) A student must demonstrate mastery of each study unit before going onto the next
(c) The written word is stressed; lectures, if used, are only for motivation and not for transmission of critical information
(d) Use of proctors permits repeated testing, immediate scoring, and significant personal interaction with the students.
These factors work together to motivate students toward a high level of achievement in a well-

The PSI method of instruction and its implementation are described in detail in the following references:

Keller, Fred S and J. Gilmour Sherman, The Keller Plan Handbook, W. A Benjamin, Inc, 1974.

Sherman, I G., ed, Personalized System of Instruction 41 Germinal Papers, W A. Benjamin, Inc, 1974
Results of applying PSI to a first course in logic design of digital systems are described in
Roth, C.H, "Continuing Effectiveness of Personalized Self-Paced Instruction in Digital Systems Engineering", Engineering Education, Vol 63, No 6, March 1973

The instructor in charge of a self-paced course will serve as course manager in addition to his role in the classroom For a small class, he may spend a good part of his time acting as proctor in the classroom, but as class size increases he will have to devote more of his time to supervision of course activities and less time to individual interaction with students. In his managerial role, the instructor is responsible for organizing the course, selection and training of proctors, supervision of proctors, and monitoring of student progress. The proctors play an important role in the success of a self-paced course, and therefore their selection, training, and supervision is very important After an initial session to discuss proper ways of grading readiness tests and interacting with students, weekly proctor meetings to discuss course procedures and problems may be appropriate

A progress chart showing the units completed by each student is very helpful in monitoring student progress through the course The instructor may wish to have individual conferences with students who fall too far behind. The instructor needs to be available in the classroom to answer individual student questions and to assist with grading of readiness tests as needed He should make a special point to speak with the weak or slow students and give them a word of encouragement. From time to time he may need to settle differences which arise between proctors and students

Various strategies for organizing a PSI course are described in the Keller Plan Handbook The procedures used for operating the self-paced digital logic course at the University of Texas are described in "Unit 0 ", which is available on the web: wwwece. utexas.edu/projects/ee316 At the first class meeting, we hand out a copy of Unit 0 The students are asked to read through Unit 0 and take a short test on the course procedures This test is immediately evaluated so that the student can complete Unit 0 before the end of the first class period In this way, the student is exposed to the basic way the course operates and is ready to proceed immediately with Unit 1 in the textbook

During a typical class period, some of the students will spend their time studying but most of the students will come prepared to take a unit test At the beginning of the period, the instructor or a proctor will be available to answer student questions on an individual basis Later in the period, most of the time will be spent evaluating unit tests. We have found that a standard 50 minute class period is not long enough for a PSI session We usually schedule sessions of $11 / 2$ or 2 hours or longer depending on class size. This allows adequate time for students to have their questions answered, take a unit test, and have their tests
graded Interactive grading of the tests with the student present is an important part of the PSI system and adequate time must be allowed for this activity If you have a large number of students and proctors, you may wish to prepare a manual for guidance of your proctors The procedures that we use for evaluating unit tests are described in a Proctor's Manual, which can be obtained by writing to Professor Charles H. Roth

### 1.4. Use of Computer Software

Three software packages are included on the CD that accompanies the textbook The first is a logic simulator program called SimUaid, the second is a basic computer-aided logic design program called LogicAid, and the third is a VHDL Simulator called DirectVHDL In addition, we use the Xilinx ISE software for synthesizing VHDL code and downloading to CPLD or FPGA circuit boards The Xilinx ISE software is available at nominal cost through the Xilinx University Program (for information, go to wwwxilinx com/univ/overviewhtml). A"Webpack" version of the Xilinx software is also available for downloading from the Xilinx com website

SimUaid provides an easy way for students to test their logic designs by simulating them We first introduce SimUaid in Unit 4, where we ask the students to design a simple logic circuit such as problem 413 or 4.14 , and simulate it SimUaid is easy to learn, and it is highly interactive so that students can flip a simulated switch and immediately observe the result In Unit 8, students design a multiple-output combinational logic circuit using NAND and NOR gates and test its operation using SimUaid Students can use the simulator to help them understand the operation of latches and flip-flops in Unit 11 In Unit 12, we ask them to design a counter and simulate it (one part of problem 12.10) In Unit 16, students use SimUaid to test their sequential circuit designs They can also generate VHDL code from theit SimUaid circuit, synthesize it, and download it to a circuit board for hardware testing In Unit 18, students can use the advanced features of SimUaid to simulate a multiplier or divider controlled by a state machine

LogicAid provides an easy way to introduce students to the use of the computer in the logic design process It enables them to solve larger, more practical design problems than they could by hand They can also use LogicAid to veify solutions that they have worked out by hand Instructors can use the program for grading homework and quizzes We first introduce LogicAid in Unit 5 The program has a Karnaugh Map Tutorial mode that is very useful in teaching students to solve Karnaugh map problems This tutorial mode helps students learn to derive minimum solutions from a Karnaugh map by informing them at each step whether that step is correct or not It also forces them to choose essential prime implicants first When in the KMap tuto mode, LogicAid prints "KMT" at the top of each output page, so you can check to see if the problems were actually solved in the tutorial mode

Students can use LogicAid to help them solve design problems in Units 8, 16, 18, 19 and other units For designing sequential circuits, they can input a state graph, convert it to a state table, reduce the state table, make a state assignment, and derive minimized logic equations for outputs and flip-flop inputs

The LogicAid State Table Checker is useful for Units 14 and 16, and for other units in which students construct state tables It allows students to check their solutions without revealing the correct answers If the solution is wrong, the program displays a short input sequence for which the student's table fails. The LogicAid folder on the CD contains encoded copies of solutions for most of the state graph problems in Fundamentals of Logic Design, 6th Ed. If you wish to create a passwordprotected solution file for other state table problems, enter the state table into LogicAid, syntax
check it, and then hold down the Cttl key while you select Save As on the file menu The Partial Graph Checker serves as a state graph tutor that allows a student to check his work at each step while constructing a state graph If the student makes a mistake, it provides feedback so that the student can correct his answer The partial graph checker works with any state graph problem for which an encoded state table solution file is provided

The DirectVHDL simulator helps students learn VHDL syntax because it provides immediate visual feedback when they make mistakes Our students use it for simulating VHDL code in Units 10 , 17, and 20. Students can simulate and debug their code at home and then bring the code into lab for synthesis and hardware testing

### 1.5. Suggested Equipment for Laboratory Exercises

Many types of logic lab equipment are available that are adequate to perform the lab exercises. Since most logic design is done today using programmable logic instead of individual ICs, we now recommend use of CPLDs or FPGAs for hardware implementation of logic circuit designs. At the University of Texas, we are presently using the XILINX Spartan-3 FPGA boards, which are available from Digilent The Spartan-3 FPGA has more than an adequate number of logic cells to implement the lab exercises in the text The board has 8 switches, 4 pushbuttons, 8 single LEDs, and four 7 -segment LEDs Information about this board and other CPLD and FPGA boards made by Digilent can be found on their website, www.digilentinc.com We use the board in conjunction with the Xiliinx ISE software mentioned earlier

## II. SOLUTIONS TO HOMEWORK PROBLEMS Unit 1 Problem Solutions

1.1 (a) $757.25_{10}$

| $16\lfloor 757$ |  | 0.25 |
| ---: | :--- | ---: |
| $16\lfloor 47$ | r5 | $\frac{16}{16\lfloor 2}$ |
| r15 $=\mathrm{F}_{16}$ | (4).00 |  |

$$
\begin{aligned}
\therefore 757.25_{10} & =2 \mathrm{~F} 5.40_{16} \\
& =\frac{0010}{2} \frac{1111}{\mathrm{~F}} \frac{0101.0100}{5} \frac{0000_{2}}{0}
\end{aligned}
$$

1.1 (c) $\quad 356.89_{10}$

| $16 \lcm{356}$ |  | 0.89 |
| :---: | :---: | :---: |
| $16 \mid 22$ | r4 | 16 |
| $16 \downharpoonright 1$ | r6 | (14). 24 |
| 0 | r1 | 16 |
|  |  | (3). 84 |
|  |  | 16 |
|  |  | (13). 44 |
|  |  | 16 |
|  |  | (7). 04 |

$$
\begin{aligned}
\therefore 356.89_{10} & =164 \cdot \mathrm{E}_{16} \\
& =\frac{0001}{1} \frac{0110}{6} \frac{0100}{4} \cdot \frac{1110}{\mathrm{E}} \frac{0011}{3} 2
\end{aligned}
$$

1.2 (a) EB1.6 $6_{16}=\mathrm{E} \times 16^{2}+\mathrm{B} \times 16^{1}+1 \times 16^{0}+6 \times 16^{-1}$ $=14 \times 256+11 \times 16+1+6 / 16=3761.375_{10}$ $111010110001.011(0)_{2}$
E B 16
$7261.3_{8}=7 \times 8^{3}+2 \times 8^{2}+6 \times 8^{1}+1+3 \times 8^{-1}$
$=7 \times 512+2 \times 64+6 \times 8+1+3 / 8=3761.375_{10}$ $\frac{111}{7} \frac{010}{2} \frac{110}{6} \frac{001}{1} \cdot \frac{011}{3}_{8}$

$$
\begin{array}{lllll}
7 & 2 & 6 & 1 & 3
\end{array}
$$

1.3

$$
\begin{aligned}
& 3 \mathrm{BA} .25_{14}= 3 \times 14^{2}+11 \times 14^{1}+10 \times 14^{0}+2 \times 14^{-1} \\
&+5 \times 14^{-2} \\
&= 588+154+10+0.1684=752.1684_{10} \\
& 6 \lcm{752} \\
& 6 \lcm{125} \text { r2 } \\
& 6 \lcm{20} \text { r5 } \\
& 6 \lcm{3} \text { r2 } \\
& 0 \text { r3 } \\
& \frac{0.1684}{(1) .0104} \\
& \\
& \frac{6}{(0) \cdot 0624} \\
& \hline(2) .3744 \\
& \hline(1) .4784
\end{aligned}
$$

$$
\therefore 3 \text { BA. } 25_{14}=752.1684_{10}=3252.1002_{6}
$$

1.1 (b) $\quad 123.17_{10}$

| $16 \lcm{123}$ |  |
| ---: | :--- | ---: |
| $16 \underline{77}$ | r11 |
| 0 | r7 |$\quad$| 0.17 |
| ---: |

$$
\begin{aligned}
\therefore 123.17_{10} & =7 \mathrm{~B} .2 \mathrm{~B}_{16} \\
& =\frac{0111}{7} \frac{1011.0010}{\mathrm{~B}} \frac{1011_{2}}{\mathrm{~B}}
\end{aligned}
$$

1.1 (d) $\quad 1063.5_{10}$

| $16 \lcm{1063}$ |  | 0.5 |
| ---: | ---: | ---: |
| $16 \lcm{66}$ | r7 | 16 |
| $16 \lcm{4}$ | r2 | $(8) .00$ |
| 0 | r4 |  |

$$
\begin{aligned}
\therefore 1063.5_{10} & =427.8_{16} \\
& =\frac{0100}{4} \frac{0010}{2} \frac{0111 \cdot 1000_{2}}{7}
\end{aligned}
$$

1.2 (b) 59D. $\mathrm{C}_{16}=5 \times 16^{2}+9 \times 16^{1}+\mathrm{D} \times 16^{0}+\mathrm{C} \times 16^{-1}$

$$
=5 \times 256+9 \times 16+13+12 / 16=
$$

$$
1437.75_{10}
$$

$$
0101 \frac{1001}{} \frac{1101}{} \cdot \frac{1100_{16}}{}
$$

$$
\begin{array}{llll}
5 & 9 & \mathrm{D} & \mathrm{C}
\end{array}
$$

$$
2635.6_{8}=2 \times 8^{3}+6 \times 8^{2}+3 \times 8^{1}+5 \times 8^{0}+6 \times 8^{-1}
$$

$$
=2 \times 512+6 \times 64+3 \times 8+5+6 / 8=
$$

$1437.75_{10}$
$\frac{010}{2} \frac{110}{6} \frac{011}{3} \frac{101}{5} \cdot \frac{110_{8}}{6}$
1.4 (b) $\quad 1457.11_{10}$
1.4 (c) $5 B 1.1 \mathrm{C}_{16}=\frac{11}{5} \frac{23}{\mathrm{~B}} \frac{01}{1} \cdot \frac{01}{1} \frac{30_{4}}{\mathrm{C}}$
1.4 (d) DEC. $A_{16}=\mathrm{D} \times 16^{2}+\mathrm{E} \times 16^{1}+\mathrm{C} \times 16^{0}+\mathrm{A} \times 16^{-1}$ $=3328+224+12+0.625=3564.625_{10}$

$$
\begin{aligned}
& \begin{array}{rlr}
16 \lcm{1457} \\
16 \underline{91} \\
16 \underline{5} & \text { r1 } & \begin{array}{r}
0.11 \\
0
\end{array} \\
\text { r11 } & \\
& \frac{16}{(1) .76} \\
& \frac{16}{(12) .16}
\end{array} \\
& \therefore 1457.11_{10}=5 B 1.1 C_{16}
\end{aligned}
$$

Unit 1 Solutions
1.5 (a)

| 111 |  |  |  |
| ---: | ---: | ---: | ---: |
| 1111 | (Add) | 1111 | (Sub) |
| $+\underline{1010}$ |  | $-\underline{1010}$ |  |
| 11001 |  | 0101 |  |

1111 (Multiply) 1.5 (b, c) See FLD p. 692 for solutions. $\times 1010$ 0000 $\underline{1111}$ 11110 $\frac{0000}{011110}$ $\frac{1111}{10010110}$
1.6, 1.7, See FLD p. 692 for solutions.
1.8

| 1.10 (a) | $1305.375_{10}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $16 \lcm{1305}$ |  | 0.375 |
|  | $16 \lcm{81}$ | r9 | 16 |
|  | 5 | r1 | (6). 000 |
|  | $\therefore 1305.375_{10}=519.600_{16}$ |  |  |
| $=\underline{0101} \underline{0001} \underline{1001.0110} \underline{000}$ |  |  |  |

1.10 (b) $11.33_{10}$

$$
\begin{aligned}
& 16 \lcm{111} \\
& 6 \\
& \mathrm{r} 15=\mathrm{F}_{16} \\
& \frac{0.33}{(5) .28} \\
& \frac{16}{(4) .48}
\end{aligned} \begin{array}{r}
\therefore 111.33_{10}=6 \mathrm{~F} .54_{16} \\
\quad=\frac{0110}{6} \frac{1111}{\mathrm{~F}} \cdot \frac{0101}{5} \frac{0100_{2}}{4}
\end{array}
$$



$$
\therefore 375.54_{8}=100101.2001_{3}
$$

## Unit 1 Solutions


Unit 1 Solutions


1.19(a)

| 101110 Quotient | 1.19(b) | 11011 Quotient |
| :---: | :---: | :---: |
| $1 0 1 \longdiv { 1 1 1 0 1 0 0 1 }$ | 1110 | )110000001 |
| 101 |  | 1110 |
| 1001 |  | 10100 |
| 101 |  | 1110 |
| 1000 |  | 11000 |
| 101 |  | 1110 |
| 110 |  | 10101 |
| 101 |  | 1110 |
| 11 Remainder |  | 111 Remainde |

Unit 1 Solutions
1.19(c)
1.20(b)
(a) $4+3$ is 10 in base 7 , i.e., the sum digit is 0 with a carry of 1 to the next column. $1+5+$ 4 is 10 in base 7. $1+6+0$ is 10 in base 7. This overflows since the correct sum is $1000{ }_{7}$.
(b) $4+3+3+3=13$ in base 10 and 23 in base 5. Try base 10. $1+2+4+1+3=11$ in base 10 so base 10 does not produce a sum digit of 2 . Try base 5. $2+2+4+1+3=22$ in base 5 so base 5 works.
(c) $4+3+3+3=31$ in base 4,21 in base 6 , and 11 in base 12. Try base 12. $1+2+4+1+3=$ B in base 12 so base 12 does not work. Try base 4 . $3+2+4+1+3=31$ in base 4 so base 4 does not work. Try base 6. $2+2+4+1+3=20$ so base 6 is correct.
1.24 (a) Expand the base $b$ number into a power series

$$
\mathrm{N}=\mathrm{d}_{3 \mathrm{k}-1} \mathrm{~b}^{3 \mathrm{k}-1}+\mathrm{d}_{3 \mathrm{k}-2} \mathrm{~b}^{3 \mathrm{k}-2}+\mathrm{d}_{3 \mathrm{k}-3} \mathrm{~b}^{3 \mathrm{k}-3}+\ldots .+
$$

$$
\mathrm{d}_{5} \mathrm{~b}^{5}+\mathrm{d}_{4} \mathrm{~b}^{4}+\mathrm{d}_{3} \mathrm{~b}^{3}+\mathrm{d}_{2} \mathrm{~b}^{2}+\mathrm{d}_{1} \mathrm{~b}^{1}+\mathrm{d}_{0} \mathrm{~b}^{0}+\mathrm{d}_{-1} \mathrm{~b}^{-1}+
$$

$$
\mathrm{d}_{-2} \mathrm{~b}^{-2}+\mathrm{d}_{-3} \mathrm{~b}^{-3}+\ldots .+\mathrm{d}_{-3 \mathrm{~m}+2^{2} \mathrm{~b}^{-3 m+2}+\mathrm{d}_{-3 \mathrm{~m}+1} \mathrm{~b}^{-3 \mathrm{~m}+1}}^{\text {- }}
$$

$+\mathrm{d}_{-3 \mathrm{~m}} \mathrm{~b}^{-3 \mathrm{~m}}$ where each $\mathrm{d}_{\mathrm{i}}$ has a value from 0 to
(b-1). (Note that 0 's can be appended to the number so that it has a multiple of 3 digits to the left and right of the radix point.) Factor $\mathrm{b}^{3}$ from each group
of 3 consecutive digits of the number to obtain

$$
\begin{aligned}
& \quad \mathrm{N}=\left(\mathrm{d}_{3 \mathrm{k}-1} \mathrm{~b}^{2}+\mathrm{d}_{3 \mathrm{k}-2} \mathrm{~b}^{1}+\mathrm{d}_{\left.3 \mathrm{k}-3 \mathrm{~b}^{0}\right)\left(\mathrm{b}^{3}\right)^{(k-1)}+\ldots}\right. \\
& +\left(\mathrm{d}_{5} \mathrm{~b}^{2}+\mathrm{d}_{4} \mathrm{~b}^{1}+\mathrm{d}_{3} \mathrm{~b}^{0}\right)\left(\mathrm{b}^{3}\right)^{1}+\left(\mathrm{d}_{2} \mathrm{~b}^{2}+\mathrm{d}_{1} \mathrm{~b}^{1}+\right. \\
& \left.\mathrm{d}_{0} \mathrm{~b}^{0}\right)\left(\mathrm{b}^{3}\right)^{0}+\left(\mathrm{d}_{-1} \mathrm{~b}^{2}+\mathrm{d}_{-2} \mathrm{~b}^{1}+\mathrm{d}_{-3} \mathrm{~b}^{0}\right)\left(\mathrm{b}^{3}\right)^{-1}+\ldots .+ \\
& \left(\mathrm{d}_{-3 \mathrm{~m}+2} \mathrm{~b}^{2}+\mathrm{d}_{-3 \mathrm{~m}+1} \mathrm{~b}^{1}+\mathrm{d}_{-3 \mathrm{~m}^{0}} \mathrm{~b}^{0}\right)\left(\mathrm{b}^{3}\right)^{-\mathrm{m}} \\
& \text { Each }\left(\mathrm{d}_{3 \mathrm{i}-1} \mathrm{~b}^{2}+\mathrm{d}_{3 \mathrm{i}-2} \mathrm{~b}^{1}+\mathrm{d}_{3 \mathrm{i}-3} \mathrm{~b}^{0}\right) \text { has a value from } \\
& 0 \text { to }\left[(\mathrm{b}-1) \mathrm{b}^{2}+(\mathrm{b}-1) \mathrm{b}^{1}+(\mathrm{b}-1) \mathrm{b}^{0}\right] \\
& \quad=(\mathrm{b}-1)\left(\mathrm{b}^{2}+\mathrm{b}^{1}+\mathrm{b}^{0}\right)=\left(\mathrm{b}^{3}-1\right)
\end{aligned}
$$

so it is a valid digit in a base $b^{3}$ number.
Consequently, the last expression is the power series expansion for a base $b^{3}$ number.
1.20(a)

110 | 10111 |
| :---: |
| $\frac{10001101}{1011}$ |
| $\frac{110}{1010}$ |
| $\frac{110}{1001}$ |
| $\frac{110}{11}$ Rematient |

1.20(c)

1010 | $\frac{1011}{1110100}$ | Quotient |
| :---: | :---: |
|  | $\frac{1010}{10010}$ |
|  | $\frac{1010}{10000}$ |
|  | $\frac{1010}{110}$ Remainder |

1.22 If the binary number has $n$ bits (to the right of the radix point), then its precision is $\left(1 / 2^{\mathrm{n}+1}\right)$. So to have the same precision, $n$ must satisfy
$\left(1 / 2^{\mathrm{n}+1}\right)<(1 / 2)\left(1 / 10^{4}\right)$ or $\mathrm{n}>4 /(\log 2)=13.28$ so $n$ must be 14 .
1.23

$$
\begin{aligned}
& .363636 \ldots \\
& \quad=\left(36 / 10^{2}\right)\left(1+1 / 10^{2}+1 / 10^{4}+1 / 10^{6}+\ldots\right) \\
& =\left(36 / 10^{2}\right)\left[1 /\left(1-1 / 10^{2}\right)\right]=\left(36 / 10^{2}\right)\left[10^{2} / 99\right] \\
& =36 / 99=4 / 11 \\
& 8(4 / 11)=2+10 / 11 \\
& 8(10 / 11)=7+3 / 11 \\
& 8(3 / 11)=2+2 / 11 \\
& 8(2 / 11)=1+5 / 11 \\
& 8(5 / 11)=3+7 / 11 \\
& 8(7 / 11)=5+1 / 11 \\
& 8(1 / 11)=0+8 / 11 \\
& 8(8 / 11)=5+9 / 11 \\
& 8(9 / 11)=6+6 / 11 \\
& 8(6 / 11)=4+4 / 11 \\
& 8(4 / 11)=2+10 / 11
\end{aligned}
$$

Repeats: .27213505642.......
1.24 (b) Expand the base $b^{3}$ number into a power series
$N=d_{k}\left(b^{3}\right)^{k}+d_{k-1}\left(b^{3}\right)^{k-1}+\ldots+d_{1}\left(b^{3}\right)^{1}+d_{0}\left(b^{3}\right)^{0}$ $+\mathrm{d}_{-1}\left(\mathrm{~b}^{3}\right)^{-1}+\ldots .+\mathrm{d}_{-\mathrm{m}}\left(\mathrm{b}^{3}\right)^{-\mathrm{m}}$
where each $d_{i}$ has a value from 0 to $\left(b^{3}-1\right)$.
Consequently, $\mathrm{d}_{\mathrm{i}}$ can be represented as a base b number in the form

$$
\left(e_{3 i-1} b^{2}+e_{3 i-2} b^{1}+e_{3 i-3} b^{0}\right)
$$

Where each $e_{j}$ has a value from 0 to ( $b-1$ ).
Substituting these expressions for the $\mathrm{d}_{\mathrm{i}}$ produces a power series expansion for a base b number.
1.25

|  | 4 | 3 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 1 |
| 6 | 1 | 0 | 1 | 0 |
| 7 | 1 | 1 | 0 | 0 |
| 8 | 1 | 1 | 0 | 1 |
| 9 | 1 | 1 | 1 | 0 |

$9154=1110000110011000$
1.26

5-3-1-1 is possible, but $6-4-1-1$ is not, because there is no way to represent 3 or 9 .

Alternate
Solutions:

|  | 5311 | (0010) |
| :---: | :---: | :---: |
| 0 | 0000 |  |
| 1 | 0001 |  |
| 2 | 0011 |  |
| 3 | 0100 |  |
| 4 | 0101 | (0110) |
| 5 | 1000 |  |
| 6 | 1001 | (1010) |
| 7 | 1011 |  |
| 8 | 1100 | (1110) |
| 9 | 1101 |  |

5-4-1-1 is not possible, because there is no way to represent 3 or $8.6-3-2-1$ is possible:

$$
\left.\begin{array}{c|c} 
& 6321 \\
\hline 0 & 0
\end{array}\right)
$$

Alternate
1.30

Solutions:
Alternate Solutions:
$\left.\begin{array}{l|llll} & 7 & 3 & 2 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 \\ \hline 2 & 0 & 0 & 1 & 0 \\ \hline 3 & 0 & 1 & 0 & 0 \\ \hline 4 & 0 & 1 & 0 & 1 \\ \hline & & 0 & 1 & 1\end{array}\right]$
(0011)
(1011)

B4A9 = 1101010111001010
Alt.: = " " 1011 "


| 1.33 (a) | In 2's complement | In 1's complement |
| :---: | :---: | :---: |
|  | $(-10)+(-11)$ | $(-10)+(-11)$ |
|  | 110110 | 110101 |
|  | 110101 | 110100 |
|  | (1)101011 (-21) | (1)101001 |
|  |  | $\longrightarrow 1$ |
|  |  | 101010 (-21) |

1.32 (b) $183.81_{10}$

| $16 \lcm{183}$ |  | 0.81 |
| ---: | :--- | ---: |
| $16 \lcm{11}$ | r7 | $\frac{16}{0}$ |
|  | r11 | $(12) .96$ |
|  |  | $\frac{16}{(15) .36}$ |

$$
\begin{aligned}
& \therefore 183.81_{10}=\mathrm{B}_{7} . \mathrm{CF}_{16} \\
& =\frac{1000010}{\mathrm{~B}} \frac{0110111}{7} \underline{0101110} \frac{1000011}{\mathrm{C}} \frac{1000110}{\mathrm{~F}}
\end{aligned}
$$

1.33 (b) In 2's complement In 1's complement
$(-10)+(-6)$ 110110
$(-10)+(-6)$ 110101
$\underline{111010}$ 111001
(1)110000 (-16) $\xrightarrow{(1) 101110}$ 101111 (-16)
1.33 (e) In 2's complement In 1's complement

| $(-11)+(-4)$ |  | $(-11)+(-4)$ |
| :---: | :---: | :---: |
| 110101 |  | 110100 |
| 111100 |  | 111011 |
| (1)110001 | (-15) | (1)101111 |
|  |  | $\longrightarrow 1$ |
|  |  | 110000 |

1.33(d) In 2's complement In 1's complement
$11+9$
001011001011
$\underline{001001} \underline{001001}$
010100 (20) 010100 (20)
1.34 (a) 01001-11010

In 2's complement 01001

In 1's complement 01001
$+\underline{00110}$
$+\underline{00101}$

1.34 (c) In 2's complement | 10110 |
| ---: |
| $+\underline{10011}$ |
| (1) 01001 |
| overflow |

In 1's complement
10110
$+\underline{10010}$
(1)01000

01001
overflow

1.34 (e)

In 2's complement

| 11100 |
| ---: |
| $+\quad 01011$ |

$+\underline{01011}$
(1)00111

In 1's complement
11100
$+\underline{01010}$


Unit 1 Solutions

| 1.35 (a) |  | In 2's complement | In 1's complement | 1.35 (b) |  | In 2's complement | In 1's complement ${ }_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 11010 | 11010 |  |  | 01011 |  |
|  |  | + $\underline{01100}$ | + $\underline{01011}$ |  |  | + $\underline{01000}$ | + $\underline{00111}$ |
|  |  | (1)00110 | (1)00101 |  |  | 10011 | 10010 |
|  |  |  | $\longrightarrow 1$ |  |  |  |  |
|  |  |  | 00110 |  |  |  |  |
| 1.35 (c) |  | In 2's complement | In 1's complement | 1.35 (d) |  | In 2's complement | In 1's complement |
|  |  | 10001 | 10001 |  |  | 10101 | 10101 |
|  |  | + 10110 | + 10101 |  |  | + $\underline{00110}$ | + $\underline{00101}$ |
|  |  | (1)00111 | (1)00110 |  |  | 11011 | 11010 |
|  |  | overflow | $\xrightarrow{\longrightarrow 0111}$ |  |  |  |  |
|  |  |  | 00111 <br> overflow |  |  |  |  |
| 1.36 | (a) | add | subt | 1.37 | (a) |  | complement |
|  |  | 101010 | 101010 |  |  | i) 00000000 (0) | 11111111 (-0) |
|  |  | + 011101 - | $\underline{011101}$ |  |  | ii) 11111110 (-1) | 00000001 (1) |
|  |  | (1)000111 | 001101 |  |  | iii) 00110011 (51) | 11001100 (-51) |
|  |  | $\longrightarrow 1$ | overflow |  |  | iv) 10000000 (-127) | 01111111 (127) |
|  |  | 001000 |  |  |  |  |  |
|  |  |  |  |  | (b) |  |  |
|  | (b) | add | subt |  |  | i) 00000000 (0) | 00000000 (0) |
|  |  | 101010 | 101010 |  |  | ii) $11111110(-2)$ | 00000010 (2) |
|  |  | + $\underline{011101}$ - | $\underline{011101}$ |  |  | iii) 00110011 (51) | 11001101 (-51) |
|  |  | (1)000111 | 001101 |  |  | iv) 10000000 (-128) | 10000000 (-128) |
|  |  |  | overflow |  |  |  |  |

## Unit 2 Problem Solutions

See FLD p. 693 for solution.
2.2 (a) In both cases, if $\mathrm{X}=0$, the transmission is 0 , and if $\mathrm{X}=1$, the transmission is 1 .

2.3 Answer is in FLD p. 693
2.4 (a) $\quad F=[(A \cdot 1)+(A \cdot 1)]+E+B C D=A+E+B C D$
2.5 (a) $\quad(A+B)(C+B)\left(D^{\prime}+B\right)\left(A C D^{\prime}+E\right)$
$=(A C+B)\left(D^{\prime}+B\right)\left(A C D^{\prime}+E\right)$ By Th. 8 D
$=\left(A C D^{\prime}+B\right)\left(A C D^{\prime}+E\right)$ By Th. 8 D
$=A C D^{\prime}+B E$ By Th. 8 D
2.6 (a) $A B+C^{\prime} D^{\prime}=\left(A B+C^{\prime}\right)\left(A B+D^{\prime}\right)$
$=\left(A+C^{\prime}\right)\left(B+C^{\prime}\right)\left(A+D^{\prime}\right)\left(B+D^{\prime}\right)$
2.6 (c) $A^{\prime} B C+E F+D E F^{\prime}=A^{\prime} B C+E\left(F+D F^{\prime}\right)$
$=A^{\prime} B C+E(F+D)=\left(A^{\prime} B C+E\right)\left(A^{\prime} B C+F+D\right)$
$=\left(A^{\prime}+E\right)(B+E)(C+E)\left(A^{\prime}+F+D\right)$

$$
(B+F+D)(C+F+D)
$$

2.6 (e) $A C D^{\prime}+C^{\prime} D^{\prime}+A^{\prime} C=D^{\prime}\left(A C+C^{\prime}\right)+A^{\prime} C$

$$
=D^{\prime}\left(A+C^{\prime}\right)+A^{\prime} C \text { By Th. 11D }
$$

$$
=\left(D^{\prime}+A^{\prime} C\right)\left(A+C^{\prime}+A^{\prime} C\right)
$$

$$
=\left(D^{\prime}+A^{\prime}\right)\left(D^{\prime}+C\right)\left(A+C^{\prime}+A^{\prime}\right) \text { By Th. 11D }
$$

$$
=\left(A^{\prime}+D^{\prime}\right)\left(C+D^{\prime}\right)
$$

2.7 (a) $\quad(\underline{A+B+C}+D)(\underline{A+B+C}+E)(\underline{A+B+C}+F)$ $=\underline{A+B+C}+D E F$

Apply second distributive law (Th. 8D) twice

2.8 (a) $\quad\left[(A B)^{\prime}+C^{\prime} D\right]^{\prime}=A B\left(C^{\prime} D\right)^{\prime}=A B\left(C+D^{\prime}\right)$

$$
=A B C+A B D^{\prime}
$$

2.8 (c) $\quad\left(\left(A+B^{\prime}\right) C\right)^{\prime}(A+B)(C+A)^{\prime}$

$$
\begin{aligned}
& =\left(A^{\prime} B+C^{\prime}\right)(A+B) C^{\prime} A^{\prime}=\left(A^{\prime} B+C^{\prime}\right) A^{\prime} B C^{\prime} \\
& =A^{\prime} B C^{\prime}
\end{aligned}
$$

2.9 (a)

$$
\begin{aligned}
F & =\left[(A+B)^{\prime}+\left(A+(A+B)^{\prime}\right)^{\prime}\right]\left(A+(A+B)^{\prime}\right)^{\prime} \\
& =\left(A+(A+B)^{\prime}\right)^{\prime}
\end{aligned}
$$

By Th. $10 D$ with $X=\left(A+(A+B)^{\prime}\right)^{\prime}=A^{\prime}(A+B)=A^{\prime} B$
2.2 (b) In both cases, if $\mathrm{X}=0$, the transmission is YZ , and if $\mathrm{X}=1$, the transmission is 1 .

2.4 (b) $Y=\left(A B^{\prime}+(A B+B)\right) B+A=\left(A B^{\prime}+B\right) B+A$

$$
=(A+B) B+A=A B+B+A=A+B
$$

2.5 (b) $\quad\left(A^{\prime}+B+C^{\prime}\right)\left(A^{\prime}+C^{\prime}+D\right)\left(B^{\prime}+D^{\prime}\right)$ $=\left(A^{\prime}+C^{\prime}+B D\right)\left(B^{\prime}+D^{\prime}\right)$
$\left\{\right.$ By Th. 8 D with $\left.X=A^{\prime}+C^{\prime}\right\}$
$=A^{\prime} B^{\prime}+B^{\prime} C^{\prime}+B^{\prime} B D+A^{\prime} D^{\prime}+C^{\prime} D^{\prime}+B D D^{\prime}$ $=A^{\prime} B^{\prime}+A^{\prime} D^{\prime}+C^{\prime} B^{\prime}+C^{\prime} D^{\prime}$
2.6 (b) $\quad W X+W Y^{\prime} X+Z Y X=X\left(W+W Y^{\prime}+Z Y\right)$
$=X(W+Z Y)$
\{By Th. 10\}
$=X(W+Z)(W+Y)$
2.6 (d) $X Y Z+W^{\prime} Z+X Q^{\prime} Z=Z\left(X Y+W^{\prime}+X Q^{\prime}\right)$
$=Z\left[W^{\prime}+X\left(Y+Q^{\prime}\right)\right]$
$=Z\left(W^{\prime}+X\right)\left(W^{\prime}+Y+Q^{\prime}\right)$ By Th. 8 D
2.6 (f) $\quad A+B C+D E$

$$
\begin{aligned}
& =(A+B C+D)(A+B C+E) \\
& =(A+B+D)(A+C+D)(A+B+E)(A+C+E)
\end{aligned}
$$

2.7 (b) $\quad W \underline{X Y Z}+V \underline{X Y Z}+U \underline{X Y Z}=\underline{X Y Z}(W+V+U)$

By first distributive law (Th. 8)

2.8 (b) $\quad\left[A+B\left(C^{\prime}+D\right)\right]^{\prime}=A^{\prime}\left(B\left(C^{\prime}+D\right)\right)^{\prime}$

$$
=A^{\prime}\left(B^{\prime}+\left(C^{\prime}+D\right)^{\prime}\right)=A^{\prime}\left(B^{\prime}+C D^{\prime}\right)
$$

$$
=A^{\prime} B^{\prime}+A^{\prime} C D^{\prime}
$$

2.9 (b) $\mathrm{G}=\left\{\left[(\mathrm{R}+\mathrm{S}+\mathrm{T})^{\prime} \mathrm{PT}(\mathrm{R}+\mathrm{S})\right]^{\prime} \mathrm{T}\right\}^{\prime}$

$$
=(R+S+T)^{\prime} P T(R+S)^{\prime}+T^{\prime}
$$

$$
=T^{\prime}+\left(R^{\prime} S^{\prime} T^{\prime}\right) P\left(R^{\prime} S^{\prime}\right) T=T^{\prime}+P R^{\prime} S^{\prime} T^{\prime} T=T^{\prime}
$$


2.10 (e)

2.11 (a) $\left(A^{\prime}+B^{\prime}+C\right)\left(A^{\prime}+B^{\prime}+C\right)^{\prime}=0 \quad$ By Th. 5D
2.11 (c) $A B+\left(C^{\prime}+D\right)(A B)^{\prime}=A B+C^{\prime}+D$

By Th. 11D
2.11 (e) $\quad\left[A B^{\prime}+(C+D)^{\prime}+E^{\prime} F\right](C+D)$

$$
=A B^{\prime}(C+D)+E^{\prime} F(C+D) \quad \text { By Th. } 8
$$

2.12 (a) $\left(X+Y^{\prime} Z\right)+\left(X+Y^{\prime} Z\right)^{\prime}=1 \quad$ By Th. 5
2.12 (c) $\left(V^{\prime} W+U X\right)^{\prime}\left(U X+Y+Z+V^{\prime} W\right)=\left(V^{\prime} W+U X\right)^{\prime}$ $(Y+Z)$ By Th. 11
2.12 (e) $\left(W^{\prime}+X\right)\left(Y+Z^{\prime}\right)+\left(W^{\prime}+X\right)^{\prime}\left(Y+Z^{\prime}\right)$

$$
=(Y+Z) \text { By Th. } 9
$$

2.13 (a) $F_{1}=A^{\prime} A+B+(B+B)=0+B+B=B$
2.13 (c) $F_{3}=\left[(A B+C)^{\prime} D\right][(A B+C)+D]$

$$
=(A B+C)^{\prime} D(A B+C)+(A B+C)^{\prime} D
$$

$$
=(A B+C)^{\prime} D \text { By Th. } 5 \mathrm{D} \& \mathrm{Th} .2 \mathrm{D}
$$

2.14 (a) $\operatorname{ACF}(B+E+D)$
2.15 (a) $f^{\prime}=\left\{\left[A+(B C D)^{\prime}\right]\left[(A D)^{\prime}+B\left(C^{\prime}+A\right)\right]\right\}^{\prime}$
$=\left[A+(B C D)^{\prime}\right]^{\prime}+\left[(A D)^{\prime}+B\left(C^{\prime}+A\right)\right]^{\prime}$
$=A^{\prime}(B C D)^{\prime \prime}+(A D)^{\prime}\left[B\left(C^{\prime}+A\right)\right]^{\prime}$
$=A^{\prime} B C D+A D\left[B^{\prime}+\left(C^{\prime}+A\right)^{\prime}\right]$
$=A^{\prime} B C D+A D\left[B^{\prime}+C^{\prime \prime} A^{\prime}\right]$
$=A^{\prime} B C D+A D\left[B^{\prime}+C A^{\prime}\right]$
2.16 (a) $f^{\mathrm{D}}=\left[A+(B C D)^{\prime}\right]\left[(A D)^{\prime}+B\left(C^{\prime}+A\right)\right]^{D}$

$$
=\left[A(B+C+D)^{\prime}\right]+\left[(A+D)^{\prime}\left(B+C^{\prime} A\right)\right]
$$

2.17 (a) $f=\left[\left(A^{\prime}+B\right) C\right]+\left[A\left(B+C^{\prime}\right)\right]$
$=A^{\prime} C+B^{\prime} C+A B+A C^{\prime}$
$=A^{\prime} C+B^{\prime} C+A B+A C^{\prime}+B C$

$$
=A^{\prime} C+C+A B+A C^{\prime}=C+A B+A=C+A
$$

2.17 (c) $f=\left(A^{\prime}+B^{\prime}+A\right)(A+C)\left(A^{\prime}+B^{\prime}+C^{\prime}+B\right)$

$$
\left(B+C+C^{\prime}\right)=(A+C)
$$

2.10 (f)

2.11 (b) $A B\left(C^{\prime}+D\right)+B\left(C^{\prime}+D\right)=B\left(C^{\prime}+D\right)$ By Th. 10
2.11 (d) $\left(A^{\prime} B F+C D^{\prime}\right)\left(A^{\prime} B F+C E G\right)=A^{\prime} B F+C D^{\prime} E G$ By Th. 8D

$$
2.11 \text { (f) } \quad A^{\prime}(B+C)\left(D^{\prime} E+F\right)^{\prime}+\left(D^{\prime} E+F\right) \text {. } \begin{aligned}
& =A^{\prime}(B+C)+D^{\prime} E+F \quad \text { By Th. 11D }
\end{aligned}
$$

2.12 (b) $\left[W+X^{\prime}(Y+Z)\right]\left[W^{\prime}+X^{\prime}(Y+Z)\right]=X^{\prime}(Y+Z)$ By Th. 9D
2.12 (d) $\left(U V^{\prime}+W^{\prime} X\right)\left(U V^{\prime}+W^{\prime} X+Y^{\prime} Z\right)=U V^{\prime}+W^{\prime} X$ By Th. 10D
2.12 (f) $\begin{gathered}\left(V^{\prime}+U+W\right)\left[(W+X)+Y+U Z^{\prime}\right]+[(W+X)+ \\ \left.U Z^{\prime}+Y\right]=(W+X)+U Z^{\prime}+Y \text { By Th. } 10\end{gathered}$
2.13 (b) $F_{2}=A^{\prime} A^{\prime}+A B^{\prime}=A^{\prime}+A B^{\prime}=A^{\prime}+B^{\prime}$
2.13 (d) $Z=[(A+B) C]^{\prime}+(A+B) C D=[(A+B) C]^{\prime}+D$

By Th. 11D with $Y=[(A+B) C]^{\prime}$ $=A^{\prime} B^{\prime}+C^{\prime}+D^{\prime}$
2.14 (b) $W+Y+Z+V U X$
2.15(b) $f^{\prime}=\left[A B^{\prime} C+\left(A^{\prime}+B+D\right)\left(A B D^{\prime}+B^{\prime}\right)\right]^{\prime}$
$=\left(A B^{\prime} C\right)^{\prime}\left[\left(A^{\prime}+B+D\right)\left(A B D^{\prime}+B^{\prime}\right]^{\prime}\right.$
$=\left(A^{\prime}+B^{\prime \prime}+C^{\prime}\right)\left[\left(A^{\prime}+B+D\right)^{\prime}+\left(A B D^{\prime}\right)^{\prime} B^{\prime \prime}\right]$
$=\left(A^{\prime}+B+C^{\prime}\right)\left[A^{\prime \prime} B^{\prime} D^{\prime}+\left(A^{\prime}+B^{\prime}+D^{\prime}\right) B\right]$
$=\left(A^{\prime}+B+C^{\prime}\right)\left[A B^{\prime} D^{\prime}+\left(A^{\prime}+B^{\prime}+D\right) B\right]$
2.16 (b) $f^{\mathrm{D}}=\left[A B^{\prime} C+\left(A^{\prime}+B+D\right)\left(A B D^{\prime}+B^{\prime}\right)\right]^{\mathrm{D}}$
$=\left(A+B^{\prime}+C\right)\left[A^{\prime} B D+\left(A+B+D^{\prime}\right) B^{\prime}\right)$
2.17 (b) $f=A^{\prime} C+B^{\prime} C+A B+A C^{\prime}=A+C$
2.18 (a) product term, sum-of-products, product-of-sums)
2.18 (b) sum-of-products
2.18 (d) sum term, sum-of-products, product-of-sums
2.19

2.20 (c) $F=D\left[\left(A^{\prime}+B^{\prime}\right) C+A C^{\prime}\right]$

$$
=D\left(A^{\prime}+B^{\prime}+A C^{\prime}\right)\left(C+A C^{\prime}\right)
$$

$$
=D\left(A^{\prime}+B^{\prime}+C^{\prime}\right)(C+A)
$$



$$
2.22 \text { (a) } \begin{aligned}
& A^{\prime} B^{\prime}+A^{\prime} C D+A^{\prime} D E^{\prime} \\
& =A^{\prime}\left(B^{\prime}+C D+D E^{\prime}\right) \\
& =A^{\prime}\left[B^{\prime}+D\left(C+E^{\prime}\right)\right] \\
& =A^{\prime}\left(B^{\prime}+D\right)\left(B^{\prime}+C+E^{\prime}\right)
\end{aligned}
$$

2.22 (b) $H^{\prime} I^{\prime}+J K$
$=\left(H^{\prime} I^{\prime}+J\right)\left(H^{\prime} I^{\prime}+K\right)$

$$
=\left(H^{\prime}+J\right)\left(I^{\prime}+J\right)\left(H^{\prime}+K\right)\left(I^{\prime}+K\right)
$$

2.22 (c) $A^{\prime} B C+A B^{\prime} C+C D^{\prime}$
$=C\left(A^{\prime} B+A B^{\prime}+D^{\prime}\right)$
$=C\left[(A+B)\left(A^{\prime}+B^{\prime}\right)+D^{\prime}\right]$
$=C\left(A+B+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+D^{\prime}\right)$
2.23 (a) $W+U^{\prime} Y V=\left(W+U^{\prime}\right)(W+Y)(W+V)$
2.23 (c) $A^{\prime} B^{\prime} C+B^{\prime} C D^{\prime}+B^{\prime} E^{\prime}=B^{\prime}\left(A^{\prime} C+C D^{\prime}+E^{\prime}\right)$

$$
\begin{aligned}
& =B^{\prime}\left[E^{\prime}+C\left(A^{\prime}+D^{\prime}\right)\right] \\
& =B^{\prime}\left(E^{\prime}+C\right)\left(E^{\prime}+A^{\prime}+D^{\prime}\right)
\end{aligned}
$$

2.18 (c) none apply
2.18 (e) product-of-sums
2.20 (a) $F=D\left[\left(A^{\prime}+B^{\prime}\right) C+A C^{\prime}\right]$
2.20 (b) $F=D\left[\left(A^{\prime}+B^{\prime}\right) C+A C^{\prime}\right]$ $=A^{\prime} C D+B^{\prime} C D+A C^{\prime} D$

2.21

| A | B | C | H | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | x |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | x |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | x |

2.22 (d) $A^{\prime} B^{\prime}+\left(C D^{\prime}+E\right)=A^{\prime} B^{\prime}+(C+E)\left(D^{\prime}+E\right)$ $=\left(A^{\prime} B^{\prime}+C+E\right)\left(A^{\prime} B^{\prime}+D^{\prime}+E\right)$ $=\left(A^{\prime}+C+E\right)\left(B^{\prime}+C+E\right)$ $\left(A^{\prime}+D^{\prime}+E\right)\left(B^{\prime}+D^{\prime}+E\right)$
2.22 (e) $A^{\prime} B^{\prime} C+B^{\prime} C D^{\prime}+E F^{\prime}=A^{\prime} B^{\prime} C+B^{\prime} C D^{\prime}+E F^{\prime}$
$=B^{\prime} C\left(A^{\prime}+D^{\prime}\right)+E F^{\prime}$
$=\left(B^{\prime} C+E F^{\prime}\right)\left(A^{\prime}+D^{\prime}+E F^{\prime}\right)$
$=\left(B^{\prime}+E\right)\left(B^{\prime}+F^{\prime}\right)(C+E)\left(C+F^{\prime}\right)$

$$
\left(A^{\prime}+D^{\prime}+E\right)\left(A^{\prime}+D^{\prime}+F^{\prime}\right)
$$

2.22 (f) $W X^{\prime} Y+W^{\prime} X^{\prime}+W^{\prime} Y^{\prime}=X^{\prime}\left(W Y+W^{\prime}\right)+W^{\prime} Y^{\prime}$

$$
=X^{\prime}\left(W^{\prime}+Y\right)+W^{\prime} Y^{\prime}
$$

$$
=\left(X^{\prime}+W^{\prime}\right)\left(X^{\prime}+Y^{\prime}\right)\left(W^{\prime}+Y+W^{\prime}\right)\left(W^{\prime}+Y+Y^{\prime}\right)
$$

$$
=\left(X^{\prime}+W^{\prime}\right)\left(X^{\prime}+Y^{\prime}\right)\left(W^{\prime}+Y\right)
$$

2.23 (b) $T W+U Y^{\prime}+V$

$$
=(T+U+Z)\left(T+Y^{\prime}+V\right)(W+U+V)\left(W+Y^{\prime}+V\right)
$$

2.23 (d) $A B C+A D E^{\prime}+A B F^{\prime}=A\left(B C+D E^{\prime}+B F^{\prime}\right)$

$$
\begin{aligned}
& =A\left[D E^{\prime}+B\left(C+F^{\prime}\right)\right] \\
& =A\left(D E^{\prime}+B\right)\left(D E^{\prime}+C+F^{\prime}\right) \\
& =A(B+D)\left(B+E^{\prime}\right)\left(C+F^{\prime}+D\right)\left(C+F^{\prime}+E^{\prime}\right)
\end{aligned}
$$

Unit 2 Solutions
2.24 (a) $\left[\left(X Y^{\prime}\right)^{\prime}+\left(X^{\prime}+Y\right)^{\prime} Z\right]=X^{\prime}+Y+\left(X^{\prime}+Y\right)^{\prime} Z$ $=X^{\prime}+Y^{\prime}+Z$ By Th. 11D with $Y=\left(X^{\prime}+Y\right)$
2.24 (c) $\begin{array}{r}\left.\left[\left(A^{\prime}+B^{\prime}\right)\right)^{\prime}+\left(A^{\prime} B^{\prime} C\right)^{\prime}+C^{\prime} D\right]^{\prime} \\ \\ =\left(A^{\prime}+B^{\prime}\right) A^{\prime} B^{\prime} C\left(C+D^{\prime}\right)=A^{\prime} B^{\prime} C\end{array}$
2.25 (a) $F(P, Q, R, S)^{\prime}=\left[\left(R^{\prime}+P Q\right) S\right]^{\prime}=R\left(P^{\prime}+Q^{\prime}\right)+S^{\prime}$ $=R P^{\prime}+R Q^{\prime}+S^{\prime}$
2.25 (c) $\quad F(A, B, C, D)^{\prime}=\left[A^{\prime}+B^{\prime}+A C D\right]^{\prime}$

$$
=\left[A^{\prime}+B^{\prime}+C D\right]^{\prime}=A B\left(C^{\prime}+D^{\prime}\right)
$$

2.26 (a) $F=\left[\left(A^{\prime}+B\right)^{\prime} B\right]^{\prime} C+B=\left[A^{\prime}+B+B^{\prime}\right] C+B$

$$
=C+B
$$

2.26 (c) $H=\left[W^{\prime} X^{\prime}\left(Y^{\prime}+Z^{\prime}\right)\right]^{\prime}=W+X+Y Z$
2.28 (a) $F=A B C+A^{\prime} B C+A B^{\prime} C+A B C^{\prime}$
$=B C+A B^{\prime} C+A B C^{\prime}($ By Th. 9)
$=C\left(B+A B^{\prime}\right)+A B C^{\prime}=C(A+B)+A B C^{\prime}$
(By Th. 11D)
$=A C+B C+A B C^{\prime}=A C+B\left(C+A C^{\prime}\right)$
$=A C+B(A+C)=A C+A B+B C$

2.24 (b) $\left(X+\left(Y^{\prime}(Z+W)^{\prime}\right)^{\prime}\right)^{\prime}=X^{\prime} Y^{\prime}(Z+W)^{\prime}=X^{\prime} Y^{\prime} Z^{\prime} W^{\prime}$
2.24 (d) $\quad(A+B) C D+(A+B)^{\prime}=C D+(A+B)^{\prime}$
$\left\{\right.$ By Th. 11D with $\left.Y=(A+B)^{\prime}\right\}$
$=C D+A^{\prime} B^{\prime}$
2.25 (b) $\quad F(W, X, Y, Z)^{\prime}=\left[X+Y Z\left(W+X^{\prime}\right)\right]^{\prime}$
$=\left[X+X^{\prime} Y Z+W Y Z\right]^{\prime}$
$=[X+Y Z+W Y Z]^{\prime}=[X+Y Z]^{\prime}$
$=X^{\prime} Y^{\prime}+X^{\prime} Z^{\prime}$
2.26 (b) $G=\left[(A B)^{\prime}(B+C)\right]^{\prime} C=\left(A B+B^{\prime} C^{\prime}\right) C=A B C$
2.27

$$
\begin{aligned}
F & =(\underline{V+X}+W)(\underline{V+X}+Y)(V+Z) \\
& =(V+X+W Y)(V+Z)=V+Z(X+W Y)
\end{aligned}
$$

$$
\text { By Th. 8D with } X=V
$$


2.28 (b) Beginning with the answer to (a):
$F=A(B+C)+B C$


Alternate solutions:
$F=A B+C(A+B)$
$F=A C+B(A+C)$

2.29 (e)

| $W X Y Z$ | $W^{\prime} X Y$ | $W Z$ | $W^{\prime} X Y+W Z$ | $W^{\prime}+Z$ | $W+X Y$ | $\left(W^{\prime}+Z\right)(W+X Y)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0000 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0001 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0010 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0011 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0100 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0101 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0110 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0111 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1000 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1001 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1010 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1011 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1100 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1101 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1110 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1111 | 0 | 1 | 1 | 1 | 1 | 1 |

$$
\begin{aligned}
F & =\left(X+Y^{\prime}\right) Z+X^{\prime} Y Z^{\prime} \\
& =\left(X+Y^{\prime}+X^{\prime} Y Z^{\prime}\right)\left(Z+X^{\prime} Y Z^{\prime}\right) \\
& =\left(X+Y^{\prime}+X^{\prime}\right)\left(X+Y^{\prime}+Y\right)\left(X+Y^{\prime}+Z^{\prime}\right)\left(Z+X^{\prime}\right)(Z+Y)\left(Z+Z^{\prime}\right) \\
& =\left(1+Y^{\prime}\right)(X+1)\left(X+Y^{\prime}+Z^{\prime}\right)\left(Z+X^{\prime}\right)(Z+Y)(1) \\
& =(1)(1)\left(X+Y^{\prime}+Z^{\prime}\right)\left(Z+X^{\prime}\right)(Z+Y)(1) \\
& =\left(X+Y^{\prime}+Z^{\prime}\right)\left(Z+X^{\prime}\right)(Z+Y)
\end{aligned}
$$

$$
G=\left(X+Y^{\prime}+Z^{\prime}\right)\left(X^{\prime}+Z\right)(Y+Z)
$$

(from the circuit) (distributive law) (distributive law) (complementation laws)
(0 and 1 operations)
(0 and 1 operations)
(from the circuit)

Unit 2 Solutions

## Unit 3 Problem Solutions

3.6 (a) $\begin{aligned} & \left(W+X^{\prime}+Z^{\prime}\right)\left(W^{\prime}+Y^{\prime}\right)\left(W^{\prime}+X+Z^{\prime}\right)\left(W+X^{\prime}\right)(W+Y+Z) \\ = & \left(W+X^{\prime}\right)\left(W^{\prime}+Y^{\prime}\right)\left(W^{\prime}+X+Z^{\prime}\right)(W+Y+Z)\end{aligned}$
$=\left(W+X^{\prime}\right)(\underbrace{W^{\prime}+Y^{\prime}})\left(\underline{W}^{\prime}+X+Z^{\prime}\right)(W+Y+Z)$
$=\left(W+X^{\prime}\right)\left[W^{\prime}+Y^{\prime}\left(X+Z^{\prime}\right)\right](W+Y+Z)$
$=\left(\underline{W+X} X^{\prime}\right)\left[W^{\prime}+Y^{\prime}\left(X+Z^{\prime}\right)\right](\underline{W+Y+Z})$
$=\left[W+X^{\prime}(Y+Z)\right]\left[W^{\prime}+Y^{\prime}\left(X+Z^{\prime}\right)\right]=W Y^{\prime}\left(X+Z^{\prime}\right)+W^{\prime} X^{\prime}(Y+Z)\left\{U \operatorname{sing}(X+Y)\left(X^{\prime}+Z\right)=X^{\prime} Y+X Z\right.$ with $\left.X=W\right\}$
$=W Y^{\prime} X+W Y^{\prime} Z^{\prime}+W^{\prime} X^{\prime} Y+W^{\prime} X^{\prime} Z$
3.6 (b)

3.7 (a)

$=\left(C^{\prime}+D\right)\left[C+\left(D^{\prime}+B^{\prime}\right)\left(D^{\prime}+D\right)\right]=\left(C^{\prime}+D\right)\left(C+D^{\prime}+B^{\prime}\right)$
3.7 (b)

$=D^{\prime}\left[\left(A^{\prime}+B\right)\left(A+C^{\prime}\right)\right]+D\left[\left(B^{\prime}+A^{\prime}\right)\left(B^{\prime}+C\right)\right] \quad\left\{U s i n g X Y+X^{\prime} Z=\left(X^{\prime}+Y\right)(X+Z)\right.$ twice inside the brackets $\}$
$=\left[D+\left(A^{\prime}+B\right)\left(A+C^{\prime}\right)\right]\left[D^{\prime}+\left(B^{\prime}+A^{\prime}\right)\left(B^{\prime}+C\right)\right]\left\{U \operatorname{sing} X Y+X^{\prime} Z=\left(X^{\prime}+Y\right)(X+Z)\right.$ with $\left.X=D\right\}$
$=\left(D+A^{\prime}+B\right)\left(D+A+C^{\prime}\right)\left(D^{\prime}+B^{\prime}+A^{\prime}\right)\left(D^{\prime}+B^{\prime}+C\right) \quad\{$ Using the Distributive Law $\}$
3.8
$F=A B \oplus[(A \equiv D)+D]=A B \oplus\left(A A^{\prime}+D^{\prime}+D\right)=A B \oplus\left(A^{\prime} D^{\prime}+D\right)=A B \oplus\left(A^{\prime}+D\right)$
$=(A B)^{\prime}\left(A^{\prime}+D\right)+A B\left(A^{\prime}+D\right)^{\prime}=\left(A^{\prime}+B^{\prime}\right)\left(A^{\prime}+D\right)+A B\left(A D^{\prime}\right)$
$=A^{\prime}+B^{\prime} D+A B D^{\prime}\{U \operatorname{sing}(X+Y)(X+Z)=X+Y Z\}=A^{\prime}+B D^{\prime}+B^{\prime} D \quad\left\{U \operatorname{sing} X+X^{\prime} Y=X+Y\right\}$
$3.9 \quad A \oplus B C=(A \oplus B)(A \oplus C)$ is not a valid distributive law. PROOF: Let $A=1, B=1, C=0$.
LHS: $A \oplus B C=1 \oplus 1 \cdot 0=1 \oplus 0=1$. RHS: $(A \oplus B)(A \oplus C)=(1 \oplus 1)(1 \oplus 0)=0 \cdot 1=0$.
3.10 (a) $(X+W)(Y \oplus Z)+X W^{\prime}$
$=(X+W)\left(Y Z^{\prime}+Y^{\prime} Z\right)+X W^{\prime}$
$=X X Z^{\prime}+X Y^{\prime} Z+\frac{W Y Z^{\prime}}{1}+\frac{W Y^{\prime} Z}{?}+X W^{\prime}$
Using Consensus Theorem
$W Y Z '+W Y^{\prime} Z+X W^{\prime}$
3.10 (b) $(A \oplus B C)+B D+A C D=A^{\prime} B C+A(B C)^{\prime}+B D+$

ACD
$=A^{\prime} B C+A\left(B^{\prime}+C^{\prime}\right)+B D+A C D$
$=A^{\prime} B C+\underline{A B^{\prime}+A C^{\prime}+\underline{B D}+A C D}$
$=A^{\prime} B C+A B^{\prime}+A C^{\prime}+A D+B D+A C D$
(Add consensus term AD, eliminate ACD)
$=A^{\prime} B C+A B^{\prime}+A C^{\prime}+B D$
(Remove consensus term AD)
3.10 (c)


## Unit 3 Solutions

3.11

$$
\begin{aligned}
& \left(\underline{A+B^{\prime}}+C+E^{\prime}\right)\left(\underline{A+B^{\prime}}+D^{\prime}+E\right)\left(B^{\prime}+C^{\prime}+D^{\prime}+E^{\prime}\right)=\left[A+B^{\prime}+\left(C+\underline{E^{\prime}}\right)\left(D^{\prime}+\underline{E}\right)\right]\left(B^{\prime}+C^{\prime}+D^{\prime}+E^{\prime}\right) \\
& =\left(A+\underline{B^{\prime}}+D^{\prime} E^{\prime}+C E\right)\left(\underline{B^{\prime}}+C^{\prime}+D^{\prime}+E^{\prime}\right)=B^{\prime}+\left(A+D^{\prime} E^{\prime}+C E\right)\left(C^{\prime}+D^{\prime}+E^{\prime}\right)
\end{aligned}
$$



$$
=B^{\prime}+A C^{\prime}+A E^{\prime}+C D^{\prime}+D^{\prime} E^{\prime}
$$

$A^{\prime} C D^{\prime} E+A^{\prime} B^{\prime} D^{\prime}+\triangle A B C E+A B D=A^{\prime} B^{\prime} D^{\prime}+A B D+B C D^{\prime} E$
Proof: $L H S: A^{\prime} X^{\prime} E+B C D^{\prime} E+A^{\prime} B^{\prime} D^{\prime}+A B \not \mathscr{C}^{\prime} E+\underline{A B D} \begin{aligned} & \text { Add consensus term to left-hand side and use it to } \\ & \text { eliminate two consensus terms }\end{aligned}$ $=B C D^{\prime} E+A^{\prime} B^{\prime} D^{\prime}+A B D \quad$ This yields the right-hand side.
$\therefore$ LHS $=$ RHS
3.13 (a) $K L M V^{\prime}+K^{\prime} L^{\prime} M N+M N^{\prime}=K^{\prime} L^{\prime} M N+M N^{\prime}=M\left(K^{\prime} L^{\prime} N+N^{\prime}\right)=M\left(N^{\prime}+K^{\prime} L^{\prime}\right)\left\{\right.$ Th. 11C with $\left.Y=N^{\prime}\right\}=M N^{\prime}+K^{\prime} L^{\prime} M$
3.13 (b) $K L^{\prime} M^{\prime}+M \underline{N^{\prime}}+L M^{\prime} \underline{N^{\prime}}=K L^{\prime} M^{\prime}+N^{\prime}\left(\underline{M}+L \underline{M^{\prime}}\right)=K L^{\prime} M^{\prime}+N^{\prime}(M+L)=K L^{\prime} M^{\prime}+M N^{\prime}+L N^{\prime}$
3.13 (c) $\left(K+\underline{L^{\prime}}\right)\left(K^{\prime}+\underline{L^{\prime}}+N\right)\left(\underline{L^{\prime}}+M+N^{\prime}\right)=L^{\prime}+\underline{K}\left(\underline{K^{\prime}}+N\right)\left(M+N^{\prime}\right)=L^{\prime}+K \underline{N}\left(M+\underline{N^{\prime}}\right)=L^{\prime}+K M N$
3.13 (d) $\left(\underline{K^{\prime}}+L+\underline{M^{\prime}}+N\right)\left(\underline{K^{\prime}}+M^{\prime}+N+E\right)\left(\underline{K^{\prime}}+M^{\prime}+N+E^{\prime}\right) K M$
$=\left[K^{\prime}+M^{\prime}+(L+N)(\underline{N+R})\left(\underline{N+R^{\prime}}\right)\right] K M \quad\left\{T h .8 N\right.$ twice with $\left.X=K^{\prime}+M^{\prime}\right\}=\left[K^{\prime}+M^{\prime}+(\underline{N}) N\right] K M$ $=\left[K^{\prime}+\underline{M^{\prime}}+N\right] \underline{K M}=K M N$
3.14 (a) $K^{\prime} L^{\prime} \underline{M}+K \underline{M^{\prime}} N+K L \underline{M}+L \underline{M^{\prime}} N^{\prime}=M^{\prime}\left(K N+L N^{\prime}\right)+M\left(K^{\prime} L^{\prime}+K L\right)$
$=M^{\prime}\left[\left(K+N^{\prime}\right)(L+N)\right]+M\left[\left(K^{\prime}+L\right)\left(K+L^{\prime}\right)\right] \quad\{T h .14$ twice with $X=N$ and $X=L\}$
$=\left[M+\left(K+N^{\prime}\right)(L+N)\right]\left[M^{\prime}+\left(K^{\prime}+L\right)\left(K+L^{\prime}\right)\right] \quad\{T h .14$ with $X=M\}$
$=\left(M+K+N^{\prime}\right)(M+L+N)\left(M^{\prime}+K^{\prime}+L\right)\left(M^{\prime}+K+L^{\prime}\right) \quad\{$ Distributive Law $\}$
3.14 (b) $K \underline{L}+K^{\prime} \underline{L^{\prime}}+\underline{L^{\prime}} M^{\prime} N^{\prime}+\underline{L} M N^{\prime}=L^{\prime}\left(K^{\prime}+M^{\prime} N^{\prime}\right)+L\left(K+M N^{\prime}\right)$
$=\left(L+K^{\prime}+M^{\prime} N^{\prime}\right)\left(L^{\prime}+K+M N^{\prime}\right) \quad\{$ Th. 14 with $X=L\}$
$=\left(L+K^{\prime}+M^{\prime}\right)\left(L+K^{\prime}+N^{\prime}\right)\left(L^{\prime}+K+M\right)\left(L^{\prime}+K+N^{\prime}\right)$
3.14 (c) $K \underline{L}+K^{\prime} \underline{L^{\prime}} M+\underline{L^{\prime}} M^{\prime} N+\underline{L} M^{\prime} N^{\prime}=L^{\prime}\left[K^{\prime} \underline{M}+\underline{M^{\prime}} N\right]+L\left[K+M^{\prime} N^{\prime}\right]=\underline{L^{\prime}}\left[(M+N)\left(M^{\prime}+K^{\prime}\right)\right]+\underline{L}\left[\left(K+M^{\prime}\right)\left(K+N^{\prime}\right)\right]$
$=\left[L+(M+N)\left(M^{\prime}+K^{\prime}\right)\right]\left[L^{\prime}+\left(K+M^{\prime}\right)\left(K+N^{\prime}\right)\right]=(L+M+N)\left(L+M^{\prime}+K^{\prime}\right)\left(L^{\prime}+K+M^{\prime}\right)\left(L^{\prime}+K+N^{\prime}\right)$
3.14 (d) $K^{\prime} M^{\prime} \underline{N}+K L^{\prime} \underline{N^{\prime}}+K^{\prime} M \underline{N^{\prime}}+L \underline{N}=N\left(K^{\prime} M^{\prime}+L\right)+N^{\prime}\left(\underline{K} L^{\prime}+\underline{K^{\prime}} M\right)=\underline{N}\left(L+K^{\prime}\right)\left(L+M^{\prime}\right)+\underline{N^{\prime}}\left(L^{\prime}+K^{\prime}\right)(K+M)$ $=\left[N^{\prime}+\left(L+K^{\prime}\right)\left(L+M^{\prime}\right)\right]\left[N+\left(L^{\prime}+K^{\prime}\right)(K+M)\right]=\left(N^{\prime}+L+K^{\prime}\right)\left(N^{\prime}+L+M^{\prime}\right)\left(N+L^{\prime}+K^{\prime}\right)(N+K+M)$
3.14 (e) $\underline{W} X \underline{Y}+\underline{W} X^{\prime} \underline{Y}+\underline{W Y Z}+X Y Z^{\prime}=W Y\left(\underline{X}+\underline{X} \underline{X}^{\prime}+Z\right)+X Y Z^{\prime}=W \underline{Y}+X \underline{Y} Z^{\prime}=Y\left(W+X Z^{\prime}\right)=Y(W+X)\left(W+Z^{\prime}\right)$
3.15 (a) $\underbrace{\left(K^{\prime}+M^{\prime}+N\right.})\left(K^{\left(K^{\prime}+M\right)\left(L+M^{\prime}+N^{\prime}\right)\left(K^{\prime}+L+M\right)(M+N)}\right.$
$=\left(M^{\prime}+N L+K^{\prime} N^{\prime}\right)\left(M+K^{\prime} N\right)=M\left(L N+K^{\prime} N^{\prime}\right)+\left(M^{\prime} K^{\prime} N\right)\left\{U \operatorname{sing} X Y+X^{\prime} Z=(X+Z)\left(X^{\prime}+Y\right)\right.$ with $\left.X=M\right\}$ $=M L N+M K^{\prime} N^{\prime}+M^{\prime} K^{\prime} N$
3.15 (b) $\underbrace{\left(K^{\prime}+L^{\prime}+M^{\prime}\right.}) \xrightarrow[\left(K^{\left(K+M+N^{\prime}\right.}\right)(K+L)\left(K^{\prime}+N\right)\left(K^{\prime}-M+N\right)]{ }$

$$
=\left[K^{\prime}+N\left(L^{\prime}+M^{\prime}\right)\right]\left[K+L\left(M+N^{\prime}\right)\right]=K N\left(L^{\prime}+M^{\prime}\right)+K^{\prime} L\left(M+N^{\prime}\right)=K N L^{\prime}+K N M^{\prime}+K^{\prime} L M+K^{\prime} L N^{\prime}
$$

3.15 (c) $\left(\underline{K^{\prime}+L^{\prime}+M}\left(\underline{K+N^{\prime}}\right)\left(K^{\prime}+L+N^{\prime}\right)(K+L)\left(K+M+N^{\prime}\right)\right.$

$$
\begin{aligned}
& =\left[K^{\prime}+\left(L^{\prime}+M\right)\left(L+N^{\prime}\right)\right]\left(K+L N^{\prime}\right)=\left(K^{\prime}+L M+L^{\prime} N^{\prime}\right)\left(K+L N^{\prime}\right) \quad\{\text { Th. } 14 \text { with } X=L\} \\
& =K\left(L M+L^{\prime} N^{\prime}\right)+K^{\prime} L N^{\prime}\{\text { By Th. } 14 \text { with } X=K\} \\
& \quad=K L M+K L^{\prime} N^{\prime}+K^{\prime} L N^{\prime}
\end{aligned}
$$

3.15 (d) $(K+L+M)\left(K^{\prime}+L^{\prime}+N^{\prime}\right)\left(K^{\prime}+L^{\prime}+M^{\prime}\right)(K+L+N)=(K+L+M N)\left(K^{\prime}+L^{\prime}+M^{\prime} N^{\prime}\right)$ $=K\left(L^{\prime}+M^{\prime} N^{\prime}\right)+K^{\prime}(L+M N)\{T h .14$ with $X=K\}=K L^{\prime}+K M^{\prime} N^{\prime}+K^{\prime} L+K^{\prime} M N$
3.15 (e) $(K+L+M)(K+M+N)\left(K^{\prime}+L^{\prime}+M^{\prime}\right)\left(K^{\prime}+M^{\prime}+N^{\prime}\right)=(K+M+L N)\left(K^{\prime}+M^{\prime}+L^{\prime} N^{\prime}\right)$ $=K\left(M^{\prime}+L^{\prime} N^{\prime}\right)+K^{\prime}(M+L N)=K M^{\prime}+K L^{\prime} N^{\prime}+K^{\prime} M+K^{\prime} L N$ Alt. soln's: $K M^{\prime}+K^{\prime} M+L^{\prime} M N^{\prime}+L M^{\prime} N($ or $) K M^{\prime}+K^{\prime} M+K^{\prime} L N+L^{\prime} M N^{\prime}($ or $) K M^{\prime}+K^{\prime} M+K L N^{\prime}+L M^{\prime} N$
3.16 (a) $(\underline{K L \oplus M})+M^{\prime} N^{\prime}=(K L)^{\prime} M+K L M^{\prime}+M^{\prime} N^{\prime}=\left(K^{\prime}+L^{\prime}\right) M+K L \underline{M^{\prime}}+\underline{M^{\prime}} N^{\prime}=\underline{M}\left(K^{\prime}+L^{\prime}\right)+\underline{M^{\prime}}\left(K L+N^{\prime}\right)$ $=\left(M^{\prime}+K^{\prime}+L^{\prime}\right)\left(M+N^{\prime}+K L\right)=\left(M^{\prime}+K^{\prime}+L^{\prime}\right)\left(M+N^{\prime}+K\right)\left(M+N^{\prime}+L\right)$
3.16 (b) $\quad M^{\prime}\left(K \oplus N^{\prime}\right)+M N+K^{\prime} N=M^{\prime}\left[K^{\prime} N^{\prime}+K N\right]+M N+K^{\prime} N=K^{\prime} M^{\prime} N^{\prime}+K M^{\prime} \underline{N}+M \underline{N}+K^{\prime} \underline{N}$ $=K^{\prime} M^{\prime} N^{\prime}+N\left(M+\underline{K} M^{\prime}+\underline{K^{\prime}}\right)$ $=K^{\prime} M^{\prime} N^{\prime}+N\left(\underline{M}+K^{\prime}+\underline{M^{\prime}}\right)=K^{\prime} M^{\prime} \underline{N^{\prime}}+\underline{N}=N+K^{\prime} M^{\prime}=\left(K^{\prime}+N\right)\left(M^{\prime}+N\right)$
(a) $x \equiv 0=x(0)+x^{\prime}(0)^{\prime}=x^{\prime}$
(b) $x \equiv 1=x(1)+x^{\prime}(1)^{\prime}=x$
(c) $x \equiv x=x(x)+x^{\prime}(x)^{\prime}=x+x^{\prime}=1$
(d) $x \equiv x^{\prime}=x\left(x^{\prime}\right)+x^{\prime}\left(x^{\prime}\right)^{\prime}=0$
(e) $x \equiv y=x y+x^{\prime} y^{\prime}=y x+y^{\prime} x^{\prime}=y \equiv x$
(f) $(x \equiv y) \equiv z=\left(x y+x^{\prime} y^{\prime}\right) \equiv z=\left(x y+x^{\prime} y^{\prime}\right) z+\left(x y^{\prime}+x^{\prime} y\right) z^{\prime}=x y z+x^{\prime} y^{\prime} z+x y^{\prime} z^{\prime}+x^{\prime} y z^{\prime}$ $=x\left(y z+y^{\prime} z^{\prime}\right)+x^{\prime}\left(y^{\prime} z+y z^{\prime}\right)=x\left(y z+y^{\prime} z^{\prime}\right)+x^{\prime}(y z+y z)^{\prime}=x \equiv\left(y z+y^{\prime} z^{\prime}\right)=x \equiv(y \equiv z)$
(g) $(x \equiv y)^{\prime}=\left(x y+x^{\prime} y^{\prime}\right)^{\prime}=\left(x^{\prime}+y^{\prime}\right)(x+y)=x^{\prime} y+x y^{\prime}=x^{\prime} \equiv y=x y^{\prime}+x^{\prime} y=x \equiv y^{\prime}$
3.18 (a) $x \oplus 0=x(0)^{\prime}+x^{\prime}(0)=x$
(b) $x \oplus 1=x(1)^{\prime}+x^{\prime}(1)^{\prime}=x^{\prime}$
(c) $x \oplus x=x(x)^{\prime}+x^{\prime}(x)=0$
(d) $x \oplus x^{\prime}=x\left(x^{\prime}\right)^{\prime}+x^{\prime}\left(x^{\prime}\right)=x+x^{\prime}=1$
(e) $x \oplus y=x y^{\prime}+x^{\prime} y=y^{\prime} x+y x^{\prime}=y \oplus x$
(f) $(x \oplus y) \oplus z=\left(x y^{\prime}+x^{\prime} y\right) \oplus z=\left(x y^{\prime}+x^{\prime} y\right) z^{\prime}+\left(x y^{\prime}+x^{\prime} y\right)^{\prime} z=x y^{\prime} z^{\prime}+x^{\prime} y z^{\prime}+x y z+x^{\prime} y^{\prime} z$ $=x\left(y z+y^{\prime} z^{\prime}\right)+x^{\prime}\left(y z^{\prime}+y^{\prime} z\right)=x\left(y z^{\prime}+y z^{\prime}\right)^{\prime}+x^{\prime}\left(y z^{\prime}+y^{\prime} z\right)=x \oplus\left(y z+y^{\prime} z^{\prime}\right)=x \oplus(y \oplus z)$
(g) $(x \oplus y)^{\prime}=\left(x y^{\prime}+x^{\prime} y\right)^{\prime}=\left(x^{\prime}+y\right)\left(x+y^{\prime}\right)=x^{\prime} y^{\prime}+x y=x^{\prime} \oplus y=x y+x^{\prime} y^{\prime}=x \oplus y^{\prime}$
3.19 (a) $x \oplus y \oplus x y=x \oplus\left[y(x y)^{\prime}+y^{\prime}(x y)\right]=x \oplus\left[y x^{\prime}\right]=x\left(y x^{\prime}\right)^{\prime}+x^{\prime}\left(y x^{\prime}\right)=x\left(y^{\prime}+x\right)+x^{\prime} y=x+x^{\prime} y=x+y$
(b) $x \equiv y \equiv x y=\left(x y+x^{\prime} y^{\prime}\right) \equiv x y=\left(x y+x^{\prime} y^{\prime}\right) x y+\left(x y+x^{\prime} y^{\prime}\right)^{\prime}(x y)^{\prime}=x y+\left(x y^{\prime}+x^{\prime} y\right)\left(x^{\prime}+y^{\prime}\right)=x y+x^{\prime} y+x y^{\prime}$ $=x+y$
3.20 (a) $x y \oplus x z=x y\left(x^{\prime}+z^{\prime}\right)+\left(x^{\prime}+y^{\prime}\right) x z=x y z^{\prime}+x y z=x\left(y z^{\prime}+y z^{\prime}\right)=x(y \oplus z)$
(b) For $y=1$, the left hand side is $x+z^{\prime}$ but the right hand side is $x^{\prime} z^{\prime}$ which are not equal.
(c) For $y=0$, the left hand side is $x z^{\prime}$ but the right hand side is $x^{\prime}+z^{\prime}$ which are not equal.
(d) $(x+y) \equiv(x+z)=(x+y)(x+z)+(x+y)^{\prime}(x+z)^{\prime}=x+y z+\left(x^{\prime} y^{\prime}\right)\left(x^{\prime} z^{\prime}\right)=x+y z+x^{\prime} y^{\prime} z^{\prime}=x+y z+y^{\prime} z^{\prime}$ $=x+(y \equiv z)$


$=W^{\prime} Y^{\prime}+W X^{\prime} Y+W X Z$
3.21 (c) $(B+\mathbb{C + D})(\underline{(A+B+C)}(\underbrace{A^{\prime}+C+D})\left(B^{\prime}+C^{\prime}+D^{\prime}\right)=(A+B+C)\left(A^{\prime}+C+D\right)\left(B^{\prime}+C^{\prime}+D^{\prime}\right)$
3.21 (d)


## Unit 3 Solutions

3.21 (e) $\frac{A^{\prime} B e^{\prime}}{4}+\underline{B C^{\prime} D^{\prime}}+\underline{A^{\prime} C D}+\underline{B^{\prime} C D}+\underline{A^{\prime} B D}=B C^{\prime} D^{\prime}+B^{\prime} C D+A^{\prime} B D$
3.21 (f)

$$
(\underbrace{A+B+C})(\underbrace{B+C^{\prime}+D})\left(A^{A+B+D}\right)\left(A^{\prime}+B^{\prime}+D^{\prime}\right)=(A+B+C)\left(B+C^{\prime}+D\right)\left(A^{\prime}+B^{\prime}+D^{\prime}\right)
$$

3.22
3.23

$$
\begin{aligned}
F= & \underline{A^{\prime}} B+\underline{A} C+\underline{B} C^{\prime} D^{\prime}+\underline{B} E F+\underline{B} D F=(A+B)\left(A^{\prime}+C\right)+B\left(C^{\prime} D^{\prime}+E F+D F\right) \\
& =\left[(A+B)\left(A^{\prime}+C\right)+B\right]\left[(A+B)\left(A^{\prime}+C\right)+C^{\prime} D^{\prime}+E F+D F\right] \\
& =(\underline{A+B})\left(A^{\prime}+C+B\right)\left(A+B+C^{\prime} D^{\prime}+E F+D F\right)\left(A^{\prime}+\underline{C}+C^{\prime} D^{\prime}+E F+D F\right) \\
& =(\underline{A+B})\left(\underline{A^{\prime}+C+B}\right)(\underline{C+B})\left(\underline{A+B+C^{\prime} D^{\prime}+E F+D F}\right)\left(A^{\prime}+C+D^{\prime}+E F+D F\right) \\
& =(A+B)(B+C)\left(A^{\prime}+C+\underline{D^{\prime}}+F E+\underline{D} F\right)=(A+B)(B+C)\left(A^{\prime}+C+D^{\prime}+\underline{F}+\underline{F E}\right) \\
& =(A+B)(B+C)\left(A^{\prime}+C+D^{\prime}+F\right) \\
& =(B+A C)\left(A^{\prime}+C+D^{\prime}+F\right) \\
& =\left(A^{\prime} B+B C^{\prime}+B D^{\prime}+B F+\underline{A C}+A C D^{\prime}+A \not C F=A^{\prime} B+B D^{\prime}+B F+A C\right.
\end{aligned}
$$

use consensus, $X+X Y=X$ where $X=A C$
$3.24 X^{\prime} Y^{\prime} Z^{\prime}+X Y Z=\left(X+Y^{\prime} Z^{\prime}\right)\left(X^{\prime}+Y Z\right)=\left(X+Y^{\prime}\right)(\underline{X+Z})\left(\underline{X^{\prime}+Y}\right)\left(X^{\prime}+Z\right)\left(Y+Z^{\prime}\right)$

$$
=\left(X+Y^{\prime}\right)\left(X+Z^{\prime}\right)(X+Y)\left(\underline{X^{\prime}+Z}\right)(\underline{Y+Z})=\left(\underline{X+Y^{\prime}}\right)\left(X^{\prime}+Z^{\prime}\right)\left(X^{\prime}+Z\right)(\underline{Y+Z})
$$

$$
=\left(X+Y^{\prime}\right)\left(X^{\prime}+Z\right)\left(Y+Z^{\prime}\right)
$$

Alt.: $\left(X^{\prime}+Y\right)\left(Y^{\prime}+Z\right)\left(X+Z^{\prime}\right)$ by adding $\left(Y^{\prime}+Z\right)$ as consensus in 3rd step
3.25 (a) $x \underline{y}+x^{\prime} y z^{\prime}+y z=y\left(\underline{x}+\underline{x}^{\prime} z^{\prime}\right)+y z=x y+y z^{\prime}+y z$ $=x y+y=y$
Alternate Solution: $x \underline{y}+x^{\prime} y z^{\prime}+y z=y\left(\underline{x}+\underline{x}^{\prime} z^{\prime}+z\right)$

$$
=y\left(x+\underline{z^{\prime}}+z\right)=y(x+\underline{1})=y
$$

3.25 (c)
$x y^{\prime}+\underline{z}+\left(x^{\prime}+y\right) \underline{z^{\prime}}$
$=x^{\prime} y+\left(x^{\prime}+y\right)\{$ By Th. 11D with $Y=z\}$
$=x \underline{y}^{\prime}+x^{\prime}+\underline{y}=\underline{x}+\underline{x^{\prime}}+y=1+y=1$
Alt.: $x y^{\prime}+z+\left(x^{\prime}+y\right) z^{\prime}=\left(x y^{\prime}+z\right)+\left(x y^{\prime}+z\right)^{\prime}=1$
3.25 (e) $w^{\prime} x^{\prime}+x^{\prime} y^{\prime}+\underline{y z}+w^{\prime} z^{\prime}+x^{\prime} z$ Add redundant term
$=\underbrace{\prime} x^{\prime}+x^{\prime} y^{\prime}+y z+\frac{w^{\prime} z^{\prime}}{冖}+\frac{x^{\prime} z}{乙}$
$=x^{\prime} y^{\prime}+y z+w^{\prime} z^{\prime}+x$ Remove redundant term
$=x^{\prime} y^{\prime}+y z+w^{\prime} z^{\prime}$
3.25 (b) $\left(x y^{\prime}+z\right)\left(x+y^{\prime}\right) z=\left(x y^{\prime}+x z+y^{\prime} z\right) z$

$$
=\underline{x} y^{\prime} \underline{z}+\underline{x} \underline{z}+y^{\prime} z=x z+y^{\prime} z
$$

Alternate Solution: $\frac{\left(x y^{\prime}+z\right)}{L}\left(x+y^{\prime}\right) \frac{z}{T}=z\left(x+y^{\prime}\right)$ $=z x+z y^{\prime}$
3.25 (d) $a^{\prime} d\left(b^{\prime}+c\right)+a^{\prime} d^{\prime}\left(b+c^{\prime}\right)+\left(\underline{b^{\prime}}+c\right)\left(\underline{b}+c^{\prime}\right)$

$$
=\underline{a^{\prime} b^{\prime} d+d^{\prime} c d+\underline{a^{\prime} b d^{\prime}} d+a^{\prime} c^{\prime} d^{\prime}+b^{\prime} c^{\prime}+b c}
$$

$$
=a^{\prime} b b^{\prime} d+a^{\prime} b d^{\prime}+b^{\prime} c^{\prime}+b c^{\prime}
$$

Other Solutions: $b^{\prime} c^{\prime}+b c+a^{\prime} c^{\prime} d^{\prime}+a^{\prime} b^{\prime} d$

$$
b^{\prime} c^{\prime}+b c+a^{\prime} c^{\prime} d^{\prime}+a^{\prime} c d
$$

$$
b^{\prime} c^{\prime}+b c+a^{\prime} b d^{\prime}+a^{\prime} c d
$$


$=A^{\prime} B D+\frac{B^{\prime} E F}{L}+C D E^{\prime} G+A^{\prime} D \mathcal{L}$ (consensus)
$=A^{\prime} B D+B^{\prime} E F+C D E^{\prime} G$
3.26 (a)

3.27

Alternate Solutions: $F=W^{\prime} Y+W X^{\prime}+W Z^{\prime}+X Y^{\prime}$

$$
\begin{aligned}
& F=Y Z^{\prime}+W^{\prime} X+X Y^{\prime}+W Y^{\prime} \\
& F=W^{\prime} X+X^{\prime} Y+X Z^{\prime}+W Y^{\prime} \\
& F=W^{\prime} X+X Y^{\prime}+W Z^{\prime}+W Y^{\prime}
\end{aligned}
$$

3.28 (b) NOT VALID. Counterexample: $\mathrm{a}=0, \mathrm{~b}=1, \mathrm{c}=0$. LHS $=0$, RHS $=1 . \therefore$ This equation is not always valid.
In fact, the two sides of the equation are complements: $[(a+b)(b+c)(c+a)]^{\prime}$ $=[(b+a c)(a+c)]^{\prime}=[a b+a c+b c]^{\prime}$ $=\left(a^{\prime}+b^{\prime}\right)\left(a^{\prime}+c^{\prime}\right)\left(b^{\prime}+c^{\prime}\right)$
3.28 (d) VALID: LHS $=x y^{\prime}+x^{\prime} z+y z^{\prime}$
consensus terms: $y^{\prime} z, x z^{\prime}, x^{\prime} y$

3.25 (g) $\left[\left(a^{\prime}+d^{\prime}+b^{\prime} c\right)\left(b+d+a c^{\prime}\right)\right]^{\prime}+b^{\prime} c^{\prime} d^{\prime}+a^{\prime} c^{\prime} d$ $=a d\left(b+c^{\prime}\right)+b^{\prime} d^{\prime}\left(a^{\prime}+c\right)+b^{\prime} c^{\prime} d^{\prime}+a^{\prime} c^{\prime} d$

$=a b d+\underbrace{\prime} b^{\prime} d^{\prime}+b^{\prime} d^{\prime}+c^{\prime} d=a b d+b^{\prime} d^{\prime}+c^{\prime} d$
3.26 (b)

3.28 (a) VALID: $a^{\prime} b+b^{\prime} c+c^{\prime} a$
$=a^{\prime} b\left(c+c^{\prime}\right)+\left(a+a^{\prime}\right) b^{\prime} c+\left(b+b^{\prime}\right) a c^{\prime}$


Alternate Solution: $a^{\prime} b+b^{\prime} c+c^{\prime} a$ Add all consensus terms: $a b^{\prime}, b c^{\prime}, c a^{\prime}$


$$
=a b^{\prime}+b c^{\prime}+c a^{\prime}
$$

3.28 (c) VALID. Starting with the right side, add consensus terms

3.28 (e) NOT VALID. Counterexample: $x=0, y=1, z=0$, then LHS $=0$, RHS $=1 . \therefore$ This equation is not always valid. In fact, the two sides of the equations are complements.
LHS $=(x+y)(y+z)(x+z)$
$=\left[(x+y)^{\prime}+(y+z)^{\prime}+(x+z)^{\prime}\right]^{\prime}$
$=\left(x^{\prime} y^{\prime}+y^{\prime} z^{\prime}+x^{\prime} z^{\prime}\right)^{\prime}=\left[x^{\prime}\left(y^{\prime}+z^{\prime}\right)+y^{\prime} z^{\prime}\right]^{\prime}$
$=\left[\left(x^{\prime}+y^{\prime} z^{\prime}\right)\left(y^{\prime}+z^{\prime}+y^{\prime} z^{\prime}\right)\right]^{\prime}$
$=\left[\left(x^{\prime}+y^{\prime}\right)\left(x^{\prime}+z^{\prime}\right)\left(y^{\prime}+z^{\prime}\right)\right]^{\prime}$
$\neq\left(x^{\prime}+y^{\prime}\right)\left(y^{\prime}+z^{\prime}\right)\left(x^{\prime}+z^{\prime}\right)$

## Unit 3 Solutions

3.29

$$
\begin{aligned}
S U M & =(X \oplus Y) \oplus C_{\mathrm{i}}=\left(X Y^{\prime}+X^{\prime} Y\right) \oplus C_{\mathrm{i}} \\
& =\left(X Y^{\prime}+X^{\prime} Y\right) C_{\mathrm{i}}^{\prime}+\left(X Y^{\prime}+X^{\prime} Y\right)^{\prime} C_{\mathrm{i}} \\
& =X Y^{\prime} C_{\mathrm{i}}^{\prime}+X^{\prime} Y C_{\mathrm{i}}^{\prime}+X^{\prime} Y^{\prime} C_{\mathrm{i}}+X Y C_{\mathrm{i}} \\
C_{\mathrm{o}}= & (X \oplus Y) C_{\mathrm{i}}+X Y \\
= & X Y^{\prime} C_{\mathrm{i}}+X^{\prime} Y C_{\mathrm{i}}+X Y \\
= & X C_{\mathrm{i}}+Y C_{\mathrm{i}}+X Y
\end{aligned}
$$

| $X$ | $Y$ | $C_{i}$ | $S U M$ | $C_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

3.30

| $A$ | $B$ | $C$ | $F$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| $F=$ | $A B+A C+B C$ |  |  |

3.31 (a) VALID:

LHS $=\left(X^{\prime}+Y^{\prime}\right)(X \oplus Z)+(X+Y)(X \oplus Z)$
$=\left(X^{\prime}+Y^{\prime}\right)\left(X^{\prime} Z^{\prime}+X Z\right)+(X+Y)\left(X^{\prime} Z+X Z '\right)$
$=\frac{X^{\prime} Z^{\prime}}{\square}+\underset{\sim}{X} X^{\prime}+\underline{X Y^{\prime} Z}+\underline{X^{\prime} Y Z}+\underline{X Z^{\prime}}+\underline{X X Z^{\prime}}$
$=\underline{X^{\prime} Z^{\prime}}+\left(\underline{X Y^{\prime}+X^{\prime} Y}\right) Z+\underline{X Z^{\prime}}$
$=\underline{Z}^{\prime}+\underline{Z}(X \oplus Y)=Z^{\prime}+(X \oplus Y)=$ RHS
3.31 (b) LHS $=\left(W^{\prime}+X+Y^{\prime}\right)\left(\underline{W}+X^{\prime}+Y\right)\left(\underline{W}+Y^{\prime}+Z\right)=\left(W^{\prime}+X+Y^{\prime}\right)\left(W+\left(X^{\prime}+\underline{Y}\right)\left(\underline{Y^{\prime}}+Z\right)\right)$
$=\left(\underline{W^{\prime}}+X+Y^{\prime}\right)\left(\underline{W}+\left(X^{\prime} Y^{\prime}+Y Z\right)\right)=\left(W^{\prime}\left(X^{\prime} Y^{\prime}+Y Z\right)+W\left(X+Y^{\prime}\right)\right)=\underbrace{W^{\prime} X^{\prime} Y^{\prime}}_{\text {consensus terms: }}+\underbrace{W}_{X Y} \underbrace{W^{\prime} Y Z}+\underbrace{W X}_{X Y Z}+W Y^{\prime}$
$=W^{\prime} X^{\prime} Y^{\prime}+W^{\prime} Y Z+W X+W Y^{\prime}+X Y Z+X^{\prime} Y^{\prime}=W \mathrm{X}^{\prime} Y^{\prime}+W^{\prime} X^{\prime} Z+W^{\prime} Y Z+X Y Z+\frac{W X+W Y^{\top}+\frac{X^{\prime} Y^{\prime}}{V^{\prime}}}{\underline{\text { W }}}$
$=W^{\prime} X Z+\frac{W^{\prime} Y Z}{Z}+X Y Z+W X+\underline{X^{\prime} Y^{\prime}}=W^{\prime} Y Z+X Y Z+W X+X^{\prime} Y^{\prime}$
3.31 (c) $\mathrm{LHS}=\underline{A} \underline{B} \underline{C}+\underline{A^{\prime}} C^{\prime} \underline{D^{\prime}}+\underline{A^{\prime}} \underline{B} \underline{D^{\prime}}+\underline{A C D}=\underline{A} C(B+D)+\underline{A^{\prime}} D^{\prime}\left(B+C^{\prime}\right)=\left(A+D^{\prime}\left(B+C^{\prime}\right)\right)\left(A^{\prime}+C(B+D)\right)$
$\left.\left.=\left(A+D^{\prime}\right) \underline{\left(A+B+C^{\prime}\right)}\left(A^{\prime}+C\right) \underline{\left(A^{\prime}+B+D\right.}\right)=\left(A+D^{\prime}\right)\left(A+B+C^{\prime}\right) \underline{\left(A^{\prime}+C\right)}\left(A^{\prime}+B+D\right) \underline{\left(B+C^{\prime}+D\right.}\right)$ consensus: $\quad B+C^{\prime}+D$

3.32 (a) VALID. $[A+B=C] \Rightarrow\left[D^{\prime}(A+B)=D^{\prime}(C)\right]$ $[A+B=C] \Rightarrow\left[A D^{\prime}+B D^{\prime}=C D^{\prime}\right]$
3.32 (c) VALID. $[A+B=C] \Rightarrow[(A+B)+D=(C)+D]$ $[A+B=C] \Rightarrow[A+B+D=C+D]$
3.32 (b) NOT VALID. Counterexample: $A=1, B=C=0$ and $D=1$ then LHS $=(0)(0)+(0)(0)=0$

$$
\text { RHS }=(0)(1)=0=\text { LHS }
$$

but $B+C=0+0=0 ; D=1 \neq B+C$
$\therefore$ The statement is false.
3.32 (d) NOT VALID. Counterexample: $C=1, A=B=0$ and $D=1$ then LHS $=0+0+1=1$

$$
\text { RHS = } 1+1 \text { = } 1 \text { = LHS }
$$

but $A+B=0+0=0 \neq D$
$\therefore$ The statement is false.

$$
\begin{aligned}
& 3.33 \text { (a) } A^{\prime} C^{\prime}+B C+A B^{\prime}+A^{\prime} B D+B^{\prime} C^{\prime} D^{\prime}+A C D^{\prime} \\
& \text { Consensus terms: (1) } B^{\prime} C^{\prime} \text { using } A^{\prime} C^{\prime}+A B^{\prime} \\
& \text { (2) } A^{\prime} B \text { using } A^{\prime} C^{\prime}+B C \text { (3) } A C \text { using } A B^{\prime}+B C \\
& \text { (4) } A B^{\prime} D^{\prime} \text { using } B^{\prime} C^{\prime} D^{\prime}+A C D^{\prime} \\
& \text { Using 1, 2, 3: } A^{\prime} C^{\prime}+B C+A B^{\prime}+A^{\prime} B D+B^{\prime} C D^{\prime} \\
& + \text { ACD' }{ }^{\prime} B^{\prime} C^{\prime}+A^{\prime} B+A C=A^{\prime} C^{\prime}+B C+A B^{\prime} \\
& \text { (Using the consensus theorem to remove the added } \\
& \text { terms since the terms that generated them are still } \\
& \text { present.) } \\
& \text { 3.34 abd'f' + b'cegh' + abd'f + acd'e + b'ce } \\
& =\left(a b d^{\prime} f^{\prime}+a b d^{\prime} f\right)+\left(b^{\prime} c e g h^{\prime}+b^{\prime} c e\right)+a c d^{\prime} e \\
& =a b d '+b^{\prime} c e+a c d ' e \\
& =a b d^{\prime}+b^{\prime} c e \text { (consensus) } \\
& =(b+c e)\left(b^{\prime}+a d^{\prime}\right) \\
& =(b+c)(b+e)\left(b^{\prime}+a\right)\left(b^{\prime}+d^{\prime}\right) \\
& 3.36 \quad a b c^{\prime}+d^{\prime} e+a c e+b^{\prime} c^{\prime} d^{\prime} \\
& =\left(d^{\prime}+a b c^{\prime}+a c e+b^{\prime} c^{\prime} d^{\prime}\right)\left(e+a b c^{\prime}+a c e+b^{\prime} c^{\prime} d^{\prime}\right) \\
& =\left(d^{\prime}+a b c^{\prime}+a c e\right)\left(e+a b c^{\prime}+b^{\prime} c^{\prime} d^{\prime}\right) \\
& =\left[d^{\prime}+a\left(b c^{\prime}+c e\right)\right]\left[e+c^{\prime}\left(a b+b^{\prime} d^{\prime}\right)\right] \\
& =\left[d^{\prime}+a(b+c)\left(c^{\prime}+e\right)\right]\left[e+c^{\prime}\left(a+b^{\prime}\right)\left(b+d^{\prime}\right)\right] \\
& =\left(d^{\prime}+a\right)\left(d^{\prime}+b+c\right)\left(d^{\prime}+c^{\prime}+e\right)\left(e+c^{\prime}\right) \\
& \left(e+a+b^{\prime}\right)\left(e+b+d^{\prime}\right) \\
& =\left(d^{\prime}+a\right)\left(d^{\prime}+b+c\right)\left(e+c^{\prime}\right) \\
& \left(e+a+b^{\prime}\right)\left(e+b+d^{\prime}\right) \\
& =\left(d^{\prime}+a\right)\left(d^{\prime}+b+c\right)\left(e+c^{\prime}\right)\left(e+a+b^{\prime}\right) \\
& \text { (consensus) }
\end{aligned}
$$

3.33 (b) $A^{\prime} C^{\prime} D^{\prime}+B C^{\prime} D+A B^{\prime} C^{\prime}+A^{\prime} B C$

Consensus terms:
(1) $A^{\prime} B C^{\prime}$ using $A^{\prime} C^{\prime} D^{\prime}+B C^{\prime} D$
(2) $A C^{\prime} D$ using $A B^{\prime} C^{\prime}+B C^{\prime} D$
(3) $B^{\prime} C^{\prime} D^{\prime}$ using $A^{\prime} C^{\prime} D^{\prime}+A B^{\prime} C^{\prime}$
(4) $A^{\prime} B D^{\prime}$ using $A^{\prime} C^{\prime} D^{\prime}+A^{\prime} B C$
(5) $A^{\prime} B D$ using $B C^{\prime} D+A^{\prime} B C$

Using 1: $A^{\prime} C^{\prime} D^{\prime}+B C^{\prime} D+A B^{\prime} C^{\prime}+A^{\prime} B C+A^{\prime} B$, which is the minimum solution.
3.35
3.37

$$
\begin{aligned}
& \text { (a) } \left.(x \equiv y)^{\prime}=\left(x y+x^{\prime} y^{\prime}\right)\right)^{\prime}=\left(x^{\prime}+y^{\prime}\right)(x+y) \\
= & x^{\prime} y+x y^{\prime}=x \oplus y \\
& \text { (b) } a^{\prime} b^{\prime} c^{\prime}+a a^{\prime} b c+a b^{\prime} c+a b c^{\prime} \\
= & a^{\prime}\left(b^{\prime} c^{\prime}+b c\right)+a\left(b^{\prime} c+b c^{\prime}\right) \\
= & a^{\prime}(b \equiv c)+a(b \equiv c)^{\prime} \\
= & a^{\prime} \equiv(b \equiv c)
\end{aligned}
$$

Unit 3 Solutions

## Unit 4 Problem Solutions

4.1 See FLD p. 695 for solution.
4.2

| $A$ | $B$ | $C$ | $\boldsymbol{D}$ | $\boldsymbol{E}$ |  | $\boldsymbol{y}$ | $\boldsymbol{z}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | (less than 10 gpm$)$ | + |  |
| 1 | 0 | 0 | 0 | 0 | (at least 10 gpm ) | + |  |
| 1 | 1 | 0 | 0 | 0 | (at least 20 gpm ) | + | + |
| 1 | 1 | 1 | 0 | 0 | (at least 30 gpm$)$ |  | + |
| 1 | 1 | 1 | 1 | 0 | (at least 40 gpm$)$ |  | + |
| 1 | 1 | 1 | 1 | 1 | (at least 50 gpm ) |  |  |

4.2 (a) $Y=A^{\prime} B^{\prime} C^{\prime} D^{\prime} E^{\prime}+A B^{\prime} C^{\prime} D^{\prime} E^{\prime}+A B C^{\prime} D^{\prime} E^{\prime}$
4.2 (b) $Z=A B C^{\prime} D^{\prime} E^{\prime}+A B C D^{\prime} E^{\prime}+A B C D E^{\prime}$
4.3
$F_{1}=\sum m(0,4,5,6) ; F_{2}=\sum m(0,3,4,6,7) ; F_{1}+F_{2}=\sum m(0,3,4,5,6,7)$
General rule: $F_{1}+F_{2}$ is the sum of all minterms that are present in either $F_{1}$ or $F_{2}$.
Proof: Let $F_{1}=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1} a_{\mathrm{i}} m_{\mathrm{i}} ; F_{2}=\sum_{\mathrm{j}=0}^{2^{\mathrm{n}}-1} b_{\mathrm{j}} m_{\mathrm{j}} ; F_{1}+F_{2}=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1} a_{\mathrm{i}} m_{\mathrm{i}}+\sum_{\mathrm{j}=0}^{2^{\mathrm{n}}-1} b_{\mathrm{j}} m_{\mathrm{j}}=a_{0} m_{0}+a_{1} m_{1}+a_{2^{\mathrm{n}}-1} m_{2}+\ldots$
$+b_{0} m_{0}+b_{1} m_{1}+b_{2} m_{2}+\ldots=\left(a_{0}+b_{0}\right) m_{0}+\left(a_{1}+b_{1}\right) m_{1}+\left(a_{2}+b_{2}\right) m_{2}+\ldots=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1}\left(a_{\mathrm{i}}+b_{\mathrm{i}}\right) m_{\mathrm{i}}$
4.4 (a) $2^{2^{n}}=2^{2^{2}}=2^{4}=16$
4.4 (b)

| $x$ | $y$ | $z_{0}$ | $z_{1}$ | $z_{2}$ | $z_{3}$ | $z_{4}$ | $z_{5}$ | $z_{6}$ | $z_{7}$ | $z_{8}$ | $z_{9}$ | $z_{10}$ | $z_{11}$ | $z_{12}$ | $z_{13}$ | $z_{14} z_{15}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

4.5

Alternate
Solutions

| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $Z$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | $\mathrm{X}^{3}$ | 1 |
| 0 | 0 | 1 | $\mathrm{X}^{2}$ | $\mathrm{X}^{2}$ | 1 | 1 |
| 0 | 1 | 0 | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | X |
| 0 | 1 | 1 | $\mathrm{X}^{2}$ | $\mathrm{X}^{2}$ | 1 | 1 |
| 1 | 0 | 0 | $\mathrm{X}^{4}$ | 0 | 0 | 0 |
| 1 | 0 | 1 | $\mathrm{X}^{2}$ | $\mathrm{X}^{2}$ | 1 | 1 |
| 1 | 1 | 0 | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | X |
| 1 | 1 | 1 | $\mathrm{X}^{4}$ | 0 | 0 | 0 |


| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ |
| :--- | :--- | :--- | :--- | :--- | :--- |

${ }^{1}$ These truth table entries were made don't cares because $A B C=110$ and $A B C=010$ can never occur
${ }^{2}$ These truth table entries were made don't cares because when $F$ is 1 , the output $Z$ of the OR gate will be 1 regardless of its other input. So changing $D$ and $E$ cannot affect $Z$.
${ }^{3}$ These truth table entries were made don't cares because when $D$ and $E$ are both 1, the output $Z$ of the OR gate will be 1 regardless of the value of $F$.
${ }^{4}$ These truth table entries were made don't cares because when one input of the AND gate is 0 , the output will be 0 regardless of the value of its other input.
4.6 (a) Of the four possible combinations of $d_{1} \& d_{5}, d_{1}=1$ and $d_{5}=0$ gives the best solution:
$F=A^{\prime} B^{\prime} C^{\prime}+A^{\prime} B^{\prime} C+A B C^{\prime}+A B C=A^{\prime} B^{\prime}+A B$
4.6 (b) By inspection, $G=C$ when both don't cares are set to 0 .

## Unit 4 Solutions

4.7 (a) Exactly one variable not complemented: $F=A^{\prime} B^{\prime} C$ $+A^{\prime} B C^{\prime}+A B^{\prime} C^{\prime}=\sum m(1,2,4)$
4.7 (b) Remaining terms are maxterms:
$F=\prod M(0,3,5,6,7)=(A+B+C)\left(A+B^{\prime}+C^{\prime}\right)$
$\left(A^{\prime}+B+C^{\prime}\right)\left(A^{\prime}+B^{\prime}+C\right)\left(A^{\prime}+B^{\prime}+C^{\prime}\right)$
4.8 (a) $\quad F(A, B, C, D)=\sum m(0,1,2,3,4,5,6,8,9,12)$

Refer to FLD p. 695 for full term expansion
4.8 (b) $\quad F(A, B, C, D)=\prod M(7,10,11,13,14,15)$

Refer to FLD p. 695 for full term expansion
4.9 (a) $F=a b c^{\prime}+b^{\prime}\left(a+a^{\prime}\right)\left(c+c^{\prime}\right)=a b c^{\prime}+a b^{\prime} c+a b^{\prime} c^{\prime}$
$+a^{\prime} b^{\prime} c+a^{\prime} b^{\prime} c^{\prime} ; F=\sum m(0,1,4,5,6)$
4.9 (b) Remaining terms are maxterms: $F=\prod M(2,3,7)$
4.9 (c) Maxterms of $F$ are minterms of $F^{\prime}$ :
$F^{\prime}=\sum m(2,3,7)$
4.9 (d) Minterms of $F$ are maxterms of $F^{\prime}$ :
$F^{\prime}=\prod M(0,1,4,5,6)$
4.10

$$
\begin{aligned}
& F(a, b, c, d)=(\underline{a+b+d})\left(\frac{a^{\prime}+c}{}\right)\left(\frac{a a^{\prime}+b^{\prime}+c^{\prime}}{}\right)\left(a+b+c^{\prime}+d^{\prime}\right) \\
= & (a+b+c+d)\left(a+b+c^{\prime}+d\right)\left(a^{\prime}+c+b b^{\prime}+d d^{\prime}\right)\left(a^{\prime}+b^{\prime}+c^{\prime}+d\right)\left(a^{\prime}+b^{\prime}+c^{\prime}+d^{\prime}\right)\left(a+b+c^{\prime}+d^{\prime}\right) \\
= & (a+b+c+d)\left(a+b+c^{\prime}+d\right)\left(a^{\prime}+b+c+d\right)\left(a^{\prime}+b+c+d^{\prime}\right)\left(a^{\prime}+b^{\prime}+c+d\right)\left(a^{\prime}+b^{\prime}+c+d^{\prime}\right) \\
& \left(a^{\prime}+b^{\prime}+c^{\prime}+d\right)\left(a^{\prime}+b^{\prime}+c^{\prime}+d^{\prime}\right)\left(a+b+c^{\prime}+d^{\prime}\right)
\end{aligned}
$$

4.10 (a) $F=\sum m(1,4,5,6,7,10,11)$
4.10 (c) $\quad F^{\prime}=\sum m(0,2,3,8,9,12,13,14,15)$
4.11 (a) difference, $d_{\mathrm{i}}=x_{\mathrm{i}} \oplus y_{\mathrm{i}} \oplus b_{\mathrm{i}} ; b_{\mathrm{i}+1}=b_{\mathrm{i}} x_{\mathrm{i}}^{\prime}+x_{\mathrm{i}}^{\prime} y_{\mathrm{i}}+b_{\mathrm{i}} y_{\mathrm{i}}$

| $x_{\mathrm{i}} y_{\mathrm{i}}$ | $b_{\mathrm{i}}$ | $b_{\mathrm{i}+1}$ | $d_{\mathrm{i}}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

4.10 (b) $F=\prod M(0,2,3,8,9,12,13,14,15)$
4.10 (d) $\quad F^{\prime}=\prod M(1,4,5,6,7,10,11)$
4.11 (b) $d_{\mathrm{i}}=s_{\mathrm{i}}{ }^{\prime} b_{\mathrm{i}+1}$ is the same as $c_{\mathrm{i}+1}$ with $x_{\mathrm{i}}$ replaced by $x_{\mathrm{i}}{ }^{\prime}$
4.12 See FLD p. 696 for solution.

Unit 4 Solutions
4.13

| $A$ | $B$ | $C$ | $D$ | $Z$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

4.14

$\begin{aligned} Z= & A^{\prime} B C^{\prime} D+A^{\prime} B C D^{\prime}+A^{\prime} B C D+ \\ & A B^{\prime} C^{\prime} D^{\prime}+A B^{\prime} C^{\prime} D+A B^{\prime} C D^{\prime}\end{aligned}$
$+A B^{\prime} C D$
$=A^{\prime} B D+A B^{\prime} C^{\prime}+A B^{\prime} C+A^{\prime} B C D^{\prime}$
$=A B^{\prime}+A^{\prime} B D+A^{\prime} B C D^{\prime}+\underline{A^{\prime} B C}$
(Added consensus terms)
$\therefore Z=A B^{\prime}+A^{\prime} B D+A^{\prime} B C$

4.15 (a) Prime digits are $1,3,5$, and 7 represented as $0010,0111,1011$ and 1110 . The minterms are $A^{\prime} B^{\prime} C D^{\prime}, A^{\prime} B C D$, $A B^{\prime} C D$ and $A B C D^{\prime}$. The don't care minterms are $A^{\prime} B^{\prime} C^{\prime} D^{\prime}, A^{\prime} B^{\prime} C D, A^{\prime} B C^{\prime} D, A^{\prime} B C D^{\prime}, A B^{\prime} C^{\prime} D, A B^{\prime} C D^{\prime}, A B C^{\prime} D^{\prime}$ and $A B C D$.
(b) Nonprime digits are $0,2,4$, and 6 represented as 0001, 0100,1000 and 1101. The maxterms are $A+B+C+D^{\prime}$, $A+B^{\prime}+C+D, A^{\prime}+B+C+D$ and $A^{\prime}+B^{\prime}+C+D^{\prime}$. The don't care maxterms are $A+B+C+D, A+B+C^{\prime}+D^{\prime}$, $A+B^{\prime}+C+D^{\prime}, A+B^{\prime}+C^{\prime}+D, A^{\prime}+B+C+D^{\prime}, A^{\prime}+B+C^{\prime}+D, A^{\prime}+B^{\prime}+C+D$ and $A^{\prime}+B^{\prime}+C^{\prime}+D^{\prime}$.

Truth Table

4.17 Truth Table


## Unit 4 Solutions

4.18 (a), Truth Table
(b)

| A B C D | WX Y Z | (a) |
| :---: | :---: | :---: |
| 0000 | 1111 |  |
| 0001 | 1110 |  |
| 0010 | 1101 | minterms of Z: $0,2,6,12,14$ |
| 0011 | 1100 | don't care minterms: 4, 5, 7, |
| 0100 | x X X X |  |
| 0101 | x x x x | (b) |
| 0110 | 1001 | maxterms of W: 9, 12, 13, |
| 0111 | x x x x | 14, 15 |
| 1000 | $\mathrm{x} \times \mathrm{x}$ X | maxterms of X: 6, 12, 13, 14, |
| 1001 | 0110 | axte |
| 1010 | x x x x | 15 |
| 1011 | $\mathrm{x} \mathrm{x} \times \mathrm{x}$ | maxterms of Z: 1, 3, 9, 13, |
| 1100 | 0011 | 15 |
| 1101 | 0010 | $8,10,11$ |
| 1110 | 0001 |  |
| 1111 | 0000 |  |

4.18 (a), Alternative Truth Table
(b)

| A B C D | WX Y Z | (a) |
| :---: | :---: | :---: |
| 0000 | 1111 |  |
| 0001 | 1110 | minterms of $\mathrm{Y}: 0,1,9,12,13$ |
| 0010 | 1101 | minterms of Z: $0,2,8,12,14$ |
| 0011 | 1100 | don't care minterms: 4, 5, 6, |
| 0100 | x $\mathrm{x} \times \mathrm{x}$ |  |
| 0101 | x x x X | (b) |
| 0110 | x x x x | maxterms of W: $8,12,13$, |
| 0111 | 1000 | 4, 1 |
| 1000 | 0111 | maxterms of X: 7, 12, 13, |
| 1001 | x x x x | maxterms of Y: 2, 3, 7, 14, |
| 1010 | x x x x | 15 |
| 1011 | $\mathrm{x} \mathrm{x} \times \mathrm{x}$ | maxterms of Z: 1, 3, 7, 13, |
| 1100 | 0011 |  |
| 1101 | 0010 | $9,10,11$ |
| 1110 | 0001 |  |
| 1111 | 0000 |  |

4.19 (a) The buzzer will sound if the key is in the ignition switch and the car door is open, or the seat belts are not fastened, $\therefore$ The two possible interpretations are: $B=K D+S^{\prime}$ and $B=K\left(D+S^{\prime}\right)$
4.19 (b) You will gain weight, if you eat too much, or you do not exercise enough and your metabolism rate is too low. $\underset{F}{E^{\prime}}$ $\therefore$ The two possible interpretations are: $W=\left(F+E^{\prime}\right) M$ and $W=F+E^{\prime} M$
4.19 (c) The speaker will be damaged, if the volume is set too high and loud music is played or the stereo is too powerful
$\therefore$ The two possible interpretations are: $D=V M+S$ and $D=V(M+S)$
4.19 (d) The roads will be very slippery, if it snows, or it rains, and there is oil on the road.
$\therefore$ The two possible interpretations are: $V=(S+R) O$ and $V=S+R O$
$Z=A B+A C+B C$
4.22 (a) $13_{10}=D_{16}=0001101 ; \therefore X=A^{\prime} B^{\prime} C^{\prime} D E F^{\prime} G$
$4.21 \quad Z=\left(A B C D E+A^{\prime} B^{\prime} C^{\prime} D^{\prime} E^{\prime}\right)^{\prime} ; Y=A^{\prime} B^{\prime} C D^{\prime} E$
4.22 (b) $10_{10}=0001010 ; \therefore Y=A^{\prime} B^{\prime} C^{\prime} D E^{\prime} F G^{\prime}$
4.22 (c) $0_{10}=0000000_{2} ; 64_{10}=1000000_{2} ; 31_{10}=0011111_{2} ; 127_{10}=1111111_{2} ; 32_{10}=0100000_{2} ; \therefore Z=\left(A^{\prime} B^{\prime}\right)^{\prime}=A+B$
4.23 $\quad F_{1} F_{2}=\prod M(0,4,5,6,7)$. General rule: $F_{1} F_{2}$ is the product of all maxterms that are present in either $F_{1}$ or $F_{2}$. Proof:

Let $F_{1}=\prod\left(a_{\mathrm{i}}+M_{\mathrm{i}}\right) ; F_{2}=\prod\left(b_{\mathrm{j}}+M_{\mathrm{j}}\right) ; F_{1} F_{2}=\prod\left(a_{\mathrm{i}}+M_{\mathrm{i}}\right) \prod\left(b_{\mathrm{j}}+M_{\mathrm{j}}\right)$
$=\left(a_{0}+M_{0}\right)\left(b_{0}+M_{0}\right)\left(a_{1}+M_{1}\right)\left(b_{1}+M_{1}\right)\left(a_{2}+M_{2}\right)\left(b_{2}+M_{2}\right) \ldots=\left(a_{0} b_{0}+M_{0}\right)\left(a_{1} b_{1}+M_{1}\right)\left(a_{2} b_{2}+M_{2}\right) \ldots$
$=\prod\left(a_{\mathrm{i}} b_{\mathrm{i}}+M_{\mathrm{i}}\right)$
Maxterm $M_{\mathrm{i}}$ is present in $F_{1} F_{2}$ iff $a_{\mathrm{i}} b_{\mathrm{i}}=0$, i.e., if either $a_{\mathrm{i}}=0$ or $b_{\mathrm{i}}=0$. Maxterm $M_{\mathrm{i}}$ is present in $F_{1}$ iff $a_{\mathrm{i}}=0$. Maxterm $M_{\mathrm{i}}$ is present in $F_{2}$ iff $b_{\mathrm{i}}=0$. Therefore, maxterm $M_{\mathrm{i}}$ is present in $F_{1} F_{2}$ iff it is present in $F_{1}$ or $F_{2}$.
$4.24 \quad F_{1}+F_{2}=\Pi M(0,4)$. General rule: $F_{1}+F_{2}$ is the product of all maxterms that are present in both $F_{1}$ and $F_{2}$.
Proof:
Let $F_{1}=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1}\left(a_{\mathrm{i}} m_{\mathrm{i}}\right) ; F_{2}=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1}\left(b_{\mathrm{j}} m_{\mathrm{j}}\right) ; F_{1}+F_{2}=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1}\left(a_{\mathrm{i}} m_{\mathrm{i}}\right)+\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1}\left(b_{\mathrm{j}} m_{\mathrm{j}}\right)$
$\left.=a_{0} m_{0}+b_{0} m_{0}+a_{1} m_{1}+b_{1} m_{1}+a_{2} m_{2}+b_{2} m_{2}\right) \ldots=\left(a_{0}+b_{0}\right) m_{0}+\left(a_{1}+b_{1}\right) m_{1}+\left(a_{2}+b_{2}\right) m_{2}+\ldots$
$=\sum_{\mathrm{i}=0}^{2^{\mathrm{n}}-1}\left(a_{\mathrm{i}}+b_{\mathrm{i}}\right) m_{\mathrm{i}}$
Minterm $m_{\mathrm{i}}$ is present in $F_{1}+F_{2}$ iff $a_{\mathrm{i}}+b_{\mathrm{i}}=1$, i.e., if either $a_{\mathrm{i}}=1$ or $b_{\mathrm{i}}=1$ so maxterm $M_{\mathrm{i}}$ is present in $F_{1}+F_{2}$ if $a_{\mathrm{i}}=$ 0 and $b_{\mathrm{i}}=0$. Therefore, maxterm $M_{\mathrm{i}}$ is present in $F_{1}+F_{2}$ iff it is present in both $F_{1}$ and $F_{2}$.
4.25

| ABCD | FGH J | $\begin{aligned} & \text { (a) } F(A, B, C, D)= \\ & \sum m(5,6,7,10,11,13,14,15) \\ & =\prod M(0,1,2,3,4,8,9,12) \end{aligned}$ |
| :---: | :---: | :---: |
| 0000 | 0100 |  |
| 0001 | 0000 |  |
| 0010 | 0100 | $\begin{aligned} & \text { (b) } G(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})= \\ & \sum m(0,2,4,6) \\ & =\prod M(1,3,5,7,8,9,10,11, \\ & 12,13,14,15) \end{aligned}$ |
| 0011 | 0000 |  |
| 0100 | 0101 |  |
| 0101 | 1000 |  |
| 0110 | 1100 | $\begin{aligned} & \text { (c) } H(A, B, C, D)= \\ & \sum m(7,11,13,14,15) \\ & =\prod M(0,1,2,3,4,5,6,8,9 \\ & \quad 10,12) \end{aligned}$ |
| 0111 | 1010 |  |
| 1000 | 0001 |  |
| 1001 | 0000 |  |
| 1010 | 1000 | $\begin{aligned} & \text { (d) } J(A, B, C, D)= \\ & \sum m(4,8,12,13,14) \\ & =\prod M(0,1,2,3,5,6,7,9,10 \\ & \quad 11,15) \end{aligned}$ |
| 1011 | 1010 |  |
| 1100 | 0001 |  |
| 1101 | 1011 |  |
| 1110 | 1011 |  |
| 1111 | 1010 |  |

4.26

| ABCD | FGH J |
| :---: | :---: |
| 0000 | 0100 |
| 0001 | 0100 |
| 0010 | 0100 |
| 0011 | 0000 |
| 0100 | 0101 |
| 0101 | 1000 |
| 0110 | 0000 |
| 0111 | 1010 |
| 1000 | 0101 |
| 1001 | 0000 |
| 1010 | 1000 |
| 1011 | 1010 |
| 1100 | 0001 |
| 1101 | 1011 |
| 1110 | 1011 |
| 1111 | 1010 |

(a) $F(A, B, C, D)=$
$\sum m(5,7,10,11,13,14,15)$
$=\prod_{12)} M(0,1,2,3,4,6,8,9$,
12)
(b) $G(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=$
$\sum m(0,1,2,4,8)$
$=$ П $M(3,5,6,7,9,10,11,12$, $13,14,15)$
(c) $H(A, B, C, D)=$
$\sum m(7,11,13,14,15)$
$=\prod M(0,1,2,3,4,5,6,8,9$, $10,12)$
(d) $J(A, B, C, D)=$
$\sum m(4,8,12,13,14)$
$=\Pi M(0,1,2,3,5,6,7,9,10$,
11, 15)
4.27 You can also work this problem using a truth table, as in problem 4.28.

$$
\begin{array}{r}
f(a, b, c)=a\left(b+c^{\prime}\right)=a b+a c^{\prime}=a b\left(c+c^{\prime}\right)+ \\
a\left(b+b^{\prime}\right) c^{\prime}=\frac{a b c}{m_{7}}+\frac{a b c^{\prime}}{m_{6}}+\frac{a b c^{\prime}}{m_{6}}+\frac{a b^{\prime} c^{\prime}}{m_{4}}
\end{array}
$$

$f=\sum m(4,6,7) \quad f=\prod M(0,1,2,3,5)$
$f^{\prime}=\sum m(0,1,2,3,5) \quad f^{\prime}=\prod M(4,6,7)$

### 4.28

| $a$ | $b$ | $c$ | $d$ | $f$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

(a) $f=\sum m(1,2,4,5,6,11,12$, $14,15)$
(b) $f=\prod M(0,3,7,8,9,10,13)$
(c) $f^{\prime}=\sum m(0,3,7,8,9,10,13)$
(d) $f^{\prime}=\prod M(1,2,4,5,6,11,12$, 14, 15)

You can also work this problem algebraically, as in problem 4.27.

## Unit 4 Solutions

4.29 (a)

$$
\begin{aligned}
& f(A, B, C, D)=A B+A^{\prime} C D=A B C^{\prime} D^{\prime}+A B C^{\prime} D \\
& +A B C D^{\prime}+A B C D+A^{\prime} B^{\prime} C D+A^{\prime} B C D \\
& =\left(A+A^{\prime} C D\right)\left(B+A^{\prime} C D\right)=(A+C)(A+D)\left(A^{\prime}+B\right) \\
& (B+C)(B+D) \\
& f(A, B, C, D)=\left(A+B^{\prime}+C+D^{\prime}\right)\left(A+B^{\prime}+C+D\right) \\
& \left(A+B+C+D^{\prime}\right)(A+B+C+D)\left(A+B^{\prime}+C^{\prime}+D\right) \\
& \left(A+B^{\prime}+C+D\right)\left(A+B+C^{\prime}+D\right)(A+B+C+D) \\
& \left(A^{\prime}+B+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+C^{\prime}+D\right)\left(A+B+C+D^{\prime}\right) \\
& \left(A^{\prime}+B+C+D\right)\left(A^{\prime}+B+C+D^{\prime}\right)\left(A^{\prime}+B+C+D\right) \\
& \left(A+B+C+D^{\prime}\right)(A+B+C+D)\left(A^{\prime}+B+C^{\prime}+D\right) \\
& \left(A^{\prime}+B+C+D\right)\left(A+B+C^{\prime}+D\right)(A+B+C+D) \\
& =\left(A+B^{\prime}+C+D^{\prime}\right)\left(A+B+C+D^{\prime}\right)\left(A+B^{\prime}+C^{\prime}+D\right) \\
& \left(A+B^{\prime}+C+D\right)\left(A+B+C^{\prime}+D\right)(A+B+C+D) \\
& \left(A^{\prime}+B+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+C^{\prime}+D\right)\left(A^{\prime}+B+C+D^{\prime}\right) \\
& \left(A^{\prime}+B+C+D\right)
\end{aligned}
$$

Note: Consensus could have been applied twice to write $f=(A+C)(A+D)\left(A^{\prime}+B\right)$ and save some work.
4.30 (a) $F(A, B, C, D)=\sum m(3,4,5,8,9,10,11,12,14)$

$$
\begin{aligned}
F= & A^{\prime} B^{\prime} C D+A^{\prime} B C^{\prime} D^{\prime}+A^{\prime} B C^{\prime} D+A B^{\prime} C^{\prime} D^{\prime}+ \\
& A B^{\prime} C^{\prime} D+A B^{\prime} C D^{\prime}+A B^{\prime} C D+A B C^{\prime} D^{\prime}+A B C D^{\prime}
\end{aligned}
$$

4.29 (b) $f(A, B, C, D)=\left(A+B+D^{\prime}\right)\left(A^{\prime}+C\right)(C+D)$

$$
=\left(A+B+D^{\prime}\right)\left(A^{\prime} D+C\right)=A C+A^{\prime} B D+B C+C D^{\prime}
$$

$$
=A C\left(B+B^{\prime}\right)\left(D+D^{\prime}\right)+A^{\prime} B D\left(C+C^{\prime}\right)
$$

$$
+B C\left(A+A^{\prime}\right)\left(D+D^{\prime}\right)+\left(A+A^{\prime}\right)\left(B+B^{\prime}\right) C D^{\prime}
$$

$=A B C D+A B C D^{\prime}+A B^{\prime} C D+A B^{\prime} C D^{\prime}+A^{\prime} B C D$ $+A^{\prime} B C^{\prime} D+A B C D+A B C D^{\prime}+A B C D+A^{\prime} B C D^{\prime}$
$+A B C D^{\prime}+A B^{\prime} C D^{\prime}+A^{\prime} B C D^{\prime}+A^{\prime} B^{\prime} C D^{\prime}$
$=A B C D+A B C D^{\prime}+A B^{\prime} C D+A B^{\prime} C D^{\prime}+A^{\prime} B C D$ $+A^{\prime} B C^{\prime} D+A^{\prime} B C D^{\prime}+A^{\prime} B^{\prime} C D^{\prime}$
$f(A, B, C, D)=\left(A+B+C C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B B^{\prime}+C+D D^{\prime}\right)$ $\left(A A^{\prime}+B B^{\prime}+C+D\right)$
$=\left(A+B+C+D^{\prime}\right)\left(A+B+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+C+D\right)$ $\left(A^{\prime}+B+C+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C+D\right)\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right)$ $(A+B+C+D)\left(A+B^{\prime}+C+D\right)\left(A^{\prime}+B+C+D\right)$ $\left(A^{\prime}+B^{\prime}+C+D\right)$
$=\left(A+B+C+D^{\prime}\right)\left(A+B+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+C+D\right)$ $\left(A^{\prime}+B+C+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C+D\right)\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right)$ $(A+B+C+D)\left(A+B^{\prime}+C+D\right)$
4.30 (b) $F(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\prod M(0,1,2,6,7,13,15)$
$F=(A+B+C+D)\left(A+B+C+D^{\prime}\right)$
$\left(A+B+C^{\prime}+D\right)\left(A+B^{\prime}+C^{\prime}+D\right)$ $\left(A+B^{\prime}+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right)$ $\left(A^{\prime}+B^{\prime}+C^{\prime}+D^{\prime}\right)$
4.31 (a) $F(A, B, C, D)=\sum m(0,3,4,7,8,9,11,12,13,14)=\frac{A^{\prime} B^{\prime} C^{\prime} D^{\prime}}{m_{0}}+\frac{A^{\prime} B^{\prime} C D}{m_{3}}+\frac{A^{\prime} B C^{\prime} D^{\prime}}{m_{4}}+\frac{A^{\prime} B C D}{m_{7}}+\underline{A B^{\prime} C^{\prime} D^{\prime}}+\underline{m_{8}}+\frac{A B^{\prime} C^{\prime} D}{m_{9}}$

$$
+\frac{A B^{\prime} C D}{m_{11}}+\frac{A B C^{\prime} D^{\prime}}{m_{12}}+\frac{A B C^{\prime} D}{m_{13}}+\frac{A B C D^{\prime}}{m_{14}}
$$

4.31 (b) $F(A, B, C, D)=\prod M(1,2,5,6,10,15)=\left(\frac{A+B+C+D^{\prime}}{M_{1}}\right)\left(\frac{A+B+C^{\prime}+D}{M_{2}}\right)\left(\frac{A+B^{\prime}+C+D^{\prime}}{M_{5}}\right)\left(\frac{A+B^{\prime}+C^{\prime}+D}{M_{6}}\right)$

$$
\left.\frac{\left(A^{\prime}+B+C^{\prime}+D\right.}{M_{10}}\right)\left(\frac{A^{\prime}+B^{\prime}+C^{\prime}+D^{\prime}}{M_{15}}\right)
$$

4.32 (a) If don't cares are changed to ( 1,1 ), respectively,

$$
\begin{aligned}
F_{1} & =A^{\prime} B^{\prime} C^{\prime}+A B C+A^{\prime} B^{\prime} C+A B^{\prime} C \\
& =A^{\prime} B^{\prime}+A C
\end{aligned}
$$

4.32 (c) If don't cares are changed to ( 1,1 ), respectively

$$
F_{3}=(A+B+C)\left(A+B+C^{\prime}\right)=A+B
$$

4.33

| $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $Z$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | $\mathrm{X}^{2}$ | 0 |
| 0 | 0 | 1 | 0 | 1 | $\mathrm{X}^{2}$ | 1 |
| 0 | 1 | 0 | 0 | $\mathrm{X}^{2}$ | 1 | 1 |
| 0 | 1 | 1 | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | X |
| 1 | 0 | 0 | 0 | 1 | $\mathrm{X}^{2}$ | 1 |
| 1 | 0 | 1 | 0 | $\mathrm{X}^{2}$ | 1 | 1 |
| 1 | 1 | 0 | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | $\mathrm{X}^{1}$ | X |
| 1 | 1 | 1 | 1 | $\mathrm{X}^{2}$ | 1 | 0 |

${ }^{1}$ These truth table entries were made don't cares because $A B C=110$ and $A B C=011$ can never occur.
${ }^{2}$ These truth table entries were made don't cares because when one input of the OR gate is 1 , the output will be 1 regardless of the value of its other input.
4.34 (a) $\quad G_{1}(A, B, C)=\sum m(0,7)=\prod M(1,2,3,4,5,6)$
4.32 (b) If don't cares are changed to ( 1,0 ), respectively

$$
F_{2}=A^{\prime} B^{\prime} C^{\prime}+A^{\prime} B C^{\prime}+A B^{\prime} C^{\prime}+A B C^{\prime}=C^{\prime}
$$

4.32 (d) If don't cares are changed to ( 0,1 ), respectively

$$
\begin{aligned}
F_{4} & =A^{\prime} B^{\prime} C^{\prime}+A ' B C+A B^{\prime} C^{\prime}+A B C \\
& =B^{\prime} C^{\prime}+B C
\end{aligned}
$$

4.34 (b) $\quad G_{2}(A, B, C)=\sum m(0,1,6,7)=\prod M(2,3,4,5)$
4.35 (a)

| ABCD | 1's | X Y Z |
| :---: | :---: | :---: |
| 0000 | 0 | 000 |
| 0001 | 1 | 001 |
| 0010 | 1 | 001 |
| 0011 | 2 | 010 |
| 0100 | 1 | 001 |
| 0101 | 2 | 010 |
| 0110 | 2 | 010 |
| 0111 | 3 | 011 |
| 1000 | 1 | 001 |
| 1001 | 2 | 010 |
| 1010 | 2 | 010 |
| 1011 | 3 | 011 |
| 1100 | 2 | 010 |
| 1101 | 3 | 011 |
| 1110 | 3 | 011 |
| 1111 | 4 | 100 |

4.35 (b) $Y=(A+B+C+D)\left(A+B+C+D^{\prime}\right)$
$\left(A+B+C^{\prime}+D\right)\left(A+B^{\prime}+C+D\right)$
$\left(A^{\prime}+B+C+D\right)\left(A^{\prime}+B^{\prime}+C^{\prime}+D^{\prime}\right)$
$Z=(A+B+C+D)\left(A+B^{\prime}+C+D^{\prime}\right)$
$\left(A+B^{\prime}+C^{\prime}+D\right)\left(A^{\prime}+B+C+D^{\prime}\right)$
$\left(A^{\prime}+B+C^{\prime}+D\right)\left(A^{\prime}+B^{\prime}+C+D\right)$
$\left(A^{\prime}+B^{\prime}+C^{\prime}+D^{\prime}\right)$

### 4.37

| $A B C D$ |  | STUV | WX Y Z |
| :---: | :---: | :---: | :---: |
| 0000 | $0 \times 5=00$ | 0000 | 0000 |
| 0001 | $1 \times 5=05$ | 0000 | 0101 |
| 0010 | $2 \times 5=10$ | 0001 | 0000 |
| 0011 | $3 \times 5=15$ | 0001 | 0101 |
| 0100 | $4 \times 5=20$ | 0010 | 0000 |
| 0101 | $5 \times 5=25$ | 0010 | 0101 |
| 0110 | $6 \times 5=30$ | 0011 | 0000 |
| 0111 | $7 \times 5=35$ | 0011 | 0101 |
| 1000 | $8 \times 5=40$ | 0100 | 0000 |
| 1001 | $9 \times 5=45$ | 0100 | 0101 |

Note: Rows 1010 through 1111 have don't care outputs.
$S=0, T=A, U=B, V=C, W=0, X=D, Y=0$, $Z=D$
4.36 (a)

| A B C D | WX Y Z | $X=A^{\prime} B^{\prime} C^{\prime} D+A^{\prime} B^{\prime} C D^{\prime}$ |
| :---: | :---: | :---: |
| 0000 | 0011 | $+A^{\prime} B C^{\prime} D+A^{\prime} B C D$ |
| 0001 | 0100 | $+A^{\prime} B C D+A B^{\prime} C^{\prime} D^{\prime}$ |
| 0010 | 0100 | $+A B^{\prime} C^{\prime} D+A B^{\prime} C D^{\prime}$ |
| 0011 | 0101 | $+A B^{\prime} C D+A B C^{\prime} D^{\prime}$ |
| 0100 | 0100 | $+A B C D$ |
| 0101 | 0101 | $\begin{aligned} Y= & A^{\prime} B^{\prime} C^{\prime} D^{\prime}+A^{\prime} B C D+ \\ & A B C^{\prime} D+A B C D^{\prime}+ \\ & A B C D \end{aligned}$ |
| 0110 | 0101 |  |
| 0111 | 0110 |  |
| 1000 | 0100 |  |
| 1001 | 0101 | $\begin{aligned} Z= & A^{\prime} B^{\prime} C^{\prime} D^{\prime}+A^{\prime} B^{\prime} C D+ \\ & A^{\prime} B C^{\prime} D+A^{\prime} B C D^{\prime}+ \\ & A B^{\prime} C^{\prime} D+A B^{\prime} C D^{\prime}+ \\ & A B^{\prime} C D+A B C^{\prime} D^{\prime}+ \\ & A B C D \end{aligned}$ |
| 1010 | 0101 |  |
| 1011 | 0110 |  |
| 1100 | 0101 |  |
| 1101 | 0110 |  |
| 1110 | 0110 |  |
| 1111 | 0111 |  |

4.36 (b) $Y=\left(A+B+C+D^{\prime}\right)\left(A+B+C^{\prime}+D\right)$
$\left(A+B+C^{\prime}+D^{\prime}\right)\left(A+B^{\prime}+C+D\right)$
$\left(A+B^{\prime}+C+D^{\prime}\right)\left(A+B^{\prime}+C^{\prime}+D\right)$
$\left(A^{\prime}+B+C+D\right)\left(A^{\prime}+B+C+D^{\prime}\right)$
$\left(A^{\prime}+B+C^{\prime}+D\right)\left(A^{\prime}+B+C^{\prime}+D^{\prime}\right)$
$\left(A^{\prime}+B^{\prime}+C+D\right)$
$Z=\left(A+B+C+D^{\prime}\right)\left(A+B+C^{\prime}+D\right)$
$\left(A+B^{\prime}+C+D\right)\left(A+B^{\prime}+C^{\prime}+D\right)$
$\left(A^{\prime}+B+C+D\right)\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right)$
$\left(A^{\prime}+B^{\prime}+C^{\prime}+D\right)$
4.38

| A B C D |  | STUV | WX Y Z |
| :---: | :---: | :---: | :---: |
| 0000 | $0 \times 4+1=01$ | 0000 | 0001 |
| 0001 | $1 \times 4+1=05$ | 0000 | 0101 |
| 0010 | $2 \times 4+1=09$ | 0000 | 1001 |
| 0011 | $3 \times 4+1=13$ | 0001 | 0011 |
| 0100 | $4 \times 4+1=17$ | 0001 | 0111 |
| 0101 | $5 \times 4+1=21$ | 0010 | 0001 |
| 0110 | $6 \times 4+1=25$ | 0010 | 0101 |
| 0111 | $7 \times 4+1=29$ | 0010 | 1001 |
| 1000 | $8 \times 4+1=33$ | 0011 | 0011 |
| 1001 | $9 \times 4+1=37$ | 0011 | 0111 |

Note: Rows 1010 through 1111 have don't care outputs.
$S=0, T=0, U=B D+B C+A$,
$V=B^{\prime} C D+B C^{\prime} D^{\prime}+A, W=B^{\prime} C D^{\prime}+B C D$,
$X=B^{\prime} C^{\prime} D+B D^{\prime}, Y=B^{\prime} C D+B C^{\prime} D^{\prime}+A, Z=1$

## Unit 4 Solutions

Notice that the sign bit $X_{3}$ of the 4-bit number is extended to the leftmost full adder as well.

4.40

| XY | Sum Cout |  |
| :---: | :---: | :---: |
| 00 | 0 | 0 |
| 01 | 1 | 0 |
| 10 | 1 | 0 |
| 11 | 0 | 1 |


4.41 (d), (d)
(e)

| $x y$ | $f$ | xy | $f$ |
| :---: | :---: | :---: | :---: |
| 00 | 0 | $a 0$ | $a$ |
| 01 | $b$ | a 1 | 1 |
| 0 a | 0 | $a \mathrm{a}$ | $a$ |
| 0 b | $b$ | $a b$ | 1 |
| 10 | $a$ | b 0 | 0 |
| 11 | 1 | b 1 | $b$ |
| $1 a$ | $a$ | $b a$ | 0 |
| 1 b | 1 | $b b$ | $b$ |

(e)
$f(x, y)$ is completely specified by the coefficients of the minterms in the sum of minterms expression.
These coefficients are
determined by the value
of the function for $\mathrm{xy}=$
$00,01,10$ and 11.
4.41 (a), (a)
(b), (c) $f=x\left(y+y^{\prime}\right)+y\left(x+x^{\prime}\right)=x y+x y^{\prime}+x^{\prime} y$
(sum-of-minterms)
$f=x+y$ already in product-of-maxterms form
(b)

$$
\begin{aligned}
f=a x & +b y=a x\left(y+y^{\prime}\right)+b y\left(x+x^{\prime}\right) \\
& =a x y+a x y^{\prime}+b x y+b x^{\prime} y=(a+b) x y+a x y^{\prime}+b x^{\prime} y \\
& =x y+a x y^{\prime}+b x^{\prime} y
\end{aligned}
$$

(c)

$$
\begin{aligned}
& f^{\prime}=\left(a++x^{\prime}\right)\left(b^{\prime}+y^{\prime}\right)=\left(b+x^{\prime}\right)\left(a+y^{\prime}\right) \\
&=a b+a x^{\prime}+b y^{\prime}+x^{\prime} y^{\prime}=a x^{\prime}\left(y+y^{\prime}\right)+b y^{\prime}\left(x+x^{\prime}\right)+x^{\prime} y^{\prime} \\
&=a x^{\prime} y+a x^{\prime} y^{\prime}+b y^{\prime} x+b y^{\prime} x^{\prime}+x^{\prime} y^{\prime} \\
&= a x^{\prime} y+b y^{\prime} x+x^{\prime} y^{\prime}(a+b+1)=a x^{\prime} y+b y^{\prime} x^{\prime}+x^{\prime} y^{\prime} \text { so } \\
& f=\left(a^{\prime}+x+y^{\prime}\right)\left(b^{\prime}+x^{\prime}+y\right)(x+y) \\
&=\left(b+x+y^{\prime}\right)\left(a+x^{\prime}+y\right)(x+y)
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
f & =a x+b y=(a+b y)(x+b y)=(a+b)(a+y)(x+b)(x+y) \\
& =\left(a+x x^{\prime}+y\right)\left(b+y y^{\prime}+x\right)(x+y) \\
& =(a+x+y)\left(a+x^{\prime}+y\right)(b+x+y)\left(b+x+y^{\prime}\right)(x+y) \\
& =[(a+x+y)(b+x+y)(x+y)]\left(a+x^{\prime}+y\right)\left(b+x+y^{\prime}\right) \\
& =(a b+x+y)\left(a+x^{\prime}+y\right)\left(b+x+y^{\prime}\right) \\
& =(x+y)\left(a+x^{\prime}+y\right)\left(b+x+y^{\prime}\right)
\end{aligned}
$$

4.42
(a) $m_{1}+m_{2}=m_{1}\left(m_{2}{ }^{\prime}+m_{2}\right)+\left(m_{1}{ }^{\prime}+m_{1}\right) m_{2}$ $=m_{1} m_{2}{ }^{\prime}+m_{1} m_{2}+m_{1}{ }^{\prime} m_{2}$
But $m_{1} m_{2}=0$, so $m_{1}+m_{2}=m_{1} m_{2}{ }^{\prime}+m_{1}{ }^{\prime} m_{2}$ $=m_{1} \oplus m_{2}$.
(b) Using part (a), any function can be written as the exclusive-or sum of its minterms. However, if a product contains a complemented literal, it can be written as the exclusive-or sum of two products without a complemented literal by using

$$
x^{\prime} p=(x \oplus 1) p=x p \oplus p
$$

By repeated application of the preceding relationship, all complemented literals can be removed from the products.

## Unit 5 Problem Solutions

5.3 (a)

5.3 (b)

$\mathrm{f}_{2}=\mathrm{d}^{\prime} \mathrm{e}^{\prime}+\mathrm{d}^{\prime} \mathrm{f}$ ' $+\mathrm{e}^{\prime} \mathrm{f}$ '
5.3 (c)

5.4 (b)

5.3 (d)

$f_{4}=x^{\prime} z+y+x z^{\prime}$
5.4 (a)

5.4 (c)

$\mathrm{F}=\left(\mathrm{A}+\mathrm{B}^{\prime}+\mathrm{D}^{\prime}\right)\left(\mathrm{B}+\mathrm{C}+\mathrm{D}^{\prime}\right)$
5.5 (a) See FLD p. 697 for solution.
5.5 (b)

5.6 (a)

$\mathrm{f}=\mathrm{a}^{\prime} \mathrm{b}^{\prime} \mathrm{c}^{\prime}+\mathrm{a}^{\prime} d+\mathrm{b}^{\prime} \mathrm{cd}+\mathrm{abd}{ }^{\prime}+\mathrm{bcd}$
Alt: $f=\underline{a b b^{\prime} c c^{\prime}}+\underline{a^{\prime} d}+\underline{b^{\prime} c d}+\underline{a b d}+a^{\prime} b c$
(*) Indicates a minterm that makes the corresponding prime implicant essential.
$\mathrm{a}^{\prime} \mathrm{d} \rightarrow \mathrm{m}_{5} ; \mathrm{a}^{\prime} \mathrm{b}^{\prime} \mathrm{c}^{\prime} \rightarrow \mathrm{m}_{0} ;$ b'cd $\rightarrow \mathrm{m}_{11} ;$ abd' $\rightarrow \mathrm{m}_{12}$

(*) Indicates a minterm that makes the corresponding prime implicant essential.
$\mathrm{bd} \rightarrow \mathrm{m}_{13}$ or $\mathrm{m}_{15} ;$ a'c $^{\prime} \rightarrow \mathrm{m}_{3} ;$ b'd' $^{\prime} \rightarrow \mathrm{m}_{8}$ or $\mathrm{m}_{10}$

## Unit 5 Solutions

5.6 (c)

$\mathrm{F}=\underline{\mathrm{a}^{\prime} \mathrm{d}^{\prime}}+\underline{\mathrm{b}^{\prime}}+\underline{\mathrm{c}^{\prime} \mathrm{d}^{\prime}}$
5.7 (a)

$\mathrm{f}=\mathrm{a}^{\prime} \mathrm{c}^{\prime} \mathrm{d}^{\prime}+\mathrm{a}$ 'cd + b'c'd' + abcd' + a'b'd' Alt: f = a'c'd' + a'cd + b'c'd' + abcd' + a'b'c
(*) Indicates a minterm that makes the corresponding prime implicant essential.
$c^{\prime} d^{\prime} \rightarrow \mathrm{m}_{12} ; \mathrm{a}^{\prime} \mathrm{d}^{\prime} \rightarrow \mathrm{m}_{6} ; \mathrm{b}^{\prime} \rightarrow \mathrm{m}_{10}$ or $\mathrm{m}_{11}$
5.7 (b)

$\mathrm{f}=\mathrm{a} \mathrm{b}^{\prime}+\mathrm{a}^{\prime} \mathrm{c}^{\prime} \mathrm{d}^{\prime}+\mathrm{abc}$
5.7 (c)

$\mathrm{f}=\mathrm{b}^{\prime} \mathrm{c}^{\prime} \mathrm{d}^{\prime}+\mathrm{ab} \mathbf{c}+\mathrm{a} \mathrm{bc}+\mathrm{bc} \mathrm{d}^{\prime}+\mathrm{ad}{ }^{\prime}$
5.7 (d)

5.8 (a)

$f=\left(c^{\prime}+d^{\prime}\right)\left(b^{\prime}+c^{\prime}\right)(a+b+c)\left(a^{\prime}+c+d\right)$
5.8 (b)


$f=a^{\prime} b c^{\prime}+a c^{\prime} d+b^{\prime} c d^{\prime}$
5.9 (a)

$\mathrm{F}=\left(\mathrm{A}^{\prime}+\mathrm{B}^{\prime}+\mathrm{C}+\mathrm{E}\right)\left(\mathrm{A}^{\prime}+\mathrm{B}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}+\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{E}\right)$
$\left(B^{\prime}+D+E\right)\left(A+C^{\prime}+D\right)\left(A^{\prime}+C+D+E^{\prime}\right)\left(A^{\prime}+B^{\prime}+C^{\prime}+E^{\prime}\right)$

$F=A^{\prime} C^{\prime} E+A^{\prime} C^{\prime} D+A^{\prime} D E+A B^{\prime} C D^{\prime}+C^{\prime} D E$

+ A B C D E' + B'C'E' + A'B'D
Alt: $F=A^{\prime} C^{\prime} E+A^{\prime} C^{\prime} D+A^{\prime} D E+A B^{\prime} C D^{\prime}+C^{\prime} D E$
+ A B CDE' + $\mathrm{B}^{\prime} \mathrm{C}^{\prime} \mathrm{E}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{E}^{\prime}$
5.9 (b)


$$
A l t:\left\{\begin{array}{l}
F=A^{\prime} C D^{\prime}+A E^{\prime} B+C D E+A B^{\prime} C^{\prime} D^{\prime}+A B^{\prime} D E^{\prime}+B^{\prime} C E \\
F=A^{\prime} C D^{\prime}+A^{\prime} B E^{\prime}+C D E+A B^{\prime} E^{\prime}+C D E+A B^{\prime} C D+B^{\prime} D^{\prime} E \\
F=A^{\prime} C D^{\prime}+A^{\prime} B E^{\prime}+C D E B^{\prime} D E^{\prime}+B^{\prime} D^{\prime} E \\
\end{array}\right.
$$

5.10 (a)


Essential prime implicants: $c^{\prime} d^{\prime} e^{\prime}\left(m_{16}, m_{24}\right)$, $a^{\prime} c e^{\prime}\left(m_{14}\right)$, ace $\left(m_{31}\right)$, a'b'de $\left(m_{3}\right)$


Prime implicants: a'b'de, a'd'e', cd'e, a'ce', ace, $a^{\prime} b^{\prime} c, b^{\prime} c e, c^{\prime} d^{\prime} e^{\prime}, a^{\prime} c d^{\prime}$

## Unit 5 Solutions

5.11

$f=\left(\underline{a^{\prime}+b+c^{\prime}}\right) \underline{\left(a^{\prime}+d^{\prime}+e\right)} \underline{\left(a+b^{\prime}+e^{\prime}\right)} \underline{\left(a+c+e^{\prime}\right)}$

$$
(a+b+c+d)\left(a^{\prime}+b^{\prime}+c+d\right)\left(c+d+e^{\prime}\right)
$$

Alt: $f=\left(\underline{\left(a^{\prime}+b+c^{\prime}\right)} \underline{\left(a^{\prime}+d^{\prime}+e\right)} \underline{\left(a+b^{\prime}+e^{\prime}\right)} \underline{\left(a+c+e^{\prime}\right)}\right.$
$(\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d})\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}+\mathrm{c}+\mathrm{e}\right)\left(\mathrm{c}+\mathrm{d}+\mathrm{e}^{\prime}\right)$
5.12 (b)

$\mathrm{F}^{\prime}=\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}+\mathrm{AB} \mathrm{D}^{\prime}+\mathrm{A} \mathrm{C}^{\prime} \mathrm{D}$
5.12 (c)

$F=\left(A^{\prime}+B^{\prime}+D\right)(A+B+C)\left(A^{\prime}+C+D^{\prime}\right)$
5.12 (a)

$\mathrm{F}=\mathrm{A} \mathrm{B}^{\prime} \mathrm{D}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{A}^{\prime} \mathrm{C}+\mathrm{CD}$

$$
\mathrm{F}=\Pi \mathrm{M}(0,1,9,12,13,14)=(A+B+C+D)
$$

$$
\left(A+B+C+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C+D\right)
$$

$$
\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C^{\prime}+D\right)
$$

$$
\left(A^{\prime}+B+C+D^{\prime}\right)
$$

5.13

Minterms $m_{0}, m_{1}, m_{2}, m_{3}, m_{4}, m_{10}$, and $m_{11}$ can be made don't cares, individually, without changing the given expression. However, if $m_{13}$ or $m_{14}$ is made a don't care, the term BC'D or the term ACD' (respectively) is not needed in the expression.
5.14 (c)

5.14 (d)


$$
\begin{aligned}
& F=A C D^{\prime}+B C^{\prime} D+B^{\prime} C+A^{\prime} C^{\prime}
\end{aligned}
$$

Unit 5 Solutions


| 5.17 (c) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{l\|llll}  \\ \mathrm{C} \mathrm{D} \mathrm{~B} & \mathrm{~A} \\ 00 & 01 & 11 & 10 \end{array}$ |  |  |  |  |
| 00 | 1 | 0 | 0 | 0 |
| 01 | 1 | 0 | 0 | 0 |
| 11 | 1 | 0 | 1 | 0 |
| 10 | 1 | 1 | 1 | 1 |

$\mathrm{F}=\left(\mathrm{B}^{\prime}+\mathrm{C}\right)\left(\mathrm{A}+\mathrm{B}^{\prime}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}\right)\left(\mathrm{A}^{\prime}+\mathrm{B}+\mathrm{D}^{\prime}\right)$
5.18 (a) \& (b)

$F=A^{\prime}+C^{\prime} D+B^{\prime} C D^{\prime}$
5.18 (c)

$\mathrm{F}=\left(\mathrm{A}^{\prime}+\mathrm{C}+\mathrm{D}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{B}^{\prime}+\mathrm{D}\right)$
Alt: $\mathrm{F}=\left(\mathrm{A}^{\prime}+\mathrm{C}+\mathrm{D}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{B}^{\prime}+\mathrm{C}^{\prime}\right)$

5.19 (a) | $C_{1} C_{2} X_{1} X_{2}$ | $Z$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$
5.19 \text { (b) }
$$


$F=\left(C_{1}+C_{2}+X_{1}\right)\left(C_{1}+X_{1}+X_{2}\right)\left(C_{1}+C_{2}^{\prime}+X_{1}^{\prime}+X_{2}^{\prime}\right)$
$\left(\mathrm{C}_{1}{ }^{\prime}+\mathrm{C}_{2}^{\prime}+\mathrm{X}_{1}+\mathrm{X}_{2}^{\prime}\right)\left(\mathrm{C}_{1}^{\prime}+\mathrm{X}_{1}^{\prime}+\mathrm{X}_{2}\right)\left\{\begin{array}{c}\left(\mathrm{C}_{2}+\mathrm{C}_{2}+\mathrm{X}_{2}\right) \\ \left(\mathrm{C}_{2}+\mathrm{X}_{1}{ }^{\prime}+\mathrm{X}_{2}\right)\end{array}\right\}$

5.20 (c)

5.20 (d)

5.20 (e)


$$
\begin{aligned}
f & =a^{\prime} b^{\prime}+a b+b ' c \text { or } \\
& =a^{\prime} b^{\prime}+a b+a c
\end{aligned}
$$

5.20 (f)

$G=D E F^{\prime}+D^{\prime} E^{\prime}$
$G=D E F^{\prime}+D^{\prime} F$
$G=D E F^{\prime}+E^{\prime} F$

$$
\begin{aligned}
& 5.21 \\
& \\
& F=a^{\prime} b^{\prime} c^{\prime}+a^{\prime} c^{\prime} d+b c d+a b c+a b b^{\prime} \\
& =\left(a^{\prime} b^{\prime} c^{\prime}+a b^{\prime}\right)+a^{\prime} c^{\prime} d+b c d+\left(a b c+a b^{\prime}\right) \\
& =\left(a^{\prime} c^{\prime}+a\right) b^{\prime}+\left(a^{\prime} c^{\prime} d+b c d\right)+a\left(b c+b b^{\prime}\right) \\
& =\left(c^{\prime}+a\right) b^{\prime}+\left(a^{\prime} c^{\prime} d+b c d+a^{\prime} b d\right)+a\left(c+b^{\prime}\right) \\
& =\left(b^{\prime} c^{\prime}+a^{\prime} b d+a^{\prime} c^{\prime} d\right)+\left(b c d+a^{\prime} b d+a c\right)+a b{ }^{\prime} \\
& =\left(b^{\prime} c^{\prime}+a c+a b^{\prime}\right)+a^{\prime} b d \\
& =b^{\prime} c^{\prime}+\mathrm{ac}+\mathrm{a} \text { 'bd }
\end{aligned}
$$

5.22 (a)


PIs: A B', B C', A D', B D', A C', A' B

$$
\begin{aligned}
\mathrm{f} & =\mathrm{A} \mathrm{~B}^{\prime}+\mathrm{B} \mathrm{D}^{\prime}+\mathrm{A} \mathrm{C}^{\prime} \text { or } \\
& =A \mathrm{~B}^{\prime}+\mathrm{B} \mathrm{C}^{\prime}+\mathrm{B} \mathrm{D}^{\prime} \text { or } \\
& =\mathrm{A} \mathrm{~B}^{\prime}+\mathrm{B} \mathrm{C}^{\prime}+\mathrm{A} \mathrm{D}^{\prime}
\end{aligned}
$$

### 5.22 (d)



PIs: A'B, B C D, A B'C', A B'D'

$$
\mathrm{f}=\mathrm{A}^{\prime} \mathrm{B}+\mathrm{BCD}
$$

### 5.22 (g)

AB

| C D |  | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | $X$ | 1 |  | $X$ |
|  |  | $X$ | $X$ | $X$ |  |
| 11 |  | $X$ | 1 |  |  |
|  |  |  |  |  |  |
|  | 10 | $X$ | $X$ |  | $X$ |

PIs: B C D, $A^{\prime} C^{\prime}, A^{\prime} D^{\prime}, A^{\prime} B, B^{\prime} D^{\prime}, B^{\prime} C^{\prime}$
$\mathrm{f}=\mathrm{BCD}+\mathrm{A}^{\prime} \mathrm{B}$ or
$\mathrm{f}=\mathrm{BCD}+\mathrm{A}^{\prime} \mathrm{D}^{\prime}$ or
$\mathrm{f}=\mathrm{BCD}+\mathrm{A}^{\prime} \mathrm{C}^{\prime}$

### 5.23 (c)



PIs: $(\mathrm{B}+\mathrm{C}+\mathrm{D}),\left(\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right),\left(\mathrm{A}^{\prime}+\mathrm{C}+\mathrm{D}\right)$,
$\left(A^{\prime}+B\right),\left(A+B^{\prime}+D^{\prime}\right),\left(A+B^{\prime}+C^{\prime}\right)$
$f=(B+C+D)\left(C^{\prime}+D^{\prime}\right)\left(A^{\prime}+C+D\right)$
5.22 (b)


PIs: B'C D, A C', A D', A B', B C'D, B C D', A'C D, A'B D, A'B C $f=B^{\prime} C D+A C^{\prime}+A D^{\prime}$
5.22 (e)


PIs: C D, A'B, A B'
$f=C D+A^{\prime} B$
5.23 (a)

| A B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 |  |  | X |
| 01 | 0 | X |  | X |
| 11 | 0 | X | 0 |  |
| 10 | 0 | X |  | X |

PIs: ( $\left.\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right),(\mathrm{A}+\mathrm{B})$,
$\left(A+D^{\prime}\right),\left(A+C^{\prime}\right)$
$f=\left(B^{\prime}+C^{\prime}+D^{\prime}\right)(A+B)$

| 5.23 (b) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | A B |  |  |  |
| C D | 00 | 01 | 11 | 10 |
|  | 0 | 0 |  | X |
| 01 | 0 | X |  | X |
| 11 |  | ( X | 0 |  |
| 10 | 0 | X |  | X |

PIs: $\left(B^{\prime}+C^{\prime}+D^{\prime}\right),(A+C),(A+D)$,

$$
(\mathrm{B}+\mathrm{D}),(\mathrm{B}+\mathrm{C}),\left(\mathrm{A}+\mathrm{B}^{\prime}\right)
$$

$$
\mathrm{f}=\left(\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)(\mathrm{A}+\mathrm{D})(\mathrm{B}+\mathrm{C}) \text { or }
$$

$$
=\left(\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)(\mathrm{A}+\mathrm{C})(\mathrm{B}+\mathrm{D}) \text { or }
$$

$$
=\left(\mathrm{B}^{\prime}+\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)(\mathrm{A}+\mathrm{C})(\mathrm{A}+\mathrm{D})
$$

5.23 (d)

\[

\]

PIs: ( B$),\left(\mathrm{A}^{\prime}+\mathrm{C}\right),\left(\mathrm{A}^{\prime}+\mathrm{D}\right),\left(\mathrm{C}+\mathrm{D}^{\prime}\right)$, $\left(C^{\prime}+D\right),\left(A+D^{\prime}\right),\left(B+D^{\prime}\right),\left(A+C^{\prime}\right)$
$\mathrm{f}=(\mathrm{B})\left(\mathrm{C}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{D}\right)$ or
$=(\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{C}\right)\left(\mathrm{C}^{\prime}+\mathrm{D}\right)$ or
$=(\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{C}\right)\left(\mathrm{A}^{\prime}+\mathrm{D}\right)$

## Unit 5 Solutions

PIs: $\left(\mathrm{C}+\mathrm{D}^{\prime}\right),\left(\mathrm{A}^{\prime}+\mathrm{D}\right),(\mathrm{B}+\mathrm{D}),\left(\mathrm{A}^{\prime}+\mathrm{C}\right)$,
PIs: (B), (D'), (A'), (C'
$(B+C),\left(C^{\prime}+D\right),\left(A+B^{\prime}+D^{\prime}\right),\left(A+B^{\prime}+C^{\prime}\right)$

$$
\begin{aligned}
\mathrm{f} & =\left(\mathrm{A}^{\prime}+\mathrm{C}\right)(\mathrm{B}+\mathrm{C})\left(\mathrm{C}^{\prime}+\mathrm{D}\right) \text { or } \\
& =\left(\mathrm{C}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{D}\right)(\mathrm{B}+\mathrm{D})
\end{aligned}
$$



Alt: $F=A B C '+B^{\prime} C D+A^{\prime} C+A^{\prime} B^{\prime} D^{\prime}+B C^{\prime} D$
5.24 (b)
C D

$F=A^{\prime} C^{\prime} D^{\prime}+B^{\prime} C^{\prime} D^{\prime}+A^{\prime} B^{\prime} D^{\prime}$
Alt: F = A'C'D' + B'C'D' + A'B'C
5.24 (b)

$$
\begin{aligned}
\mathrm{f} & =(\mathrm{B})\left(\mathrm{A}^{\prime}\right) \text { or } \\
& =(\mathrm{B})\left(\mathrm{C}^{\prime}\right) \text { or } \\
& =(\mathrm{B})\left(\mathrm{D}^{\prime}\right)
\end{aligned}
$$

\[

\]

PIs: (B), (D'), ( $\mathrm{A}^{\prime}$ ), ( $\mathrm{C}^{\prime}$ )

Alt: $\mathrm{F}=\mathrm{A}^{\prime} \mathrm{C}^{\prime} \mathrm{D}^{\prime}+\mathrm{B}^{\prime} \mathrm{C}^{\prime} \mathrm{D}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}$
5.24 (e)

$\mathrm{F}=\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{CD}+\mathrm{BD}^{\prime}+\mathrm{AD}^{\prime}+\mathrm{AB}$

$\mathrm{f}=\mathrm{b}^{\prime} \mathrm{c}^{\prime} \mathrm{d}+\mathrm{cd}$ ' +bd ' $+\mathrm{bc}+\mathrm{ab}$

Alt: $\left\{\begin{array}{l}f=x^{\prime} y^{\prime}+w y^{\prime}+w^{\prime} z+w z^{\prime} \\ f=x^{\prime} y^{\prime}+w y^{\prime}+w^{\prime} z+w x^{\prime}\end{array}\right.$

$f=x^{\prime} y^{\prime}+w^{\prime} z+y^{\prime} z+w z^{\prime}$

$$
\begin{aligned}
& 5.23 \text { (e) } \\
&
\end{aligned}
$$

5.23 (g)

\[

\]

PIs: $(\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{D}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}\right)\left(\mathrm{A}+\mathrm{C}^{\prime}\right)$ $\left(\mathrm{C}^{\prime}+\mathrm{D}\right)\left(\mathrm{C}+\mathrm{D}^{\prime}\right)\left(\mathrm{A}+\mathrm{D}^{\prime}\right)$

$$
\begin{aligned}
\mathrm{f} & =(\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{C}\right)\left(\mathrm{C}^{\prime}+\mathrm{D}\right) \text { or } \\
& =(\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{D}\right)\left(\mathrm{C}+\mathrm{D}^{\prime}\right) \text { or } \\
& =(\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{D}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}\right)
\end{aligned}
$$

### 5.24 (c)

| A B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 |  | X |  | X |
| 01 | 1 | 1 | $1)$ |  |
| 11 |  | 1 |  |  |
| 10 |  | 1 |  |  |

$$
F=A^{\prime} C^{\prime} D+A^{\prime} B+B C^{\prime} D
$$

5.25 (a)
cd
$\mathrm{f}=\mathrm{a}$ 'd + a'bc' + c'd + bd


5.26 (b)

Alt: $F=\left(\underline{B^{\prime}+C}\right) \underline{\left(A^{\prime}+B+C^{\prime}\right)} \underline{(A+D)}\left(B^{\prime}+D\right)$
5.27 (a)

5.27 (b)


F $=b^{\prime} d^{\prime}+a^{\prime} d+c^{\prime} d$
Notice that $a b c d=0101$ and 1111 never occur, so minterms 5 and 15 are don't cares.


$$
\mathrm{f}=\mathrm{C}^{\prime} \mathrm{D}+\mathrm{AB}^{\prime} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{A}^{\prime} \mathrm{D}
$$


$f=\left(w+x^{\prime}+z\right)\left(w+y^{\prime}+z\right)\left(w^{\prime}+y^{\prime}+z^{\prime}\right)$
Alt: $f=\left(w+x^{\prime}+z\right)\left(w+y^{\prime}+z\right)\left(w^{\prime}+x^{\prime}+y^{\prime}\right)$

|  |
| :--- | :--- | :--- | :--- | :--- | :--- |

$F=A B^{\prime} D^{\prime}+A^{\prime} B+A^{\prime} C+C D$
$\mathrm{F}=\Pi \mathrm{M}(0,1,9,12,13,14)$

$$
=(A+B+C+D)\left(A+B+C+D^{\prime}\right)
$$

$$
\left(A^{\prime}+B+C+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C+D\right)
$$

$$
\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C^{\prime}+D\right)
$$

5.29 (b)

$F^{\prime}=A B D^{\prime}+A^{\prime} B^{\prime} C^{\prime}+A C^{\prime} D$

## Unit 5 Solutions

5.29 (c)

$F=\left(A^{\prime}+B^{\prime}+D\right)(A+B+C)\left(A^{\prime}+C+D^{\prime}\right)$
5.31 Prime implicants for f ': abc'e, ac'd', ab'e', a'ce, b'c'de', c'd'e, a'd'e

Prime implicants for f: a'd'e', ace, a'ce', bde', abc, bce', b'c'de, a'c'de, a'bc'd, ab'de
5.34 (a)

| ab |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | X | 1 | x |  |
| 01 |  | X |  | X |
| 11 | X | 1 | X | 1 |
| 10 | 1 | X | 1 | X |

5-variable mirror image map


Essential PIs: ab'c'e', a'bc'e, a'b'cd

Other PIs: ab'd', b cd'e', bc'd'e, a'bd'e, b'cde
5.30

5.32 For F: b'c'de', a'ce, ab'e', ac'd', abc'e, c'd'e, a'd'e For G: ab'ce, a'bcd, a'bde', cde, b'de, a'b'c'd, a'c'e'

5-variable diagonal map



PIs: $\left(\mathrm{c}+\mathrm{d}^{\prime}\right),(\mathrm{a}+\mathrm{c}),(\mathrm{b}+\mathrm{c}),\left(\mathrm{a}+\mathrm{b}+\mathrm{d}^{\prime}\right)$, $\left(a+b^{\prime}+c^{\prime}+d\right),\left(a^{\prime}+b^{\prime}+d^{\prime}\right),\left(a^{\prime}+b+d\right)$
$f=\left(c+d^{\prime}\right)\left(a^{\prime}+c\right)$ or
$=(b+c)\left(c+d^{\prime}\right)$ or $=(b+c)\left(a^{\prime}+c\right)$
5.35 (a), 5-variable mirror image map
(b) \&
(c)


PIs: A, C'D', B'E, C E, B D', D'E, B C'E', B'C D $\mathrm{F}=\mathrm{A}+\mathrm{B}^{\prime} \mathrm{E}+\mathrm{BD}^{\prime}$ or
$=A+C^{\prime} D^{\prime}+C E$
5.35 (d), 5-variable mirror image map
\& (e)


PIs: $\left(A^{\prime}+B^{\prime}\right),\left(A^{\prime}+C^{\prime}\right),\left(A^{\prime}+D+E^{\prime}\right),\left(B^{\prime}+D^{\prime}\right),\left(B^{\prime}+C+E^{\prime}\right)$,
$\left(C^{\prime}+E\right),\left(D^{\prime}+E\right),\left(B+C^{\prime}+D\right),\left(A+C+D^{\prime}\right),(A+B+E)$
$F=\left(B^{\prime}+D^{\prime}\right)(A+B+E)$ or
$=\left(C^{\prime}+E\right)\left(A+C+D^{\prime}\right)$
5.36 (a), 5-variable mirror image map
(b) \&
(c)


PIs: A B, A D, A C'E, B C, B D'E, C D, D E', C E', $A^{\prime} C, B^{\prime} D, C^{\prime} D^{\prime} E, A^{\prime} D^{\prime} E, B^{\prime} C^{\prime} E, A^{\prime} B^{\prime}$

$$
\begin{aligned}
\mathrm{F} & =\mathrm{A}^{\prime} \mathrm{C}+\mathrm{B}^{\prime} \mathrm{D}+\mathrm{AB} \text { or } \\
& =\mathrm{B}^{\prime} \mathrm{D}+\mathrm{AB}+\mathrm{BC} \text { or }
\end{aligned}
$$

$$
=A^{\prime} \mathrm{C}+\mathrm{AB}+\mathrm{AD}
$$

5.36 (d), 5-variable mirror image map
(e)


PIs: $\left(A^{\prime}+B^{\prime}+C^{\prime}\right),\left(A^{\prime}+B^{\prime}+D^{\prime}\right),\left(A^{\prime}+B^{\prime}+E\right),\left(A^{\prime}+C^{\prime}+E^{\prime}\right)$,
$\left(A^{\prime}+C^{\prime}+D\right),\left(B^{\prime}+D^{\prime}+E^{\prime}\right),\left(B^{\prime}+C+D^{\prime}\right),(D+E)$,
$(A+B+E),(A+C),(B+D),(C+E)$
$\mathrm{F}=(\mathrm{A}+\mathrm{C})(\mathrm{B}+\mathrm{D})$

5-variable diagonal map


5-variable diagonal map


5-variable diagonal map


5-variable diagonal map


## Unit 5 Solutions

5.37 (a),
(b) \&
(c)


PIs: A B C D E', B C D'E, A C'D E, A B'D'E', A B'C',
A'B C'E', A'B D', A'B'C D, A'D'E, A'C E, B'D E, A'B'E,
$B^{\prime} C ' E, C^{\prime} D^{\prime} E^{\prime}, A^{\prime} C^{\prime} D^{\prime}, B^{\prime} C^{\prime} D^{\prime}$
$F=A^{\prime} B^{\prime} E+A^{\prime} B D^{\prime}+A B^{\prime} C^{\prime}$ or
$=A^{\prime} C^{\prime} D^{\prime}+A^{\prime} C E+A B^{\prime} C^{\prime}$ or
$=A^{\prime} D^{\prime} E+B^{\prime} D E+C^{\prime} D^{\prime} E^{\prime}$ or
$=A^{\prime} B D^{\prime}+B^{\prime} C^{\prime} D^{\prime}+B^{\prime} D E$ or
$=A^{\prime} C E+B^{\prime} C^{\prime} E+C^{\prime} D^{\prime} E^{\prime}$
5.38 (a)

(*) Indicates a minterm that makes the corresponding prime implicant essential.

$$
\mathrm{a}^{\prime} \mathrm{b}^{\prime} \mathrm{d}^{\prime} \rightarrow \mathrm{m}_{1} ; \text { cd'e' } \rightarrow \mathrm{m}_{28} ; \text { bc'd'e } \rightarrow \mathrm{m}_{25} ; \text { b'cd' } \rightarrow \mathrm{m}_{21}
$$

5.39 (a)

5.37 (d),
\& (e)


PIs: $\left(B^{\prime}+C+E^{\prime}\right),\left(C^{\prime}+E\right),\left(D^{\prime}+E\right),\left(A^{\prime}+D+E^{\prime}\right)$,
( $\left.A^{\prime}+C^{\prime}\right),\left(B^{\prime}+D^{\prime}\right),\left(A^{\prime}+B^{\prime}\right),\left(A+C+D^{\prime}\right),(A+B+E)$
$\mathrm{F}=\left(\mathrm{A}^{\prime}+\mathrm{D}+\mathrm{E}^{\prime}\right)\left(\mathrm{C}^{\prime}+\mathrm{E}\right)\left(\mathrm{D}^{\prime}+\mathrm{E}\right)\left(\mathrm{B}^{\prime}+\mathrm{D}^{\prime}\right)$ or $=(A+B+E)\left(B^{\prime}+D^{\prime}\right)\left(A^{\prime}+B^{\prime}\right)\left(A^{\prime}+C^{\prime}\right)$ or $=\left(A+C+D^{\prime}\right)\left(C^{\prime}+E\right)\left(A^{\prime}+B^{\prime}\right)\left(A^{\prime}+C^{\prime}\right)$ or $=\left(B+C^{\prime}+D\right)\left(D^{\prime}+E\right)\left(B^{\prime}+D^{\prime}\right)\left(A^{\prime}+B^{\prime}\right)$ or $=\left(B^{\prime}+C+E^{\prime}\right)\left(C^{\prime}+E\right)\left(D^{\prime}+E\right)\left(A^{\prime}+C^{\prime}\right)$
5.38 (b)

a'b'd', cd'e', bc'd'e, b'cd', ac'de', ab'ce', ab'de', a'c'd'e, a'bc'e, a'bc'd, bc'de', a'bde', a'bce'
5.39 (b)

$f=\underline{a^{\prime} b ' c ' e}+\underline{a^{\prime} b c^{\prime} d^{\prime}}+\underline{b c d e}+\underline{a b ' d ' e '}+\underline{a b d e}+\underline{a c d^{\prime} e^{\prime}}+$ a'b'd'e + ab'ce

5.40

5.42 (a)

5.43 (a)

5.41

5.42 (b)


$$
F=\frac{(X+Y+Z)}{\left(V+X^{\prime}+Y^{\prime}\right)\left(V+Y^{\prime}+Z^{\prime}\right)\left(V+X^{\prime}+Z^{\prime}\right)}
$$

5.43 (b)


$$
\mathrm{F}=\left(\mathrm{c}+\mathrm{d}^{\prime}\right)\left(\mathrm{a}+\mathrm{d}^{\prime}+\mathrm{e}^{\prime}\right)\left(\mathrm{b}^{\prime}+\mathrm{c}+\mathrm{e}^{\prime}\right)\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}+\mathrm{c}^{\prime}+\mathrm{d}\right)
$$

$$
(a+c+e)\left(b+c^{\prime}+e\right)
$$

Alt: $\quad F=\left(c+d^{\prime}\right)\left(a+d^{\prime}+e^{\prime}\right)\left(a+b^{\prime}+c\right)\left(b^{\prime}+c+e^{\prime}\right)$
$\left(a^{\prime}+b^{\prime}+c^{\prime}+d\right)\left(b+c^{\prime}+e\right)$

## Unit 5 Solutions

5.44 (a)

$F=\left(v^{\prime}+w^{\prime}+x^{\prime}+y+z^{\prime}\right)\left(w+y^{\prime}+z^{\prime}\right)\left(v+y^{\prime}\right)(w+x+y)$ $\left(v^{\prime}+x+y+z\right) \quad\left(w^{\prime}+x+y\right.$ ' $)$
Alt: $\left\{\begin{aligned} F= & \left(v^{\prime}+w^{\prime}+x^{\prime}+y+z^{\prime}\right)\left(w+y^{\prime}+z^{\prime}\right)\left(v+y^{\prime}\right)(w+x+y) \\ & \left(v^{\prime}+w^{\prime}+x+z\right)\left(w^{\prime}+x+y^{\prime}\right) \\ F= & \left(v^{\prime}+w^{\prime}+x^{\prime}+y+z^{\prime}\right)\left(w+y^{\prime}+z^{\prime}\right)\left(v+y^{\prime}\right)(w+x+y) \\ & \left(v^{\prime}+w^{\prime}+x+z\right)\left(x+y^{\prime}+z^{\prime}\right)\end{aligned}\right.$
5.45 (a)

$\mathrm{F}=\mathrm{ACD}+\mathrm{BC}^{\prime} \mathrm{D}+\mathrm{B}^{\prime} \mathrm{C}+\mathrm{A}^{\prime} \mathrm{C}^{\prime}$
$\mathrm{m}_{4}, \mathrm{~m}_{13}$, or $\mathrm{m}_{14}$ change the minimum sum of products, removing $\mathrm{A}^{\prime} \mathrm{C}^{\prime}, \mathrm{BC}^{\prime} \mathrm{D}$, or ACD', respectively.

### 5.46 (a)


$F=\frac{V^{\prime} X Y^{\prime}}{\mathrm{m}_{4}}+\frac{V^{\prime} W Z^{\prime}}{\mathrm{m}_{8}}+\frac{X Y Z}{\mathrm{~m}_{31}}+V W^{\prime} X^{\prime} Y^{\prime}+V W^{\prime} Y Z^{\prime}+W^{\prime} X Z$
$F=\underline{V^{\prime} X Y^{\prime}}+\underline{V^{\prime} W Z} \underline{Z}^{\prime}+\underline{X Y Z}+V W^{\prime} X^{\prime} Z^{\prime}+V W^{\prime} X Y+W^{\prime} Y^{\prime} Z$
$F=\underline{V^{\prime} X Y^{\prime}}+\underline{V^{\prime} W Z^{\prime}}+\underline{X Y Z}+V W^{\prime} X^{\prime} Y^{\prime}+V W^{\prime} Y Z^{\prime}+W^{\prime} Y^{\prime} Z$
$F=\underline{V^{\prime} X Y^{\prime}}+\underline{V^{\prime} W Z^{\prime}}+\underline{X Y Z}+V W^{\prime} X^{\prime} Z^{\prime}+V W^{\prime} Y Z^{\prime}+W^{\prime} Y^{\prime} Z$
5.44 (b)

$F=(c+d+e)\left(a^{\prime}+c^{\prime}+d^{\prime}\right)\left(a^{\prime}+b+c+d\right)$

$$
\left(a+c^{\prime}+d\right)\left(b+d^{\prime}+e\right)(a+c+e)
$$

5.45 (b)


Changing $m_{1}$ to a don't care removes $C^{\prime} D$ from the solution.
5.46 (b) $\mathrm{V}^{\prime} \mathrm{WZ}^{\prime} \rightarrow \mathrm{m}_{8} ; \mathrm{XYZ}^{2} \rightarrow \mathrm{~m}_{31} ; \mathrm{V}^{\prime} \mathrm{XY}^{\prime} \rightarrow \mathrm{m}_{4}$

## Unit 6 Problem Solutions

6.2 (a) | 1 | $0001 \checkmark$ | 1,5 | $0-01$ | a'c'd |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 5 | $0101 \checkmark$ | 1,9 | -001 | b'c'd |
|  | $1001 \checkmark$ | 5,7 | $01-1$ | a'bd |  |
|  | 12 | $1100 \checkmark$ | 9,11 | $10-1$ | ab'd |
|  | 7 | $0111 \checkmark$ | 12,14 | $11-0$ | abd' |
| 11 | $1011 \checkmark$ | 7,15 | -111 | bcd |  |
|  | 14 | $1110 \checkmark$ | 11,15 | $1-11$ | acd |
|  | 15 | $1111 \checkmark$ | 14,15 | $111-$ | abc |

Prime implicants: $a^{\prime} c^{\prime} d, b^{\prime} c^{\prime} d, a^{\prime} b d, a b^{\prime} d$, $a b d ', b c d, a c d, a b c$

Prime implicants: $a^{\prime} b^{\prime} c^{\prime}, b^{\prime} c^{\prime} d^{\prime}, a b^{\prime} d^{\prime}, a c d^{\prime}, a ' d, b c$
6.3 (a)


$$
\begin{aligned}
& f=a b d^{\prime}+a^{\prime} c^{\prime} d+a b ' d+b c d \\
& f=a b d^{\prime}+b^{\prime} c^{\prime} d+a ' b d+a c d
\end{aligned}
$$

6.3 (b)

|  | $\begin{array}{llllllllll}0 & 1 & 3 & 5 & 6 & 7 & 8 & 10 & 14 & 15\end{array}$ |  |
| :---: | :---: | :---: |
| 1,3,5, $7 \quad$ a'd | $\uparrow \otimes$ |  |
| 6, 7, 14, 15 bc | $\otimes \otimes+$ | $\begin{aligned} & f=a^{\prime} d+b c+a^{\prime} b^{\prime} c^{\prime}+a b^{\prime} d^{\prime} \\ & f=a^{\prime} d+b c+b^{\prime} c^{\prime} d^{\prime}+a b^{\prime} d^{\prime} \end{aligned}$ |
| 0,1 a'b'c' | * | $f=a^{\prime} d+b c+b^{\prime} c^{\prime} d^{\prime}+a c d^{\prime}$ |
| 0,8 b'c'd' | $\times$ |  |
| 8,10 ab'd' | $\cdots$ |  |
| 10, 14 acd' |  |  |

Unit 6 Solutions
6.4

| 1 | $0001 \checkmark$ | 1,3 | 00-1 | 1, 3, 5, 7 | 0--1 a'd |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 0010 ${ }^{\text {d }}$ | 1, 5 | 0-01 $\downarrow$ | 1, 5, 3, 7 | $0-1$ |
| 4 | 0100 $\checkmark$ | 1, 9 | -001 $\checkmark$ | 1, 5, 9, 13 | --01 c'd |
| 3 | $0011 \checkmark$ | 2, 3 | 001- $\checkmark$ | 1, 9, 5, 13 | -01 |
| 5 | $0101 \checkmark$ | 2, 6 | 0-10 | 2, 3, 6, 7 | 0-1- a'c |
| 6 | 0110 $\checkmark$ | 2, 10 | -010 b'cd' | 2, 6, 3, 7 | 0-1- |
| 9 | 1001 $\checkmark$ | 4, 5 | 010-V | 4, 5, 6, 7 | 01-- a'b |
| 10 | 1010 ${ }^{\text {d }}$ | 4, 6 | 01-0 ${ }^{\text {- }}$ | 4, 5, 12, 13 | -10- bc' |
| 12 | 1100 $\checkmark$ | 4, 12 | -100 | 4,6,5,7 | O1- |
| 7 | 0111 ${ }^{\text {d }}$ | 3, 7 | 0-11 | 4,12,5,13 | -10- |
| 13 | $1101 \checkmark$ | 5, 7 | 01-1 $\downarrow$ | 5, 7, 13, 15 | -1-1 bd |
| 15 | 1111 | 5,13 | -101 | 5, 13, 7, 15 | -1-1 |
|  |  | 6, 7 | 011- $\checkmark$ |  |  |
|  |  | 9, 13 | 1-01 $\checkmark$ |  |  |
|  |  | 12, 13 | 110- $\checkmark$ |  |  |
|  |  | 7, 15 | -111 $\checkmark$ |  |  |
|  |  | 13, 15 | 11-1V |  |  |
|  |  | 13, 15 | 11-1 $\checkmark$ |  |  |

Prime implicants: $b^{\prime} c d^{\prime}, a^{\prime} d, c^{\prime} d, a^{\prime} c, a^{\prime} b, b c^{\prime}$, bd

6.5

| 1 | $0001 \checkmark$ | 1, 5 | 0-01 | 1, 5, 9, 13 | --01 C'D |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 0100 $\sqrt{ }$ | 1, 9 | -001 $\checkmark$ | 1, 9, 5, 13 | --01 |
| 8 | $1000 \checkmark$ | 4, 5 | 010- $\checkmark$ | 4, 5, 12, 13 | -10-BC' |
| 5 | $0101 \checkmark$ | 4, 12 | -100 $\checkmark$ | 4,12, 5, 13 | -10- |
| 9 | 1001 $\checkmark$ | 8, 9 | 100- $\checkmark$ | 5, 7, 13, 15 | -1-1 BD |
| 12 | 1100 $\checkmark$ | 8, 12 | 1-00 ${ }^{\text {d }}$ | 5, 13, 7, 15 | -1-1 |
| 7 | 0111 | 5, 7 | 01-1 $\checkmark$ | 8, 9, 12, 13 | 1-0- $A C^{\prime}$ |
| 11 | 1011 | 5, 13 | -101 $\checkmark$ | 8, 12, 9, 13 | 1-0- |
| 13 | 1101 | 9, 11 | 10-1V | 9, 11, 13, 15 | 1--1 $A D$ |
| 14 | 1110 $\checkmark$ | 9, 13 | 1-01 $\downarrow$ | 9, 13, 11, 15 | 1--1 |
| 15 | 1111 | 12, 13 | 110-V | 12, 13, 14, 15 | 11-- $A B$ |
|  |  | 12, 14 | 11-0V | 12, 14, 13, 15 | 11-- |
|  |  | 7, 15 | -111 $\checkmark$ |  |  |
|  |  | 11, 15 | 1-11 $\checkmark$ |  |  |
|  |  | 13, 15 | 11-1V |  |  |
|  |  | 14, 15 | 111-V |  |  |

Prime implicants: $C^{\prime} D, B C^{\prime}, B D, A C^{\prime}, A D, A B$

$$
\begin{aligned}
& \left.\begin{array}{lll|llll}
\text { 6.5 } \\
\text { (contd) }
\end{array}\right) \quad \begin{array}{llllll} 
& & & 9 & 12 & 13 \\
\hline
\end{array} \\
& (P 1+P 4+P 5)(P 2+P 4+P 6)(P 1+P 2+P 3+P 4+P 5+P 6)(P 3+P 5+P 6) \\
& =(P 4+P 1 P 2+P 1 P 6+P 2 P 5+P 5 P 6)(P 3+P 5+P 6) \\
& =P 3 P 4+P 4 P 5+P 4 P 6+P 1 P 2 P 3+P 1 P 2 P 5+P 1 P 2 P 6+P 1 P 3 P G \\
& +P 1 P 5 P 6+P 1 P 6+P 2 P 3 P 5+P 2 P 5+P 2 P 5 P 6+P 3 P 5 P 6+P 5 P 6=1 \\
& \left.\left.\left.\left.\left.\left.F=\underset{\text { P4 }}{\left(A C^{\prime}\right.}+\underset{\text { P3 }}{B D}\right) \text { or } \underset{\text { P5 }}{(A D}+\underset{\text { P2 }}{B C^{\prime}}\right) \text { or } \underset{\text { P5 }}{(A D}+\underset{\text { P4 }}{A C^{\prime}}\right) \text { or } \underset{\text { P6 }}{(A B}+\underset{\text { P5 }}{A D}\right) \text { or } \underset{\text { P6 }}{(A B}+\underset{\text { P4 }}{A C^{\prime}}\right) \text { or } \underset{\text { P6 }}{(A B}+\underset{\text { P1 }}{C^{\prime} D}\right)
\end{aligned}
$$

6.6 (a)

| A B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 1 | 1 |  |  |
| 01 | E | 1 |  | 1 |
| 11 |  | 1 | E | X |
| 10 |  | X |  |  |

$F=M S_{0}+E M S_{1}=A^{\prime} B+A^{\prime} C^{\prime} D^{\prime}+$
$A B^{\prime} D+E\left(A^{\prime} C^{\prime}+A C D\right)$
or $E\left(A^{\prime} C^{\prime}+B C D\right)$
C D

$\mathrm{MS}_{0}=\mathrm{A}^{\prime} \mathrm{C}^{\prime} \mathrm{D}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{A} \mathrm{B}^{\prime} \mathrm{D}$
D B $\quad E=1$

|  | $\begin{array}{llll}00 & 01 & 11 & 10\end{array}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | X | X |  |  |
| 01 | 1 | X |  | X |
| 11 |  | X | 1 | X |
| 10 |  | X |  |  |

$\mathrm{MS}_{1}=\mathrm{A}^{\prime} \mathrm{C}^{\prime}+\mathrm{ACD}$
$\mathrm{MS}_{1}=\mathrm{A}^{\prime} \mathrm{C}^{\prime}+\mathrm{BCD}$
6.6 (b)

C D


$$
\mathrm{MS}_{0}=\mathrm{A}^{\prime} \mathrm{B}^{\prime}+\mathrm{ABD}
$$


$\mathrm{MS}_{1}=\mathrm{B}^{\prime} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{C}$
$\mathrm{MS}_{1}=\mathrm{B}^{\prime} \mathrm{C}^{\prime}+\mathrm{BC}$

$\mathrm{MS}_{2}=\mathrm{AB}$
C D

$\mathrm{MS}_{3}=\mathrm{A}$ 'D or C'D or BD

$$
\begin{aligned}
Z= & A^{\prime} B^{\prime}+A B D+E\left(B^{\prime} C^{\prime}+A^{\prime} C\right)+ \\
& F(A B)+G\left(A^{\prime} D\right)
\end{aligned}
$$

## Unit 6 Solutions

6.7 (a) $\quad$| 0 | $0000 \checkmark$ | 0,4 | $0-00$ | a'c'd' |
| :--- | :--- | :--- | :--- | :--- |
| 4 | $0100 \checkmark$ | 4,5 | $010-$ | a'bc' |
| 3 | $0011 \checkmark$ | 3,7 | $0-11$ | a'cd |
| 5 | $0101 \checkmark$ | 3,11 | -011 | b'cd |
| 9 | $1001 \checkmark$ | 5,7 | $01-1$ | a'bd |
| 7 | $0111 \checkmark$ | 5,13 | -101 | bc'd |
| 11 | $1011 \checkmark$ | 9,11 | $10-1$ | ab'd |
| 13 | $1101 \checkmark$ | 9,13 | $1-01$ | ac'd |

Prime implicants: $a^{\prime} c^{\prime} d^{\prime}, a^{\prime} b c^{\prime}, a^{\prime} c d$, $b^{\prime} c d, a ' b d, b c^{\prime} d, a b ' d, a c^{\prime} d$
6.7 (b)

| 2 | 0010 | $\begin{aligned} & 2,6 \\ & 2,10 \end{aligned}$ | $\begin{aligned} & 0-10 \text { a'cd' } \\ & -010 \text { b'cd' } \\ & 010-\checkmark \end{aligned}$ | $\begin{aligned} & 4,5,12,13 \\ & 4,12,5,13 \\ & \hline \end{aligned}$ | $\begin{aligned} & -10-\quad \text { bc' } \\ & -10- \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 0100 ${ }^{\text {d }}$ |  |  |  |  |
| 5 | 0101 |  |  | 9, 11, 13, 15 | 1--1 ad |
| 6 | 0110 | 4, 6 | 01-0 a'bd' | 9, 13, 11, 15 | 1--1 |
| 9 | 1001 | 4, 12 | -100 |  |  |
| 10 | 1010 | 5, 13 | -101 |  |  |
| 12 | 1100 | 9, 11 | 10-1V |  |  |
| 11 | 1011 | 9, 13 | 1-01 |  |  |
| 13 | 1101 $\downarrow$ | 10, 11 | 101- ab'c |  |  |
| 15 1111 |  | 12, 13 | 110-V |  |  |
|  |  | 11, 15 | 1-11 $\downarrow$ |  |  |
|  |  | 13, 15 | 11-1 $\downarrow$ |  |  |

Prime implicants: $a d, b c^{\prime}, a^{\prime} c d^{\prime}, b^{\prime} c d^{\prime}, a^{\prime} b d^{\prime}, a b^{\prime} c$
6.8 (a)

6.9 (b)

| 0 | $0000 \checkmark$ | 0, 1 | 000- $\checkmark$ | 0, 1, 8, 9 | -00- b'c' |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $0001 \checkmark$ | 0, 8 | -000 $\checkmark$ | 0,8,1,9 | -00- |
| 8 | 1000 $\checkmark$ | 1, 5 | 0-01 | 1, 5, 9, 13 | --01 c'd |
| 5 | $0101 \checkmark$ | 1,9 | -001 | 1,9,5,13 | -01 |
| 6 | 0110 | 8, 9 | 100- $\checkmark$ | 8, 9, 10, 11 | 10-- ab' |
| 9 | 1001 | 8, 10 | 10-0 | 8,10, 9, 11 | 10-- |
| 10 | 1010 ${ }^{\text {d }}$ | 8,12 | 1-00 | 8, 9, 12, 13 | 1-0- ac' |
| 12 | 1100 ${ }^{\text {d }}$ | 5,7 $01-1$ a'bd <br> 5,13 $-101 \checkmark$ |  | $8,12,9,13$ | 1-0- |
| 7 | 0111 |  |  |  |  |
| 11 | 1011 | 6, 7 | 011- a'bc |  |  |
| 13 | 1101 | 9, 11 | 10-1 $\downarrow$ |  |  |
|  |  | 9, 13 | 1-01 |  |  |
|  |  | 10, 11 | 101-V |  |  |
|  |  | 12, 13 | 110-V |  |  |



## Unit 6 Solutions

6.10 Prime implicants: $a b c^{\prime}, b c^{\prime} d, a^{\prime} b d, b^{\prime} c d, a^{\prime} c, a^{\prime} b^{\prime} d^{\prime}$

$$
\begin{aligned}
& f=a b c^{\prime}+b^{\prime} c d+a^{\prime} c+a^{\prime} b^{\prime} d^{\prime}+a^{\prime} b d \\
& f=a b c^{\prime}+b^{\prime} c d+a^{\prime} c+a^{\prime} b^{\prime} d^{\prime}+b c^{\prime} d
\end{aligned}
$$

6.11

| 0 | 00000 | 0, 2 | 000-0V | 0, 2, 4, 6 | 00--0V | 0, 2, 4, 6, 8, 10, 12, 14 | 0---0 A'E' |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 00010 | 0, 4 | 00-00 | 0, 2, 8, 10 | 0-0-0 | 0, 2, 8, 10, 4, 6, 12, 14 | $\theta-\theta$ |
| 4 | 00100 | 0, 8 | 0-000 | 0, 2, 16, 18 | -00-0 B'C'E' | 0, 4, 8, 12, 2, 6, 10, 14 | $\theta-\theta$ |
| 8 | 01000 | 0, 16 | -0000 | 0, 4, 2, 6 | 00--0 |  |  |
| 16 | 10000 | 2, 6 | 00-10 | 0, 4, 8, 12 | 0--00 |  |  |
| 6 | 00110 | 2, 10 | 0-010 | 0, 8, 2, 10 | $\theta-0-\theta$ |  |  |
| 9 | 01001 | 2, 18 | -0010 | 0, 8, 4, 12 | 0--00 |  |  |
| 10 | 01010 | 4, 6 | 001-0 | 0,16,2, 18 | -00-0 |  |  |
| 12 | 01100 | 4, 12 | 0-100 | 2, 6, 10, 14 | 0--10 |  |  |
| 18 | 10010 | 8, 9 | 0100- $\checkmark$ | 2, 10, 6, 14 | $\theta-10$ |  |  |
| 7 | 00111 | 8, 10 | 010-0 | 4, 6, 12, 14 | 0-1-0 |  |  |
| 11 | 01011 | 8, 12 | 01-00V | 4, 12, 6, 14 | 0-1-0 |  |  |
| 13 | 01101 | 16, 18 | 100-0V | 8, 9, 10, 11 | 010-- A'BC' |  |  |
| 14 | 01110 | 6, 7 | 0011- A'B'CD | 8, 9, 12, 13 | 01-0- A'BD' |  |  |
| 19 | 10011 | 6, 14 | 0-110 | 8, 10, 9, 11 | 010-- |  |  |
| 21 | 10101 | 9, 11 | 010-1V | 8, 10, 12, 14 | 01--0V |  |  |
| 29 | 11101 | 9, 13 | 01-01 | 8,12, 9, 13 | 01-0- |  |  |
| 30 | 11110 | 10, 11 | 0101- $\checkmark$ | 8, 12, 10, 14 | 01--0 |  |  |
|  |  | 10, 14 | 01-10 |  |  |  |  |
|  |  | 12, 13 | 0110- $\checkmark$ |  |  |  |  |
|  |  | 12, 14 | 011-0V |  |  |  |  |
|  |  | 18, 19 | 1001- AB'C'D |  |  |  |  |
|  |  | 13, 29 | -1101 BCD'E |  |  |  |  |
|  |  | 14, 30 | -1110 BCDE' |  |  |  |  |
|  |  | 21, 29 | 1-101 ACD'E |  |  |  |  |


6.12 (a)

6.12 (b)

| $000000 \sqrt{ }$ | 0,1 | 0000-* |
| :---: | :---: | :---: |
| 1 00001 $\sqrt{ }$ | 0, 2 | 000-0* |
| $200010 \sqrt{ }$ | 0, 4 | 00-00* |
| 4 00100V | 0, 8 | 0-000* |
| 8 01000V | 1,17 | -0001* |
| 17 10001 $\sqrt{ }$ | 2, 18 | -0010* |
| 18 10010V | 4, 20 | -0100* |
| 20 10100 $\sqrt{ }$ | 17, 21 | 10-01* |
| 11 01011 $\sqrt{ }$ | 18, 26 | 1-010* |
| 13 01101 $\sqrt{ }$ | 20, 21 | 1010-* |
| 14 10110V | 11, 15 | 01-11 $\sqrt{ }$ |
| 21 10101 $\sqrt{ }$ | 11, 27 | -1011 $\sqrt{ }$ |
| 26 11010 $\sqrt{ }$ | 13, 15 | 011-1* |
| 15 01111 $\sqrt{ }$ | 14, 15 | 0111-V |
| 27 11011 $\sqrt{ }$ | 14, 30 | -1110 V |
| 30 11110 $\sqrt{ }$ | 26, 27 | 1101-V |
| 31 11111 $\sqrt{ }$ | 26, 30 | 11-10V |
|  | 15, 31 | -1111 $\sqrt{ }$ |
|  | 27, 31 | 11-11 $\sqrt{ }$ |
|  | 30, 31 | 1111-V |


| $11,15,27,31$ | $-1-11^{*}$ |
| :--- | :--- |
| $14,15,30,31$ | $-111^{*}$ |
| $26,27,30,31$ | $11-1-*$ |

Prime Implicants: A'B'D'E', AB'DE, AB'CE, ACD'E, AB'CD, ACDE', ABCD'. ABCE', $C^{\prime} D E, A^{\prime} C^{\prime}$


Essential prime implicants: $A^{\prime} C^{\prime} D^{\prime} E^{\prime}, B D E, A^{\prime} B C E, B C D$
Petrick's Method for remaining minterms: $(Q+T)(R+U)(S+V)(T+W)(U+X)(V+Y)(W+Y)(X+Z)$
$=(Q W+T)(R X+U)(S Y+V)(W+Y)(X+Z)=(Q W+T W+T Y)(R X+U X+U Z)(S Y+V)$
$=(Q S W Y+S T Y+Q V W+T V W+T V Y)(R X+U X+U Z)$ There are four minimal choices from the first parenthesis. In the second parenthesis only UZ is minimal since Z has fewer literals than the other two PI's. The minimal solutions are $(S T Y+Q V W+T V W+T V Y)(U Z)$

## Unit 6 Solutions

```
6.12 (b) \(f=B C D+A^{\prime} B C E+B D E+A^{\prime} C^{\prime} D^{\prime} E^{\prime}+A B D+B^{\prime} C^{\prime} D E^{\prime}+A B^{\prime} C D^{\prime}+B^{\prime} C^{\prime} D^{\prime} E+A^{\prime} B^{\prime} D^{\prime} E^{\prime}\)
(contd) \(f=B C D+A^{\prime} B C E+B D E+A^{\prime} C^{\prime} D^{\prime} E^{\prime}+A B D+B^{\prime} C^{\prime} D E^{\prime}+A B^{\prime} D^{\prime} E+B^{\prime} C D^{\prime} E^{\prime}+A^{\prime} B^{\prime} C^{\prime} D^{\prime}\)
\(f=B C D+A^{\prime} B C E+B D E+A^{\prime} C^{\prime} D^{\prime} E^{\prime}+A B D+B^{\prime} C^{\prime} D E^{\prime}+A B^{\prime} D^{\prime} E+B^{\prime} C D^{\prime} E^{\prime}+B^{\prime} C^{\prime} D^{\prime} E\)
\(f=B C D+A^{\prime} B C E+B D E+A^{\prime} C^{\prime} D^{\prime} E^{\prime}+A B D+B^{\prime} C^{\prime} D E^{\prime}+A B^{\prime} C D^{\prime}+B^{\prime} C D^{\prime} E^{\prime}+B^{\prime} C^{\prime} D^{\prime} E\)
```

6.13 (a)

$f^{\prime}(A, B, C, D, E)=A B^{\prime} D^{\prime} E^{\prime}+B C D E+A C^{\prime} D^{\prime}+A C^{\prime} E^{\prime}+A^{\prime} C E+A^{\prime} C D+A^{\prime} B C$
$f(A, B, C, D, E)=\left(A^{\prime}+B+D+E\right)\left(B^{\prime}+C^{\prime}+D^{\prime}+E^{\prime}\right)\left(A^{\prime}+C+D\right)\left(A^{\prime}+C+E\right)\left(A+C^{\prime}+E^{\prime}\right)\left(A+C^{\prime}+D^{\prime}\right)\left(A+B^{\prime}+C^{\prime}\right)$
6.13 (b)

$f^{\prime}=A^{\prime} B C^{\prime} D E^{\prime}+A C^{\prime} D^{\prime} E^{\prime}+A^{\prime} B^{\prime} C E+B C^{\prime} D^{\prime} E+B C D^{\prime} E^{\prime}+B^{\prime} D E+B^{\prime} C D+A B D^{\prime}$
$f=\left(A^{\prime}+B^{\prime}+D\right)\left(A^{\prime}+C+D+E\right)\left(B^{\prime}+C^{\prime}+D+E\right)\left(B^{\prime}+C+D+E^{\prime}\right)\left(B+C^{\prime}+D^{\prime}\right)\left(A+B+C^{\prime}+E^{\prime}\right)\left(B+D^{\prime}+E^{\prime}\right)\left(A+B^{\prime}+C+D^{\prime}+E\right)$
6.14 (a)

| 1 | $0001 \sqrt{ }$ | 1,3 | $00-1^{*}$ |
| :--- | :--- | :--- | :--- |
| 4 | $0100 \sqrt{ }$ | 1,5 | $0-01 \sqrt{ }$ |
| 8 | $1000 \sqrt{ }$ | 1,9 | $-001 \sqrt{ }$ |
| 3 | $0011 \sqrt{ }$ | 4,5 | $010-\sqrt{ }$ |
| 5 | $0101 \sqrt{ }$ | 4,6 | $01-0 \sqrt{ }$ |
| 6 | $0110 \sqrt{ }$ | 4,12 | $-100 \sqrt{ }$ |
| 9 | $1001 \sqrt{ }$ | 8,9 | $100-\sqrt{ }$ |
| 10 | $1010 \sqrt{ }$ | 8,10 | $10-0 \sqrt{ }$ |
| 12 | $1100 \sqrt{ }$ | 8,12 | $1-00 \sqrt{ }$ |
| 13 | $1101 \sqrt{ }$ | 5,13 | $-101 \sqrt{ }$ |
| 14 | $1110 \sqrt{ }$ | 6,14 | $-110 \sqrt{ }$ |
| 15 | $1111 \sqrt{ }$ | 9,13 | $1-01 \sqrt{ }$ |
|  |  | 10,14 | $1-10 \sqrt{ }$ |
|  |  | 12,13 | $110-\sqrt{ }$ |
|  |  | 12,14 | $11-0 \sqrt{ }$ |
|  | 13,15 | $11-1 \sqrt{ }$ |  |
|  | 14,15 | $111-\sqrt{2}$ |  |


| $1,5,9,13$ | $--01^{*}$ |
| :--- | :--- |
| $4,5,12,13$ | $-10-*$ |
| $4,6,12,14$ | $-1-0^{*}$ |
| $8,9,12,13$ | $1-0$ - $^{*}$ |
| $8,10,12,14$ | $1-0^{*}$ |
| $12,13,14,15$ | $11-{ }^{*}$ |

Prime implicants: $A^{\prime} B^{\prime} D, A B, A C^{\prime}, C^{\prime} D, A D^{\prime}$, $B D^{\prime}, B C^{\prime}$


Essential Prime Implicants: $A B, B D^{\prime}, A^{\prime} B^{\prime} D$
$f=A B+B D^{\prime}+A^{\prime} B^{\prime} D+C^{\prime} D+A D^{\prime}$
$f=A B+B D^{\prime}+A^{\prime} B^{\prime} D+A C^{\prime}+C^{\prime} D$
$f=A B+B D^{\prime}+A^{\prime} B^{\prime} D+A C^{\prime}+B C^{\prime}$

### 6.14 (b)

| 0 | $0000 \sqrt{ }$ | 0,2 | $00-0^{*}$ |
| :--- | :--- | :--- | :--- |
| 2 | $0010 \sqrt{ }$ | 0,4 | $0-00^{*}$ |
| 4 | $0100 \sqrt{ }$ | 2,10 | $-010^{*}$ |
| 10 | $1010 \sqrt{ }$ | 10,11 | 101 -* $^{*}$ |
| 7 | $0111^{*}$ |  |  |
| 11 | $1011 \sqrt{ }$ |  |  |
| 13 | $1101^{*}$ |  |  |

Prime Implicants of $f^{\prime}: A^{\prime} B C D, A^{\prime} B^{\prime} D^{\prime}, A B C^{\prime} D$, $A B^{\prime} C, B^{\prime} C D^{\prime}, A^{\prime} C^{\prime} D^{\prime}$


Essential Prime Implicants: $A B^{\prime} C, A^{\prime} B C D$

$$
f^{\prime}=A B^{\prime} C+A^{\prime} B C D+A^{\prime} B^{\prime} D^{\prime}
$$

### 6.15 (a)

| 1 | 00001 V | 1, 5 | 00-01 $\sqrt{ }$ |
| :---: | :---: | :---: | :---: |
| 2 | 00010 $\sqrt{ }$ | 1, 9 | 0-001 $\sqrt{ }$ |
| 4 | 00100 $\sqrt{ }$ | 1, 17 | -0001 $\sqrt{ }$ |
| 5 | $00101 \sqrt{ }$ | 2, 6 | 00-10* |
| 6 | 00110 $\sqrt{ }$ | 4, 5 | 0010-V |
| 9 | 01001 V | 4, 6 | 001-0 $\sqrt{ }$ |
| 12 | 01100 V | 4, 12 | 0-100 $\sqrt{ }$ |
| 17 | 10001 V | 4, 20 | -0100 V |
| 20 | 10100 V | 5, 7 | 001-1 $\sqrt{ }$ |
| 7 | 00111 V | 5, 13 | $0-101 \sqrt{ }$ |
| 13 | 01101 $\sqrt{ }$ | 6, 7 | 0011- V |
| 22 | 10110V | 6, 22 | -0110 ${ }^{\text {V }}$ |
| 25 | 11001 V | 9, 13 | 01-01 $\sqrt{ }$ |
| 28 | 11100 V | 9, 25 | -1001 $\sqrt{ }$ |
| 15 | 01111 V | 12, 13 | 0110-V |
| 30 | $11110 \sqrt{ }$ | 12, 28 | -1100 $\sqrt{ }$ |
|  |  | 17, 25 | 1-001 $\sqrt{ }$ |
|  |  | 20, 22 | 101-0 ${ }^{\text {d }}$ |
|  |  | 20, 28 | 1-100 V |
|  |  | 7, 15 | $0-111 \mathrm{~V}$ |
|  |  | 13, 15 | 011-1 $\sqrt{ }$ |
|  |  | 22, 30 | 1-110 ${ }^{\text {d }}$ |
|  |  | 28, 30 | 111-0 V |

Prime Implicants: a c $e^{\prime}, a^{\prime} c$ e, c $d^{\prime} e^{\prime}, a^{\prime} c d^{\prime}, a^{\prime} b^{\prime} c$, b'c e', a'b'd e', c'd'e, a'd'e


Essential Prime Implicants: a c $e^{\prime}, c^{\prime} d^{\prime} e, a^{\prime} c e, a^{\prime} b^{\prime} d e^{\prime}$

$$
\begin{aligned}
& f=a c e^{\prime}+c^{\prime} d^{\prime} e+a^{\prime} c e+a^{\prime} b^{\prime} d e^{\prime}+a^{\prime} c d^{\prime} \\
& f=a c e^{\prime}+c^{\prime} d^{\prime} e+a^{\prime} c e+a^{\prime} b^{\prime} d e^{\prime}+c d^{\prime} e^{\prime}
\end{aligned}
$$

Unit 6 Solutions
6.15 (b)

| $000000 \sqrt{ }$ | $\begin{aligned} & 0,8 \\ & 0,16 \end{aligned}$ | $\begin{aligned} & 0-000 \sqrt{ } \\ & -0000 \sqrt{ } \end{aligned}$ |
| :---: | :---: | :---: |
| 8 01000V |  |  |
| $1610000 \sqrt{ }$ | 8, 10 | 010-0V |
| $300011 \sqrt{ }$ | 8, 24 | -1000 V |
| 10 01010 $\sqrt{ }$ | 16, 18 | 100-0V |
| 18 10010 $\sqrt{ }$ | 16, 24 | 1-000 $\sqrt{ }$ |
| $2411000 \sqrt{ }$ | 3, 11 | 0-011 $\sqrt{ }$ |
| 11 01011 $\sqrt{ }$ | 3, 19 | -0011 $\sqrt{ }$ |
| 14 01110V | 10, 11 | 0101-V |
| 19 10011 $\sqrt{ }$ | 10, 14 | 01-10* |
| 21 10101 $\sqrt{ }$ | 10, 26 | -1010 V |
| 26 11010 $\sqrt{ }$ | 18, 19 | 1001-V |
| 23 10111 $\sqrt{ }$ | 18, 26 | 1-010 $\sqrt{ }$ |
| 2711011 | 24, 26 | 110-0 $\sqrt{ }$ |
| 29 11101 $\sqrt{ }$ | 11, 27 | -1011 $\sqrt{ }$ |
| 31 11111 $\sqrt{ }$ | 19, 23 | 10-11 $\sqrt{ }$ |
|  | 19, 27 | 1-011 $\sqrt{ }$ |
|  | 21, 23 | 101-1 $\sqrt{ }$ |
|  | 21, 29 | 1-101 $\sqrt{ }$ |
|  | 26, 27 | 1101-V |
|  | 23, 31 | 1-111 $\sqrt{ }$ |
|  | 27, 31 | 11-11 $\sqrt{ }$ |
|  | 29, 31 | 111-1 $\sqrt{ }$ |

Prime Implicants of $f^{\prime}$ : ace, ade, ac'd, ac'e', bc'd, a'bde', bc'e', c'de, c'd'e'

6.16 (a) $G=\underline{A B^{\prime} C^{\prime} D E F}+\underline{A B C D E F}+\underline{A^{\prime} C^{\prime} D^{\prime} F}+\underline{A^{\prime} C^{\prime} D^{\prime} E}+\underline{A C^{\prime} D^{\prime} E^{\prime} F^{\prime}}+A^{\prime} B C^{\prime} D^{\prime}+A^{\prime} B D^{\prime} E F^{\prime}$
$G=\underline{A B^{\prime} C^{\prime} D E F}+\underline{A B C D E F}+\underline{A^{\prime} C^{\prime} D^{\prime} F}+\underline{A^{\prime} C^{\prime} D^{\prime} E}+\underline{A C^{\prime} D^{\prime} E^{\prime} F^{\prime}}+A^{\prime} B C^{\prime} D^{\prime}+A^{\prime} B C E F^{\prime}$
6.16 (b) Essential prime implicants are underlined in 6.16 (a).
6.16 (c) If there were no don't cares, prime implicants $15,(26,30),(28,29)$, and $(28,30)$ are omitted. There is only one minimum solution. Same as (a), except delete the second equation.
6.17 (a)

| 1 | 000001 $\sqrt{ }$ | 1,33 | -00001* | 11, 15, 43, 47 | -01-11* |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 001100* | 33, 35 | 1000-1* |  |  |
| 33 | 100001V | 7,15 | 00-111* |  |  |
| 7 | 000111 $\sqrt{ }$ | 11, 15 | 001-11 $\sqrt{ }$ |  |  |
| 11 | 001011 $\sqrt{ }$ | 11, 43 | -01011 $\sqrt{ }$ |  |  |
| 35 | 100011 V | 35, 43 | 10-011* |  |  |
| 50 | 110010 V | 50, 54 | 110-10* |  |  |
| 15 | 001111 $\sqrt{ }$ | 50, 58 | 11-010* |  |  |
| 30 | 011110* | 15, 47 | -01111 $\sqrt{ }$ |  |  |
| 43 | 101011 V | 43, 47 | 101-11 $\sqrt{ }$ |  |  |
| 54 | 110110V | 43, 59 | 1-1011* |  |  |
| 58 | $111010 \sqrt{ }$ | 58, 59 | 11101-* |  |  |
| 60 | 111100* |  |  |  |  |
| 47 | 101111 V |  |  |  |  |
| 59 | $111011 \sqrt{ }$ |  |  |  |  |

Prime Implicants: $A^{\prime} B^{\prime} C D E^{\prime} F^{\prime}, A^{\prime} B C D E F^{\prime}, A B C D E^{\prime} F^{\prime}$, $B^{\prime} C^{\prime} D^{\prime} E^{\prime} F, A B^{\prime} C^{\prime} D^{\prime} F, A^{\prime} B^{\prime} D E F, A B^{\prime} D^{\prime} E F, A B C^{\prime} E F^{\prime}$, ABD'EF', ACD'EF, ABCD'E, B'CEF

Unit 6 Solutions
6.17 (a)
(contd)

Essential Prime Implicants: $A^{\prime} B^{\prime} C D E^{\prime} F^{\prime}$, ABCDE'F', B'C'D'E'F, A'B'DEF, B'CEF

$$
\begin{aligned}
G= & A^{\prime} B^{\prime} C D E^{\prime} F^{\prime}+A B C D E^{\prime} F^{\prime}+B^{\prime} C^{\prime} D^{\prime} E^{\prime} F+ \\
& A^{\prime} B^{\prime} D E F+B^{\prime} C E F+A C D^{\prime} E F+A B^{\prime} C^{\prime} D^{\prime} F \\
G= & A^{\prime} B^{\prime} C D E^{\prime} F^{\prime}+A B C D E^{\prime} F^{\prime}+B^{\prime} C^{\prime} D^{\prime} E^{\prime} F+ \\
& A^{\prime} B^{\prime} D E F+B^{\prime} C E F+A C D^{\prime} E F+A B^{\prime} D^{\prime} E F \\
G= & A^{\prime} B^{\prime} C D E^{\prime} F^{\prime}+A B C D E^{\prime} F^{\prime}+B^{\prime} C^{\prime} D^{\prime} E^{\prime} F+ \\
& A^{\prime} B^{\prime} D E F+B^{\prime} C E F+A B C D^{\prime} E+A B^{\prime} C^{\prime} D^{\prime} F \\
G= & A^{\prime} B^{\prime} C D E^{\prime} F^{\prime}+A B C D E^{\prime} F^{\prime}+B^{\prime} C^{\prime} D^{\prime} E^{\prime} F+ \\
& B^{\prime} C E F+A B C D^{\prime} E+A B^{\prime} D^{\prime} E F
\end{aligned}
$$

$$
\text { (a) }-0-1=(1,3,9,11),-01-=(2,3,10,11) \text {, }
$$ $--11=(3,7,11,15), 1--1=(9,11,13,15)$

(b) maxterms $=0,4,5,6,8,12,14$
(c) don't cares $=1,10,15$
(d) $\mathrm{B}^{\prime} \mathrm{C}, \mathrm{CD}, \mathrm{AD}$
6.19


Using Petrick's method:

$$
\begin{aligned}
& (\mathrm{C} 1+\mathrm{C} 3)(\mathrm{C} 2+\mathrm{C} 3+\mathrm{C} 5)(\mathrm{C} 1+\mathrm{C} 4)(\mathrm{C} 1+\mathrm{C} 5) \\
& (\mathrm{C} 2+\mathrm{C} 3)(\mathrm{C} 2+\mathrm{C} 3+\mathrm{C} 4)(\mathrm{C} 3+\mathrm{C} 4) \\
& =(\mathrm{C} 1 \mathrm{C} 2+\mathrm{C} 1 \mathrm{C} 5+\mathrm{C} 3)(\mathrm{C} 1+\mathrm{C} 4 \mathrm{C} 5)(\mathrm{C} 2 \mathrm{C} 4+\mathrm{C} 3) \\
& =(\mathrm{C} 1 \mathrm{C} 2+\mathrm{C} 1 \mathrm{C} 5+\mathrm{C} 1 \mathrm{C} 3+\mathrm{C} 3 \mathrm{C} 4 \mathrm{C} 5)(\mathrm{C} 2 \mathrm{C} 4+\mathrm{C} 3) \\
& = \\
& \mathrm{C} 1 \mathrm{C} 2 \mathrm{C} 4+\mathrm{C} 1 \mathrm{C} 3+\mathrm{C} 3 \mathrm{C} 4 \mathrm{C} 5
\end{aligned}
$$

Each product term specifies a nonredundant combination of carts that can be used to deliver the packages. The minimal cart solution, using carts C1 and C3, costs $\$ 6$. However, using the three carts C1, C 2 and C 4 costs only $\$ 5$ so it is the minimal cost solution desired by the stockroom manager.
6.20


Prime implicants: $A C, A D^{\prime}, A B, C D, B D, A^{\prime} D$
Minimum solutions: $\left(A D^{\prime}+C D\right)$; $\left(A D^{\prime}+B D\right)$;

$$
(A B+B D) ;(A B+C D) ;\left(A B+A^{\prime} D\right)
$$

6.22 (a)

$\left.F=A_{M C^{\prime} D+B C D^{\prime}+A^{\prime} D_{1}^{\prime}}^{M E} \frac{\left(A^{\prime} B^{\prime} C^{\prime}+B D_{1}^{\prime}\right.}{M S_{1}}\right)$
6.23 (a) Each minterm of the four variables $A, B, C, D$ expands to two minterms of the five variables $A, B, C, D, E$. For example,

$$
\begin{aligned}
& m_{4}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\mathrm{A}^{\prime} \mathrm{BC}^{\prime} \mathrm{D}^{\prime} \\
& =\mathrm{A}^{\prime} \mathrm{BC} \mathrm{D}^{\prime} \mathrm{E}^{\prime}+\mathrm{A}^{\prime} \mathrm{BC}^{\prime} \mathrm{D}^{\prime} \mathrm{E} \\
& =m_{8}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}, \mathrm{E})+m_{9}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}, \mathrm{E})
\end{aligned}
$$


6.22 (b)

$Z=\underset{M S_{0}}{C^{\prime} D+A^{\prime} C D^{\prime}}+\frac{E\left(B C^{\prime}+B^{\prime} D\right)}{M S_{1}}+\underset{M S_{2}}{F\left(C D^{\prime}\right)}+\underset{M S_{3}}{G\left(A^{\prime} C^{\prime}\right)}$
6.23 (b) Prime implicants: $A^{\prime} C^{\prime} D^{\prime}, A^{\prime} B, A B^{\prime} D, A^{\prime} C^{\prime} E, A C D E$, BCDE, B'C'DE
$F=\underline{A^{\prime} C^{\prime} D^{\prime}}+\underline{A^{\prime} B}+\underline{A B^{\prime} D}+A^{\prime} C^{\prime} E+A C D E$
$F=\underline{A^{\prime} C^{\prime} D^{\prime}}+\underline{A^{\prime} B}+\underline{A B^{\prime} D}+A^{\prime} C^{\prime} E+B C D E$
6.24


* This square contains $1+B$, which reduces to 1 .
$G=\underset{M S_{0}}{C^{\prime} E^{\prime} F+D E F}+A \underset{M S_{1}}{\left(\underset{M S^{\prime} F^{\prime}}{\left(D^{\prime}\right)}\right.}+B \underset{M S_{2}}{(D F)}$

Unit 6 Solutions

## Unit 7 Problem Solutions

7.1 (a)

$\mathrm{f}=\mathrm{a} \mathrm{b}^{\prime} \mathrm{d}^{\prime}+\mathrm{a} \mathrm{b}^{\prime} \mathrm{c}^{\prime}+\mathrm{a}$ 'b $\mathrm{c}+\mathrm{a} \mathrm{a}^{\prime} \mathrm{d}$ '

Sum of products solution requires 5 gates, 16 inputs

| a b |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | $\bigcirc$ | 1 | 0 | 1 |
| 01 | 0 | © | 0 | 1 |
| 11 | $\bigcirc$ | 1 | © | 0 |
| 10 | © | 1 | 0 | 1 |

$\mathrm{f}=\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}\right)(\mathrm{a}+\mathrm{b})\left(\mathrm{a}+\mathrm{c}+\mathrm{d}^{\prime}\right)\left(\mathrm{b}+\mathrm{c}^{\prime}+\mathrm{d}^{\prime}\right)$
$\mathrm{f}=\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}\right)(\mathrm{a}+\mathrm{b})\left(\mathrm{b}+\mathrm{c}^{\prime}+\mathrm{d}^{\prime}\right)\left(\mathrm{b}^{\prime}+\mathrm{c}+\mathrm{d}^{\prime}\right)$
$f=\left(a^{\prime}+b^{\prime}\right)(a+b)\left(a+c+d^{\prime}\right)\left(a^{\prime}+c^{\prime}+d^{\prime}\right)$
$f=\left(a^{\prime}+b^{\prime}\right)(a+b)\left(b^{\prime}+c+d^{\prime}\right)\left(a^{\prime}+c^{\prime}+d^{\prime}\right)$
Product of sums solution requires 5 gates, 14 inputs, so product of sums solution is minimum.
7.1 (b) Beginning with the minimum sum of products solution, we can get


5 gates, 12 inputs
So sum of products solution is minimum.
7.2 (a)

$$
\begin{aligned}
7.2 \text { (a) } \quad A C^{\prime} D & +A D E^{\prime}+B E^{\prime}+B C^{\prime}+A^{\prime} D^{\prime} E^{\prime} \\
& =E^{\prime}(A D+B)+A^{\prime} D^{\prime} E^{\prime}+C^{\prime}(A D+B)
\end{aligned}
$$



4 levels, 6 gates, 13 inputs

Beginning with a minimum product of sums solution, we can get


6 gates, 14 inputs
7.2 (b) $A E+B D E+B C E+B C F G+B D F G+A F G$
$=A E+A F G+B E(C+D)+B F G(C+D)$


4 levels, 6 gates, 12 inputs

## Unit 7 Solutions

7.3 $\quad F(a, b, c, d) n=a^{\prime} b d+a c^{\prime} d$ or $d\left(a^{\prime} b+a c^{\prime}\right)=d(a+b)\left(a^{\prime}+c^{\prime}\right)$

You can obtain this equation in the product of sums form using a Karnaugh map, as shown below:



$\left(F^{\prime}\right)^{\prime}=\left[\left(a+b^{\prime}+d^{\prime}\right)\left(a^{\prime}+c+d^{\prime}\right)\right]^{\prime}$

$\left(F^{\prime}\right)^{\prime}=\left(a+b^{\prime}+d^{\prime}\right)^{\prime}+\left(a^{\prime}+c+d^{\prime}\right)^{\prime}$
$\left(F^{\prime}\right)^{\prime}=\left[\left(a^{\prime} b d+a c^{\prime} d\right)^{\prime}\right]^{\prime}$

$F=d(a+b)\left(a^{\prime}+c^{\prime}\right)$
$\left(F^{\prime}\right)^{\prime}=\left[d^{\prime}+(a+b)^{\prime}+\left(a^{\prime}+c^{\prime}\right)^{\prime}\right]^{\prime}$
$\left(F^{\prime}\right)^{\prime}=\left[d^{\prime}+a^{\prime} b^{\prime}+a c\right]^{\prime}$

$\left(F^{\prime}\right)^{\prime}=d\left(a^{\prime} b^{\prime}\right)^{\prime}(a c)^{\prime}$
$\left.\left(F^{\prime}\right)^{\prime}=\left[d(a+b)\left(a^{\prime}+c^{\prime}\right)\right)^{\prime}\right]^{\prime}$

$\mathrm{F}=\mathrm{AC} \mathrm{D}+\mathrm{A}^{\prime} \mathrm{BD}$

$\mathrm{F}=(\mathrm{A}+\mathrm{B})(\mathrm{D})\left(\mathrm{A}^{\prime}+\mathrm{C}^{\prime}\right)$
7.4 $\quad F(A, B, C, D)=\sum m(5,10,11,12,13)$


$$
F=A B C^{\prime}+B C^{\prime} D+A B^{\prime} C=B C^{\prime}(A+D)+A B^{\prime} C
$$

$$
\mathrm{F}=\mathrm{BC} \mathrm{~B}^{\prime}(\mathrm{A}+\mathrm{D})+\mathrm{AB}^{\prime} \mathrm{C}
$$



4 gates, 10 inputs




4 gates, 10 inputs

7.6

$$
\begin{aligned}
Z= & A B C+A D+C^{\prime} D^{\prime} \\
& =A(B C+D)+C^{\prime} D^{\prime}
\end{aligned}
$$


7.7

$$
\begin{aligned}
Z= & A E+B D E+B C E F \\
& =E(A+B D+B C F) \\
& =E[A+B(D+C F)]
\end{aligned}
$$

For the solution to 7.8, see
7.9 FLD p. 700.

$\mathrm{f}_{1}=\underline{\mathrm{acd}}+\mathrm{ad}+\underline{\underline{a^{\prime} \mathrm{b}^{\prime} d}}$

6 gates

$\mathrm{f}_{2}=\mathrm{a}^{\prime} \mathrm{d}^{\prime}+\underline{\underline{\mathrm{a}^{\prime} \mathrm{b}^{\prime} \mathrm{d}}}+\underline{\mathrm{acd}}$
6 gates
7.10 $f_{1}(A, B, C, D)=\sum m(3,4,6,9,11)$ $f_{2}(A, B, C, D)=\sum m(2,4,8,10,11,12)$ $f_{3}(A, B, C, D)=\sum m(3,6,7,10,11)$

$\mathrm{f}_{1}=\mathrm{ab}$ d $+\underline{\underline{b^{\prime} c d}}+\mathrm{a}^{\prime} \mathrm{bd}{ }^{\prime}$

$\mathrm{f}_{2}=\underline{\mathrm{ab}^{\prime} \mathrm{c}}+\mathrm{b}^{\prime} \mathrm{cd}^{\prime}+\mathrm{bc} \mathrm{c}^{\prime}+\mathrm{ac} \mathrm{d}^{\prime}$

$$
\mathrm{f}_{2}=\underline{\mathrm{ab}^{\prime} \mathrm{c}}+\mathrm{b}^{\prime} \mathrm{cd}{ }^{\prime}+\mathrm{bc}^{\prime} \mathrm{d}^{\prime}+\mathrm{ab} \mathrm{~d}^{\prime}
$$

$\overline{11}$ gates


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## Unit 7 Solutions

7.11

| \ab |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | (0) | 0 | 1 |
| 01 | 0 | 0 | 0 | 1 |
| 11 | 1 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 |

$F_{1}=(a+c)\left(a+b^{\prime}\right)\left(a^{\prime}+b^{\prime}+c\right)\left(\underline{\left.\underline{a^{\prime}+b+c^{\prime}}\right)}\right.$

$\mathrm{F}_{2}=\left(\mathrm{b}^{\prime}+\mathrm{c}+\mathrm{d}\right)\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}+\mathrm{c}\right)\left(\mathrm{a}+\mathrm{c}^{\prime}\right)\left(\underline{\underline{(a+}+\mathrm{b}+\mathrm{c}^{\prime}}\right)$
$F_{2}=\left(a+b^{\prime}+d\right)\left(a^{\prime}+b^{\prime}+c\right)\left(a+c^{\prime}\right)\left(\underline{\left.\underline{a^{\prime}+b+c^{\prime}}\right)}\right.$

8 gates
7.12

$\mathrm{f}_{1}=(\underline{\mathrm{A}+\mathrm{B}+\mathrm{C}})\left(\mathrm{B}^{\prime}+\mathrm{D}\right)$

$\mathrm{f}_{2}=(\underline{A+B+C})\left(\underline{B^{\prime}+C+D}\right)\left(A^{\prime}+C\right)$
9 gates
7.13 (a) Using $F=\left(F^{\prime}\right)^{\prime}$ from Equations (7-23(b)), p. 206:
$f_{1}=\left[\left(A^{\prime} B D\right)^{\prime}(A B D)^{\prime}\left(A B^{\prime} C^{\prime}\right)^{\prime}\left(B^{\prime} C\right)^{\prime}\right]^{\prime} ; f_{2}=\left[C^{\prime}\left(A^{\prime} B D\right)^{\prime}\right]^{\prime} ; f_{3}=\left[(B C)^{\prime}\left(A B^{\prime} C^{\prime}\right)^{\prime}(A B D)^{\prime}\right]^{\prime}$


7.13 (b) Using $F=\left(F^{\prime}\right)^{\prime}$ from Equations derived in problem 7.12:
$f_{1}=\left[(A+B+C)^{\prime}+\left(B^{\prime}+D\right)^{\prime}\right]^{\prime}$
$f_{2}=\left[(A+B+C)^{\prime}+\left(B^{\prime}+C+D\right)^{\prime}+\left(A^{\prime}+C\right)^{\prime}\right]^{\prime}$
$f_{3}=\left[\left(B^{\prime}+C+D\right)^{\prime}+(A+C)^{\prime}+\left(B+C^{\prime}\right)^{\prime}\right]^{\prime}$


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7.14 (a)

$\mathrm{f}=(\mathrm{a}+\mathrm{b}+\mathrm{c})\left(\mathrm{a}+\mathrm{b}+\mathrm{d}^{\prime}\right)\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}+\mathrm{d}^{\prime}\right)\left(\mathrm{a}^{\prime}+\mathrm{b}^{\prime}+\mathrm{c}^{\prime}\right)$
5 gates, 16 inputs
and $f=a^{\prime} b+a b^{\prime}+b^{\prime} c d^{\prime}+a c^{\prime} d^{\prime}$
$f=a^{\prime} b+a b^{\prime}+a^{\prime} c d^{\prime}+b c^{\prime} d^{\prime}$
(two other minimun solutions)
5 gates, 14 inputs minimal

7.15 (a) From K-maps:
$F=a^{\prime} c+b c^{\prime} d+a c^{\prime} d-4$ gates, 11 inputs
$F=(a+b+c)(c+d)\left(a^{\prime}+c^{\prime}\right)-4$ gates, 10 inputs, minimal

7.15 (c) From K-maps:
$F=a d+a^{\prime} c d^{\prime}+b c d$
$=a d+a^{\prime} c d^{\prime}+a^{\prime} b c-4$ gates, 11 inputs
$F=(a+c)\left(a^{\prime}+d\right)\left(a+b+d^{\prime}\right)-4$ gates, 10
inputs, minimal

7.14 (b) Beginning with the sum of products solution, we get

$$
\begin{aligned}
& f=a^{\prime} b+a b^{\prime}+d^{\prime}\left(a^{\prime} c+a c^{\prime}\right) \\
& =a^{\prime} b+a b^{\prime}+d^{\prime}\left(a^{\prime}+c^{\prime}\right)(a+c)-6 \text { gates, } \\
& 14 \text { inputs }
\end{aligned}
$$

But, beginning with the product of sums solution above, we get
$f=\left(a+b+c d^{\prime}\right)\left(a^{\prime}+b^{\prime}+c^{\prime} d^{\prime}\right)-5$ gates, 12 inputs, which is minimum

7.15 (b) From K-maps:
$F=c d+a c+b^{\prime} c^{\prime}-4$ gates, 9 inputs
$F=\left(b^{\prime}+c\right)\left(a+c^{\prime}+d\right)-3$ gates, 7 inputs,
minimal

7.15 (d) From K-maps:
$F=a^{\prime} b+a c+b d^{\prime}-4$ gates, 9 inputs, minimal $F=(a+b)\left(a^{\prime}+c+d^{\prime}\right)\left(a^{\prime}+b+c\right)$
$=(a+b)\left(a^{\prime}+c+d^{\prime}\right)(b+c+d)-4$ gates, 11 inputs


## Unit 7 Solutions

7.16 (a) In this case, multi-level circuits do not improve the solution. From K-maps:
$F=A B C^{\prime}+A C D+A^{\prime} B C+A^{\prime} C^{\prime} D-5$ gates, 16 inputs, minimal
$F=\left(A^{\prime}+B+C\right)(A+C+D)\left(A^{\prime}+C^{\prime}+D\right)$ $\left(A+B+C^{\prime}\right)-5$ gates, 16 inputs, also minimal
Either answer is correct.

7.16 (b) Too many variables to use a K-map; use algebra. Add $A C E$ by consensus, then use $X+X Y=X$

$F=A B E(F+G)+A C\left(D^{\prime}+E\right)$


5 gates, 13 inputs, minimal


$=\mathrm{ABEF}+\mathrm{ACD}^{\prime}+\mathrm{ABEG}+\mathrm{ACE}$

7.17 (a)

|  | $A B C D$ | $F$ | $\begin{aligned} & \\ & 7.17 \text { (b) } \\ & \mathrm{F} \\ & \mathrm{F}\end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 0 | 0000 | 0 |  |
| 1 | 0001 | 0 |  |
| 2 | 0010 | 0 |  |
| 3 | 0011 | 1 |  |
| 4 | 0100 | 0 |  |
| 5 | 0101 | 1 |  |
| 6 | 0110 | 1 |  |
| 7 | 0111 | 1 |  |
| 8 | 1000 | 0 |  |
| 9 | 1001 | 1 |  |
| 10 | 1010 | 1 |  |
| 11 | 1011 | 1 |  |
| 12 | 1100 | 1 |  |
| 13 | 1101 | 1 |  |
| 14 | 1110 | 1 |  |
| 15 | 1111 | 1 |  |



$$
\begin{aligned}
F= & (A+C+D)(A+B+C) \\
& (A+B+D)(B+C+D) \\
= & (A+D+B C)(B+C+A D) \underline{\text { or }} \\
= & (A+C+B D)(B+D+A C) \underline{\text { or }} \\
= & (C+D+A B)(A+B+C D)
\end{aligned}
$$

This solution has 5 gates, 12 inputs. Beginning with the sum of products requires 6 gates.

7.18 (a) $F(w, x, y, z)=\left(x+y^{\prime}+z\right)\left(x^{\prime}+y+z\right) w$


From Karnaugh map: $F=w x y+w x^{\prime} y^{\prime}+w z$

7.18 (b) $\quad F(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d})=\sum m(4,5,8,9,13)$

From Kmap:
$F=a^{\prime} b c^{\prime}+a b^{\prime} c^{\prime}+b c^{\prime} d$
$F=a^{\prime} b c^{\prime}+a b^{\prime} c^{\prime}+a c^{\prime} d$
$F=c^{\prime}(a+b)\left(a^{\prime}+b^{\prime}+d\right)$




## Unit 7 Solutions

7.19 (a)


From Kmap:

$$
F=\left(y^{\prime}+z\right)\left(x^{\prime}+y+z^{\prime}\right)
$$


7.19 (b)

7.20 (a) Using OR and NOR gates:

$\mathrm{f}=\mathrm{a}^{\prime} \mathrm{b}+\mathrm{cd}$

7.20 (b) Using NOR gates only:

7.21 (a) NAND gates:
$F=D^{\prime}+B^{\prime} C+A^{\prime} B$

NOR gates:
$F=\left(A^{\prime}+B^{\prime}+D^{\prime}\right)\left(B+C+D^{\prime}\right)$
7.21 (b) NAND gates:
$f=a^{\prime} b c^{\prime}+a c^{\prime} d^{\prime}+b^{\prime} c d$

NOR gates:
$f=\left(b^{\prime}+c^{\prime}\right)\left(c^{\prime}+d\right)(a+b+c)\left(a^{\prime}+c+d^{\prime}\right)$
7.21 (c) NAND gates:
$f=a^{\prime} b^{\prime} d^{\prime}+b c^{\prime} d+c d^{\prime}$
NOR gates:
$f=\left(b+d^{\prime}\right)\left(b^{\prime}+d\right)\left(a^{\prime}+c\right)\left(b^{\prime}+c^{\prime}\right)$
$f=\left(b+d^{\prime}\right)\left(b^{\prime}+d\right)\left(a^{\prime}+c\right)\left(c^{\prime}+d^{\prime}\right)$
7.21 (e) NAND gates:
$F=A C D^{\prime}+A B E^{\prime}+C D E+A^{\prime} B^{\prime} C^{\prime} D^{\prime}+B^{\prime} D^{\prime} E+A^{\prime} B^{\prime} D E^{\prime}$
$F=A C D^{\prime}+A B E^{\prime}+C D E+A^{\prime} B^{\prime} C^{\prime} E^{\prime}+A^{\prime} B^{\prime} C D+B^{\prime} D^{\prime} E$
$F=A C D^{\prime}+A B E^{\prime}+C D E+A^{\prime} B^{\prime} D E^{\prime}+A^{\prime} B^{\prime} C^{\prime} E^{\prime}+B^{\prime} D^{\prime} E$
$F=A C D^{\prime}+A B E^{\prime}+C D E+A^{\prime} B^{\prime} D E^{\prime}+A^{\prime} B^{\prime} C^{\prime} E+B^{\prime} C E$

NOR gates:
$F=\left(A+C^{\prime}+D+E\right)\left(C+D^{\prime}+E^{\prime}\right)\left(A+B^{\prime}+E\right)$

$$
\left(A^{\prime}+B+D^{\prime}+E\right)\left(A^{\prime}+B+C\right)\left(B^{\prime}+D+E^{\prime}\right)
$$

7.21 (g) NAND gates:
$f=x^{\prime} y^{\prime}+w y^{\prime}+w^{\prime} z^{\prime}+w z$
$f=x^{\prime} y^{\prime}+w y^{\prime}+w x^{\prime}+w^{\prime} z^{\prime}$
$f=x^{\prime} y^{\prime}+w y^{\prime}+y^{\prime} z^{\prime}+w z$
NOR gates:
$f=\left(w+x^{\prime}+z^{\prime}\right)\left(w+y^{\prime}+z^{\prime}\right)\left(w^{\prime}+y^{\prime}+z\right)$
$f=\left(w+x^{\prime}+z^{\prime}\right)\left(w+y^{\prime}+z^{\prime}\right)\left(w^{\prime}+x^{\prime}+y^{\prime}\right)$
7.21 (d) NAND gates:

$$
\begin{aligned}
F= & A^{\prime} B^{\prime} C D^{\prime}+A C^{\prime} E+C^{\prime} D E+A D E+A^{\prime} B C D E^{\prime}+ \\
& A C^{\prime} D+B^{\prime} C^{\prime} E^{\prime}+A B^{\prime} D \\
F= & A^{\prime} B^{\prime} C D^{\prime}+A C^{\prime} E+C^{\prime} D E+A D E+A^{\prime} B C D E^{\prime}+ \\
& A C^{\prime} D+B^{\prime} C^{\prime} E^{\prime}+A B^{\prime} E^{\prime}
\end{aligned}
$$

NOR gates:
$F=\left(B^{\prime}+D+E\right)\left(A^{\prime}+C^{\prime}+D\right)\left(A+B+C^{\prime}+D^{\prime}\right)$
$\left(A+B^{\prime}+C+E\right)\left(A^{\prime}+B^{\prime}+C^{\prime}+E\right)$
$\left(A+C+D+E^{\prime}\right)\left(A+B^{\prime}+C^{\prime}+E^{\prime}\right)$
7.21 (f) NAND gates:
$f=c^{\prime} d^{\prime}+a^{\prime} b+a^{\prime} d^{\prime}+a b^{\prime} c^{\prime}$
NOR gates:
$f=\left(a+b+d^{\prime}\right)\left(a^{\prime}+b^{\prime}+d^{\prime}\right)\left(a^{\prime}+c^{\prime}\right)$
7.22
(a) F is 0 if any 3 (or 4 ) of the inputs are 1 so

$$
\begin{aligned}
F= & \left(A+B^{\prime}+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C+D^{\prime}\right) \\
& \left(A^{\prime}+B^{\prime}+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B^{\prime}+C^{\prime}+D\right) \\
& \left(A^{\prime}+B+C^{\prime}+D^{\prime}\right) \\
= & \left(A^{\prime}+B^{\prime}+C^{\prime}\right)\left(A^{\prime}+B^{\prime}+D^{\prime}\right)\left(A^{\prime}+C^{\prime}+D^{\prime}\right) \\
& \left(B^{\prime}+C^{\prime}+D^{\prime}\right)
\end{aligned}
$$

(b) $F=\left(A^{\prime}+B^{\prime}+C^{\prime} D^{\prime}\right)\left(A^{\prime} B^{\prime}+C^{\prime}+D^{\prime}\right)$ or $F=\left(A^{\prime}+C^{\prime}+B^{\prime} D^{\prime}\right)\left(A^{\prime} C^{\prime}+B^{\prime}+D^{\prime}\right)$

7.23 (a)

| ab |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 |
| 00 | 0 | 1 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 1 | 0 | 0 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## Unit 7 Solutions

| 7.23 (b) |  | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 1 | 0 | 1 | 1 |
|  | 01 | 1 | 0 | 1 | 1 |
|  | 11 | 0 | 0 | 0 | 0 |
|  | 10 | 1 | 0 | 1 | 1 |
|  |  | $\mathrm{f}^{\prime}=\left(\mathrm{a}+\mathrm{b}^{\prime}\right)\left(\mathrm{c}^{\prime}+\mathrm{d}^{\prime}\right)$ |  |  |  |




7.24 (c) $f=A\left(B^{\prime}+C^{\prime}\right)+A^{\prime} B C+B^{\prime} C^{\prime}$

7.24 (b) $f=A(A B)^{\prime}+(A B)^{\prime}\left[A B+B^{\prime}+C\right] B+$
$\left[A B+B^{\prime}+C\right] C^{\prime}$
$=A B^{\prime}+\left(A^{\prime}+B^{\prime}\right)[A B+B C]+A C^{\prime}+B^{\prime} C^{\prime}$
$=A B^{\prime}+A^{\prime} B C+A C^{\prime}+B^{\prime} C^{\prime}$

7.25 (b) $f=A\left(B^{\prime}+C^{\prime}\right)+A^{\prime} B C+B^{\prime} C^{\prime}$
$=\left[A+A^{\prime} B C+B^{\prime} C^{\prime}\right]\left[B^{\prime}+C^{\prime}+A^{\prime} B C+B^{\prime} C^{\prime}\right]$
$=\left[A+B C+B^{\prime} C^{\prime}\right]\left[A^{\prime}+B^{\prime}+C^{\prime}\right]$
$=\left[A+B+B^{\prime} C^{\prime}\right]\left[A+C+B^{\prime} C^{\prime}\right]\left[A^{\prime}+B^{\prime}+C^{\prime}\right]$
$=\left[A+B+C^{\prime}\right]\left[A+C+B^{\prime}\right]\left[A^{\prime}+B^{\prime}+C^{\prime}\right]$
7.25 (c) $f=\left[A+B+C^{\prime}\right]\left[A+C+B^{\prime}\right]\left[A^{\prime}+B^{\prime}+C^{\prime}\right]$
$=\left[A+\left(B+C^{\prime}\right)\left(B^{\prime}+C\right]\left[A^{\prime}+B^{\prime}+C^{\prime}\right]\right.$
$=\left[A+B C+B^{\prime} C^{\prime}\right]\left[A^{\prime}+B^{\prime}+C^{\prime}\right]$


7.27 (a)

7.27 (b) $f=\left(A^{\prime}+B\right)\left(C^{\prime}+C D\right)\left(D^{\prime}+C D\right)+B^{\prime}\left(C^{\prime}+D^{\prime}\right) C$
$=\left(A^{\prime}+B\right)\left(C^{\prime}+D\right)\left(D^{\prime}+C\right)+B^{\prime} C D^{\prime}$
$=\left(A^{\prime}+B\right)\left(C^{\prime} D^{\prime}+C D\right)+B^{\prime} C D^{\prime}$
$=A^{\prime} C^{\prime} D^{\prime}+A^{\prime} C D+B C^{\prime} D^{\prime}+B C D+B^{\prime} C D^{\prime}$
7.28 (a)

$f=\left(b+c^{\prime}+d\right)\left(b+c^{\prime}+e^{\prime}\right)\left(b^{\prime}+d^{\prime}+e\right)$ $(a+c+d)\left(a+b^{\prime}+d^{\prime}\right)$
7.29

$$
\begin{aligned}
& f=\left(a^{\prime}+d\right)\left(a^{\prime}+b+c\right)\left(a+b^{\prime}\right) \\
& =\left(a+b^{\prime}\right)\left[a^{\prime}+d(b+c)\right] \\
& =\left(a+b^{\prime}\right)\left(a^{\prime}+b d+c d\right)
\end{aligned}
$$

7.28 (b) $f=\left(b^{\prime}+d^{\prime}+a e\right)\left(b+c^{\prime}+d e^{\prime}\right)(a+c+d)$

7.30 (a) $Z=a b e ' f+c^{\prime} e^{\prime} f+d ' e ' f+g h$

$$
=e^{\prime} f\left(a b+c^{\prime}+d^{\prime}\right)+g h
$$


7.31

$$
\begin{aligned}
F= & a b d e^{\prime}+a^{\prime} b^{\prime}+c \\
& =\left(a+b^{\prime}\right)\left(a^{\prime}+b d e^{\prime}\right)+c \\
& =\left(a+b^{\prime}+c\right)\left(a^{\prime}+c+b d e^{\prime}\right)
\end{aligned}
$$



Alternate: $F=\left(a^{\prime}+b+c\right)\left(b^{\prime}+c+a d e^{\prime}\right)$
7.33 (a)


Draw AND-OR circuit and replace all gates with NANDs.
7.33 (c) $F=B\left(A^{\prime} D+C^{\prime} D\right)+B^{\prime}\left(A^{\prime} D^{\prime}+C D\right)$

7.30 (b) $Z=\left(a^{\prime}+b+e+f\right)\left(c^{\prime}+a^{\prime}+b\right)\left(d^{\prime}+a^{\prime}+b\right)(g+h)$
$=\left[a^{\prime}+b+c^{\prime} d^{\prime}(e+f)\right](g+h)$

7.32

$$
\begin{aligned}
f= & x^{\prime} y z+x v y^{\prime} w^{\prime}+x v y^{\prime} z^{\prime} \\
& =x^{\prime} y z+x v y^{\prime}\left(z^{\prime}+w^{\prime}\right)
\end{aligned}
$$


7.33 (b)

$$
\begin{aligned}
& F=\left(B+C+D^{\prime}\right)\left(B^{\prime}+D\right)\left(A^{\prime}+D\right)\left(A^{\prime}+B^{\prime}+C^{\prime}\right)
\end{aligned}
$$

Draw OR-AND circuit and replace all gates with NORs.

Alternative:

$$
\begin{aligned}
F= & A^{\prime}\left(B^{\prime} D^{\prime}+B D\right)+D\left(B^{\prime} C+B C^{\prime}\right) \\
& =D\left(A^{\prime} B+B C^{\prime}\right)+B^{\prime}\left(A^{\prime} D^{\prime}+C D\right) \\
& =A^{\prime}\left(B^{\prime} D^{\prime}+C D\right)+D\left(B^{\prime} C+B C^{\prime}\right) \\
& =D\left(A^{\prime} C+B C^{\prime}\right)+B^{\prime}\left(A^{\prime} D^{\prime}+C D\right)
\end{aligned}
$$

7.34 (a)


$$
\begin{aligned}
F= & A B C D+A B C^{\prime} D^{\prime}+A B^{\prime} C^{\prime} D+A^{\prime} B C D^{\prime}+ \\
& A^{\prime} B^{\prime} C D+A^{\prime} B^{\prime} C^{\prime} D^{\prime}
\end{aligned}
$$

7.34 (b)


$$
\begin{aligned}
F= & \left(A+C+D^{\prime}\right)\left(B+C^{\prime}+D\right)\left(A+B^{\prime}+C\right) \\
& \left(A+B^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+D\right)\left(A^{\prime}+B+C^{\prime}\right) \\
& \left(B^{\prime}+C+D^{\prime}\right)\left(A^{\prime}+C^{\prime}+D\right)
\end{aligned}
$$

7.34 (c) Many solutions exist. Here is one, drawn with alternate gate symbols.
$F=A^{\prime}\left(B^{\prime} C^{\prime} D^{\prime}+B^{\prime} C D+B C D^{\prime}\right)+A\left(B^{\prime} C^{\prime} D+B C^{\prime} D^{\prime}+B C D\right)$
$=A^{\prime}\left(B^{\prime}\left(C^{\prime} D^{\prime}+C D\right)+B C D^{\prime}\right)+A\left(B\left(C^{\prime} D^{\prime}+C D\right)+B^{\prime} C^{\prime} D\right)$

7.35 (a) $F=A^{\prime} B C^{\prime}+B D+A C+B^{\prime} C D^{\prime}$

$$
=B\left(D+A^{\prime} C^{\prime}\right)+C\left(A+B^{\prime} D^{\prime}\right)
$$




Many NOR solutions exist. Here is one.

$$
\begin{aligned}
F & =(B+C)\left(A^{\prime}+C+D\right)\left(A+B+D^{\prime}\right)\left(A+B^{\prime}+C^{\prime}+D\right) \\
& =(B+C)\left[A+\left(B+D^{\prime}\right)\left(B^{\prime}+C^{\prime}+D\right)\right]\left(A^{\prime}+C+D\right) \\
& =(B+C)\left[A(C+D)+A^{\prime}\left(B+D^{\prime}\right)\left(B^{\prime}+C^{\prime}+D\right)\right] \\
& =(B+C)\left[A(C+D)+A^{\prime}\left(B\left(C^{\prime}+D\right)+B^{\prime} D^{\prime}\right)\right]
\end{aligned}
$$



## Unit 7 Solutions

7.35 (b)


$F=(A+C)(B+D)\left(A^{\prime}+B+C^{\prime}\right)\left(B^{\prime}+C+D^{\prime}\right)$




$$
F=\left(B^{\prime}+C+D^{\prime}\right)\left(A^{\prime}+B+C^{\prime}\right)(B+D)(A+C)
$$

$$
=\left(C+A\left(B^{\prime}+D^{\prime}\right)\right)\left(B+D\left(A^{\prime}+C^{\prime}\right)\right)
$$

$F=\sum m(0,1,2,3,4,5,7,9,11,13,14,15)$
$F=D+A^{\prime} B^{\prime}+A^{\prime} C^{\prime}+A B C$
$=D+A^{\prime}\left(B^{\prime}+C^{\prime}\right)+A B C$

Alternate solution:
$F=D+\left(A^{\prime}+B C\right)\left(A+B^{\prime}+C^{\prime}\right)$
7.36


$$
F=A^{\prime} B^{\prime}+A^{\prime} C^{\prime}+D+A B C
$$

7.36 (b)

7.36 (c)

7.37
$Z=A\left[B C^{\prime}+D+E\left(F^{\prime}+G H\right)\right]$
7.38

(a) No-NOR-AND is equivalent to NOT-AND-AND.
(b) Yes-NOR-OR is equivalent to OR-AND-NOT.
(c) No-NOR-NAND is equivalent to OROR.
(d) Yes-NOR-XOR is equivalent to NOT-AND-XOR.

(e) Yes-NAND-AND is equivalent to NOT-OR-AND.
(f) No-NAND-OR is equivalent to NOT-OR-OR..
(g) No-NAND-NOR is equivalent to ANDAND.
(h) Yes-NAND-XOR is equivalent to ANDXOR or AND-XOR-NOT.
7.39

$\mathrm{f}_{1}=\underline{\mathrm{b} \mathrm{c}^{\prime} \mathrm{d}^{\prime}}+\mathrm{ac}$

$\mathrm{f}_{2}=\mathrm{b}^{\prime} \mathrm{c}+\underline{\mathrm{b} \mathrm{c}^{\prime} \mathrm{d}^{\prime}}$

$\mathrm{f}_{3}=\mathrm{ab}+\mathrm{ad}+\underline{\mathrm{b}^{\prime} \mathrm{d}^{\prime}}$

## Unit 7 Solutions

7.40
ab

$\mathrm{f}_{1}=\mathrm{a}^{\prime} \mathrm{c}+\underline{\underline{a^{\prime} b d}}+\underline{\underline{\mathrm{a} b^{\prime} d^{\prime}}}$
c d

$f_{2}=a^{\prime} b^{\prime}+\underline{a^{\prime} b d}+\underline{\underline{a b} b^{\prime}}$

6 gates
7.41


$$
\mathrm{f}_{2}=\underline{\underline{\underline{\underline{\underline{\prime}}}}} \underline{\underline{x^{\prime} \mathrm{yz}}+\underline{\underline{x y z}}}
$$

8 gates
7.42 (a)

$\mathrm{f}_{1}$

$\mathrm{f}_{2}$

$f_{1}=\left(\underline{a^{\prime}+b+d}\right)\left(\underline{\underline{b^{\prime}+c^{\prime}+d^{\prime}}}\right)\left(b^{\prime}+d\right)-6$ gates
$f_{2}=\left(\underline{a^{\prime}+b+d}\right)\left(\underline{\underline{b^{\prime}+c^{\prime}+d^{\prime}}}\right)\left(b+d^{\prime}\right)$
7.42 (b)


Circle 1's to get sum-of-products expressions:
$f_{1}=\underline{b c^{\prime} d}+\underline{\underline{a^{\prime} b b^{\prime}}}{ }^{\prime}+b^{\prime} d-6$ gates
$f_{2}=\underline{b c^{\prime} d}+\underline{\underline{a^{\prime} b^{\prime} d^{\prime}}}+b d^{\prime}$
Then convert directly to NAND gates.
7.43 (a) Circle 0's

$f_{1}=(\underline{a+c+d})\left(b^{\prime}+c^{\prime}\right)\left(c^{\prime}+d\right)$
c d

|  | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | © | © | 0 | 0 |
| 01 | 1 | 1 | $\bigcirc$ | 0 |
| 11 | 1 | 1 | $\bigcirc$ | 1 |
| 10 | 1 | 1 | 1 | 1 |

$f_{2}=(\underline{a+c+d})\left(a^{\prime}+c\right)\left(a^{\prime}+b^{\prime}+d^{\prime}\right)$


7 gates
7.43 (b) Circle 1's to get sum-of-products expressions:

$\mathrm{f}_{2}=\mathrm{a}^{\prime} \mathrm{d}+\mathrm{cd}+\underline{\mathrm{b}^{\prime} \mathrm{cd}}$

Then convert directly to NAND gates

7.44 (a)


| $c$ | $d$ |
| :--- | :--- | $\begin{array}{llll}a b & 01 & 11 & 10\end{array}$


$f_{2}=b d^{\prime}+\underline{b^{\prime} c d}+\underline{\underline{a b c ' d}}$

7.44 (b)


Unit 7 Solutions
7.45 (a)

$\mathrm{f}_{1}=\mathrm{a}^{\prime} \mathrm{d}^{\prime}+\underline{\underline{a^{\prime} b c d}+\underline{\underline{a c d}}}$

| ab$00$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 1 | 1 | 0 | 0 |
| 01 | 1 | 1 | 0 | 0 |
| 11 | 0 | (1) | 0 | 0 |
| 10 | 0 | 0 | 1 | $1)$ |

$\mathrm{f}_{2}=\mathrm{a}^{\prime} \mathrm{c}^{\prime}+\underline{\mathrm{a}^{\prime} \mathrm{bcd}}+\underline{\underline{\mathrm{acd}^{\prime}}}$
7.45 (b)

7.46 (a) The circuit consisting of levels 2, 3, and 4 has OR gate outputs. Convert this circuit to NAND gates in the usual way, leaving the AND gates at level 1 unchanged. The result is:

7.46 (b) One solution would be to replace the two AND gates in (a) with NAND gates, and then add inverters at the output. However, the following solution avoids adding inverters at the outputs:

$$
\begin{aligned}
& F_{1}=\left[\left(a+b^{\prime}\right) c+d\right]\left(e^{\prime}+f\right) \\
& =a c e^{\prime}+b^{\prime} c e^{\prime}+d e^{\prime}+a c f+b^{\prime} c f+d f \\
& =\underline{c e^{\prime}\left(a+b^{\prime}\right)}+d\left(e^{\prime}+f\right)+\underline{c f\left(a+b^{\prime}\right)} \\
& F_{2}=\left[\left(a+b^{\prime}\right) c+g^{\prime}\right]\left(e^{\prime}+f\right) h \\
& =h\left(a c e^{\prime}+b^{\prime} c e^{\prime}+a c f+b^{\prime} c f\right)+g^{\prime} h\left(e^{\prime}+f\right) \\
& =h\left[c e^{\prime}\left(a+b^{\prime}\right)+\underline{\left.c f\left(a+b^{\prime}\right)\right]+g^{\prime} h\left(e^{\prime}+f\right)}\right.
\end{aligned}
$$



## Unit 8 Problem Solutions

8.1

8.2 (a) (contd)


Static 0-hazards are: $0001 \leftrightarrow 0011$ and $1000 \leftrightarrow 1001$
8.2 (c)


$$
F^{t}=\left(A+C^{\prime}\right)\left(A^{\prime}+C+D\right)\left(B+C+D^{\prime}\right)
$$

$$
\left(A^{\prime}+B+C\right)\left(A+B+D^{\prime}\right)
$$

8.3 (b) Modified circuit (to avoid hazards)
8.2 (a)


Static 1-hazards: $1101 \leftrightarrow 1111$ and $0100 \leftrightarrow 0101$
8.2 (b)

$F^{t}=A^{\prime} C^{\prime} D^{\prime}+A C+B C^{\prime} D+\underline{A^{\prime} B C^{\prime}}+\underline{A B D}$
8.3 (a)

(static '1' hazard)
8.4 $A=1 ; B=Z ; C=1 \cdot Z=X ; D=1+Z=1$; $E=X^{\prime}=X ; F=1^{\prime}=0 ; G=X \cdot 0=0$; $H=X+0=X$
See FLD Table 8-1, p. 231.

## Unit 8 Solutions

$A=B=0, C=D=1$
So $F=A B^{\prime} D+B C^{\prime} D^{\prime}+B C D=0$

But in the figure, gate 4 outputs $\mathrm{F}=1$, indicating something is wrong. For the last NAND gate, $\mathrm{F}=0$ only when all its inputs are 1 . But the output of gate 3 is 0 . Therefore, gate 4 is working properly, but gate 3 is connected incorrectly or is malfunctioning.
8.6 (a)


The circuit has three static 0 -hazards: $0001 \leftrightarrow 0011,1001 \leftrightarrow 1011$ and $1000 \leftrightarrow 1010$. Two sum terms are needed to eliminate the hazards: ( $\left.\mathrm{A}^{\prime}+\mathrm{B}\right)\left(\mathrm{B}+\mathrm{D}^{\prime}\right)$
8.7 (a)

$f=\left(a+d^{\prime}\right)\left(b^{\prime}+c+d\right)\left(a^{\prime}+c^{\prime}+d^{\prime}\right)\left(b^{\prime}+c^{\prime}+d\right)$
The static- 0 hazards are $0100 \leftrightarrow 0101$, $0100 \leftrightarrow 0110,0111 \leftrightarrow 0110,1100 \leftrightarrow 1110$, $1111 \leftrightarrow 1110,0011 \leftrightarrow 1011$ and $0111 \leftrightarrow 1111$.
8.7 (b) The minimal POS expression for f is $\mathrm{f}(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d})$ $=\left(a+d^{\prime}\right)\left(b^{\prime}+d\right)\left(c^{\prime}+d^{\prime}\right)$ but $\left(a+b^{\prime}\right)$ and $\left(b^{\prime}+c^{\prime}\right)$ must be added to eliminate the static-0 hazards.


Static-1 Hazards: $0000 \leftrightarrow 0010,1101 \leftrightarrow 1001$
8.6 (b)

8.8 (b)
8.8 (a) contd

$F=B D+A^{\prime} C+A B^{\prime} C^{\prime}+A^{\prime} B^{\prime} D^{\prime}$.


Static-1 Hazards: $0000 \leftrightarrow 1000,1101 \leftrightarrow 1001$


Static-1 Hazards: $0000 \leftrightarrow 0010,1000 \leftrightarrow 1001$

Hazard-free AND-OR circuit function: $f(A, B, C, D)=B D+A^{\prime} C+A C^{\prime} D+B^{\prime} C^{\prime} D^{\prime}+$ $A^{\prime} B^{\prime} D^{\prime}+A B^{\prime} C^{\prime}$


$$
\begin{aligned}
F= & \left(A+B+C+D^{\prime}\right)\left(B^{\prime}+C+D\right) \\
& \left(A^{\prime}+B+C^{\prime}\right)\left(A^{\prime}+B^{\prime}+D\right)
\end{aligned}
$$

Static-0 Hazard: $1110 \leftrightarrow 1010$


$$
\text { tatic-0 Hazard: } 1110 \leftrightarrow 1010
$$

Unit 8 Solutions


Static-0 Hazard: $1100 \leftrightarrow 1110$

Hazard-free OR-AND circuit function:
$f(A, B, C, D)=\left(A+B+C+D^{\prime}\right)\left(B^{\prime}+C+D\right)$

$$
\left(A^{\prime}+B+C^{\prime}\right)\left(A^{\prime}+B^{\prime}+D\right)\left(A^{\prime}+C^{\prime}+D\right)
$$

8.9 (a)

8.9 (b) Since a circuit with NOR gates is desired, start with POS expressions for f that corresponds to a hazardfree OR-AND (NOR-NOR) circuit. From the Karnaugh map, all prime implicants are required, $f=\left(A^{\prime}+C^{\prime}\right)\left(A+B^{\prime}\right)(A+D)\left(C^{\prime}+D\right)\left(B^{\prime}+C^{\prime}\right)$.

$$
\mathrm{f}=(\mathrm{A}+\mathrm{D})\left(\mathrm{A}+\mathrm{B}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}^{\prime}\right)\left(\mathrm{C}^{\prime}+\mathrm{D}\right)\left(\mathrm{B}^{\prime}+\mathrm{C}^{\prime}\right)
$$


$f=\left(A^{\prime} \mathrm{B}^{\prime}+A C^{\prime}\right)(A+D)=A A^{\prime} \mathrm{B}^{\prime}+A C^{\prime}+A^{\prime} \mathrm{B}^{\prime} D+$
$\left.A C^{\prime} D\right)=A A^{\prime} B^{\prime}+A C^{\prime}+A^{\prime} B^{\prime} D$
static-1 hazard: $0001 \leftrightarrow 1001$
static-0 hazard: $0010 \leftrightarrow 1010$
potential dynamic hazards:
$0000 \leftrightarrow 1000$ and $0011 \leftrightarrow 1011$
dynamic hazard: $0000 \leftrightarrow 1000$
(Note the $0011 \leftrightarrow 1011$ change only propagates over one path in the circuit and is not a dynamic hazard.)
f can be multiplied out as $f=\left(A^{\prime} B^{\prime} D+C^{\prime}\right)\left(A+B^{\prime} D\right)$. When this expression is expanded to a POS, it does not contain any sum of the form $\left(X+X^{\prime}+\beta\right)$ so the corresponding circuit is free of hazards. The three level NOR circuit is.


It is possible to start with a SOP that is free of hazards, namely, $f=A C^{\prime}+A^{\prime} B^{\prime} D+B^{\prime} C^{\prime} D$, and then factor it, e.g., the same result as above is obtained by $f=\left(A+B^{\prime} D\right) C^{\prime}+A^{\prime} B^{\prime} D=\left(A^{\prime} B^{\prime} D+C^{\prime}\right)\left(A+B^{\prime} D\right)$.
8.10
8.11 (a) contd

$f=A B^{\prime} C^{\prime}+A B D^{\prime}+B D^{\prime}$.
8.11 (a) $f=(A+B)\left(B^{\prime} C^{\prime}+B D^{\prime}\right)$
$=A B^{\prime} C^{\prime}+A B D^{\prime}+B B^{\prime} C^{\prime}+B D^{\prime}$
$=(A+B)\left(B^{\prime}+B\right)\left(B^{\prime}+D^{\prime}\right)\left(B+C^{\prime}\right)\left(C^{\prime}+D^{\prime}\right)$
From the Karnaugh map and the $B B^{\prime} C^{\prime}$ term
static-1 hazard: $1100 \leftrightarrow 1000$
static-0 hazard: $0001 \leftrightarrow 0101$
potential dynamic hazards:

$$
0000 \leftrightarrow 0100 \text { and } 1101 \leftrightarrow 1001
$$

The circuit shows that only $0000 \leftrightarrow 0100$ propagates over three paths.

8.11 (a) From the Karnaugh map for f, it is seen that a hazard-free POS expression for f requires all prime implicants.
$f=(A+B)\left(B^{\prime}+D^{\prime}\right)\left(B+C^{\prime}\right)\left(C^{\prime}+D^{\prime}\right)\left(A+D^{\prime}\right)$ $f$ can be multiplied out as $f=(A+B)\left(B^{\prime}+D^{\prime}\right)\left(B+C^{\prime}\right)\left(C^{\prime}+D^{\prime}\right)\left(A+D^{\prime}\right)=\left(A C^{\prime}+B\right)\left(A B^{\prime} C^{\prime}+D^{\prime}\right)$

$$
f=(A+B)\left(B+C^{\prime}\right)\left(B^{\prime}+D^{\prime}\right)\left(C^{\prime}+D^{\prime}\right)\left(A+D^{\prime}\right)
$$


8.13
8.12



Static 1-hazards lie between $1000 \leftrightarrow 1010$ and $0010 \leftrightarrow 0011$

Without hazards: $Z^{t}=A C^{\prime} D^{\prime}+A^{\prime} C D+B^{\prime} C D^{\prime}+$ $A^{\prime} B^{\prime} C+A B^{\prime} D^{\prime}$

## Unit 8 Solutions

8.14

$$
\begin{aligned}
A= & \mathrm{Z} ; B=0 ; C=\mathrm{Z}^{\prime}=\mathrm{X} ; D=\mathrm{Z} \cdot 0=0 ; \\
& E=\mathrm{Z} ; F=0+0+\mathrm{X}=\mathrm{X} ; G=(0 \cdot \mathrm{Z})^{\prime}=0^{\prime}=1 ; \\
& H=(\mathrm{X}+1)^{\prime}=1^{\prime}=0
\end{aligned}
$$

8.15 $A=B=C=1$, so $F=\left(A+B^{\prime}+C^{\prime}\right)\left(A^{\prime}+B+C^{\prime}\right)$ $\left(A^{\prime}+B^{\prime}+C\right)=1$
But, in the figure, gate 4 outputs $\mathrm{F}=0$, indicating something is wrong. For the last NOR gate, $\mathrm{F}=$ 1 only when all its inputs are 0 . But the output of gate 1 is 1 . Therefore, gate 4 is working properly, but gate 1 is connected incorrectly or is malfunctioning.
8.16 (a) $F(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum m(0,2,5,6,7,8,9,12,13,15)$

There are 3 different minimum AND-OR solutions to this problem. The problem asks for any two of these.

$F=B D+A C^{\prime}+A^{\prime} C D^{\prime}+B^{\prime} C^{\prime} D^{\prime}$
Solution 1: 1-hazards are between $0000 \leftrightarrow 0010$ and $0111 \leftrightarrow 0110$

$F=B D+A C^{\prime}+A^{\prime} B^{\prime} D^{\prime}+A^{\prime} B C$
Solution 2: 1-hazards are between $0010 \leftrightarrow 0110$ and $0000 \leftrightarrow 1000$

$F=B D+A C^{\prime}+A^{\prime} B^{\prime} D^{\prime}+A^{\prime} C D^{\prime}$
Solution 3: 1-hazards are between $0111 \leftrightarrow 0110$ and $0000 \leftrightarrow 1000$ Without hazards:
$F^{t}=B D+A C^{\prime}+B^{\prime} C^{\prime} D^{\prime}+A^{\prime} C D^{\prime}+$ $A^{\prime} B^{\prime} D^{\prime}+A^{\prime} B C$
8.16 (b)

$F=\left(A+B+D^{\prime}\right)\left(A+B^{\prime}+C+D\right)\left(A^{\prime}+C^{\prime}+D\right)$
$\left(A^{\prime}+B+C^{\prime}\right)$
0 -hazard is between $1011 \leftrightarrow 0011$

$F=\left(A+B+D^{\prime}\right)\left(A+B^{\prime}+C+D\right)\left(A^{\prime}+C^{\prime}+D\right)$ $\left(B+C^{\prime}+D^{\prime}\right)$
0 -hazard is between $1011 \leftrightarrow 1010$

Either way, without hazard:
$F^{t}=\left(A+B+D^{\prime}\right)\left(A+B^{\prime}+C+D\right)\left(A^{\prime}+C^{\prime}+D\right)$
$\left(B+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+C^{\prime}\right)$
$\left(B+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+B+C^{\prime}\right)$

## Unit 9 Problem Solutions

9.1 See FLD p. 703 for solution.
9.3 See FLD p. 704 for solution.
9.5

| $y_{0} y_{1} y_{2}$ | $y_{3}$ | $a$ | $b$ | $c$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| X | 1 | 0 | 0 | 0 | 0 | 1 |
| X | 1 | 0 | 1 | 1 | 0 | 1 |
| X | X | 1 | 1 | 1 | 1 | 1 |


$a=y_{3}+y_{2}$
9.2 See FLD p. 703 for solution.
9.4 See FLD p. 704 and Figure 4-4 on FLD p.105.

$b=y_{3}+y_{2} y_{1}$

$c=y_{3}+y_{2}+y_{1}+y_{0}$
9.6 See FLD p. 705 for solution.
9.8 See FLD p. 705-706 for solution.
9.10 Note: $A_{6}=A_{4}^{\prime}$ and $A_{5}=A_{4}$. Equations for $A_{4}$ through $\mathrm{A}_{0}$ can be found using Karnaugh maps. See FLD p. 707-708 for answers.
9.11 (a) $F=C^{\prime} D^{\prime}+B C^{\prime}+A^{\prime} C \rightarrow$ Use 3 AND gates
$F^{\prime}=\left[C^{\prime} D^{\prime}+B C^{\prime}+A^{\prime} C\right]^{\prime}=\left[C^{\prime}\left(B+D^{\prime}\right)+C A^{\prime}\right]^{\prime}$
$=\left[\left(C+B+D^{\prime}\right)\left(A^{\prime}+C^{\prime}\right)\right]^{\prime}$
$=B^{\prime} C^{\prime} D+A C \rightarrow$ Use 2 AND gates
9.12 (a) See FLD p. 708, use the answer for 9.12 (b), but leave off all connections to 1 and $1^{\prime}$.
9.7 See FLD p. 705 for solution.
9.9 The equations derived from Table 4-6 on FLD p. 107 are:
$D=x^{\prime} y^{\prime} b_{\text {in }}+x^{\prime} y b_{\text {in }}{ }^{\prime}+x y^{\prime} b_{\text {in }}{ }^{\prime}+x y b_{\text {in }}$ bout $=x^{\prime} b_{\text {in }}+x^{\prime} y+y b_{\text {in }}$
See p. p. 706 for PAL diagram.
9.11 (b) $F=A^{\prime} B^{\prime}+C^{\prime} D^{\prime} \rightarrow$ Use 2 AND gates
$F^{\prime}=\left(A^{\prime} B^{\prime}+C^{\prime} D^{\prime}\right)^{\prime}$
$=(A+B)(C+D)$
$=A C+A D+B C+B D \rightarrow$ Use 4 AND gates
9.12 (b) See FLD p. 708 for solution.
9.13 Using Shannon's expansion theorem:
$F=a b^{\prime} c d e^{\prime}+b c^{\prime} d^{\prime} e+a^{\prime} c d^{\prime} e+a c^{\prime} d e^{\prime}$
$=b^{\prime}\left(a c d e^{\prime}+a^{\prime} c d^{\prime} e+a c^{\prime} d e^{\prime}\right)+b\left(c^{\prime} d^{\prime} e+a^{\prime} c d^{\prime} e+a c^{\prime} d e^{\prime}\right)$
$=b^{\prime}\left[a d e^{\prime}\left(c+c^{\prime}\right)+a^{\prime} c d^{\prime} e\right]+b\left[\left(c^{\prime}+a^{\prime} c\right) d^{\prime} e+a c^{\prime} d e^{\prime}\right]$
$=b^{\prime}\left(a d e^{\prime}+a^{\prime} c d^{\prime} e\right)+b\left(c^{\prime} d^{\prime} e+a^{\prime} d^{\prime} e+a c^{\prime} d e^{\prime}\right)$
The same result can be obtained by splitting a Karnaugh map, as shown to the right.


## Unit 9 Solutions

### 9.14 (a) $R=a b^{\prime} h^{\prime}+b c h^{\prime}+e g^{\prime} h+f g h$ <br> $$
=\left(a b^{\prime}+b c\right) h^{\prime}+\left(\mathrm{e} g^{\prime}+f g\right) h
$$ <br> $$
=\left[(a) b^{\prime}+(c) b\right] h^{\prime}+\left[(e) g^{\prime}+(f) g\right] h
$$ <br> 

9.14 (b)

9.15 There are many solutions. For example:
9.16


### 9.16

 contd


9.18 Since the decoder outputs are negative, NAND gates are required. The excess-3 outputs are $\Sigma \mathrm{m}(5,6,7,8,9), \Sigma \mathrm{m}(1,2,3,4,9), \Sigma \mathrm{m}(0,3,4,7,8)$, and $\Sigma \mathrm{m}(0,2,4,6,8)$ so four 5 -input NAND gates are needed with inputs corresponding to the minterms of the excess- 3 outputs.

9.20
$f(a, b, c, d, e)=a^{\prime} b^{\prime} c d e^{\prime}+a^{\prime} b^{\prime} c d e+a^{\prime} b c^{\prime} d^{\prime} e+$ $a^{\prime} b c^{\prime} d e+a^{\prime} b c d^{\prime} e^{\prime}+a^{\prime} b c d^{\prime} e+a b^{\prime} c^{\prime} d^{\prime} e^{\prime}+a b^{\prime} c^{\prime} d^{\prime} e+$ $a b ' c ' d e^{\prime}+a b^{\prime} c d^{\prime} e^{\prime}+a b ' c d^{\prime} e+a b ' c d e+a b c^{\prime} d^{\prime} e+$ $a b c d^{\prime} e^{\prime}$
$=a\left(b^{\prime} c^{\prime} d^{\prime} e^{\prime}\right)+a\left(b^{\prime} c^{\prime} d^{\prime} e\right)+a\left(b^{\prime} c^{\prime} d e^{\prime}\right)+a\left(b^{\prime} c d^{\prime} e^{\prime}\right)$
$+a\left(b^{\prime} c d^{\prime} e\right)+a^{\prime}\left(b^{\prime} c d e^{\prime}\right)+\left[a^{\prime}\left(b^{\prime} c d e\right)+a\left(b^{\prime} c d e\right)\right]+$ $\left[a^{\prime}\left(b c^{\prime} d^{\prime} e\right)+a\left(b c^{\prime} d^{\prime} e\right)\right]+a^{\prime}\left(b c^{\prime} d e\right)+\left[a^{\prime}\left(b c d^{\prime} e^{\prime}\right)+\right.$ $\left.a\left(b c d^{\prime} e^{\prime}\right)\right]+a^{\prime}\left(b c d^{\prime} e\right)$
$\mathrm{I} 0=\mathrm{a}, \mathrm{I} 1=\mathrm{a}, \mathrm{I} 2=\mathrm{a}, \mathrm{I} 3=0, \mathrm{I} 4=\mathrm{a}, \mathrm{I} 5=\mathrm{a}, \mathrm{I} 6=\mathrm{a}^{\prime}$, $\mathrm{I} 7=1, \mathrm{I} 8=0, \mathrm{I} 9=1, \mathrm{I} 10=0, \mathrm{I} 11=\mathrm{a}, \mathrm{I} 12=1, \mathrm{I} 13$ $=\mathrm{a}^{\prime}, \mathrm{I} 14=0, \mathrm{I} 15=0$
9.19 Using S1 = w and S0 $=\mathrm{z}, \mathrm{I} 0=\mathrm{x}, \mathrm{I} 1=1, \mathrm{I} 2=\mathrm{y}$ and I3 $=0$ which does not require any gates.


Other answers: Using S1 $=\mathrm{w}$ and $\mathrm{S} 0=\mathrm{y}, \mathrm{I} 0=\mathrm{x}, \mathrm{I} 1$ $=\mathrm{z}, \mathrm{I} 2=0$ and $\mathrm{I} 3=\mathrm{z}$ ' which requires one inverter. Using S1 = w and S0 = x, I0 = z, I1 = 1, I2 = yz' and I3 = y which requires one inverter and one AND gate.


## Unit 9 Solutions

9.21 (a) | $x$ | $y$ | $c_{\text {in }}$ | Sum $C_{\text {out }}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |


9.21 (b)


### 9.21(c)


9.22 (a)

| $x$ | $y$ | $b_{\text {in }}$ | Diff |
| :--- | :--- | :--- | :--- |
| 0 | $B_{\text {out }}$ |  |  |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

$\rightarrow$ Diff
$\rightarrow B_{\text {out }}$
9.22 (b)

9.22 (c)


For a positive number $\mathrm{A},|\mathrm{A}|=\mathrm{A}$ and for a negative number $A,|A|=-A$. Therefore, if the number is negative, that is $\mathrm{A}[3]$ is 1 , then the output should be the 2's complement (that is, invert and add 1 ) of the input A .


9.25

9.26 (a)


$$
D=\sum m(1,2,4,7) ; \text { Bout }=\sum m(1,2,3,7)
$$

9.26 (b)



If any of the inputs $y_{0}$ through $y_{7}$ is 1 , then $d$ of the 8 -to- 3 decoder should be 1 . But in that case, $c_{1}$ or $c_{2}$ of one of the 4-to-2 decoders will be 1. So $d=c_{1}+c_{2}$.

If one of the inputs $y_{4}, y_{5}, y_{6}$, and $y_{7}$ is 1 , then $a$ should be 1 , and $b$ and $c$ should correspond to $a_{2}$ and $b_{2}$, respectively. Otherwise, $a$ should be 0 , and $b$ and $c$ should corresond to $a_{1}$ and $b_{1}$, respectively. So $a=c_{2}, b=c_{2} a_{2}+c_{2}{ }^{\prime} a_{1}$, and $c=c_{2} b_{2}+c_{2}{ }^{\prime} b_{1}$.
9.28


| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $S_{3}$ | $S_{2}$ | $S_{1}$ | $S_{0}$ | $C_{\text {out }}$ | Meaning |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | X | X | X | X | X | $(0000$ is a not valid input $)$ |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | $(0+0=0)$ |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | $(2+3=5)$ |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | $(7+4=11)$ |

9.29 (a)

| RSTU | VW Y Z |
| :---: | :---: |
| 0000 | 0000 |
| 0001 | 0001 |
| 0010 | 0010 |
| 0011 | 0100 |
| 0100 | 0101 |
| 0101 | 0110 |
| 0110 | 1000 |
| 0111 | 1001 |
| 1000 | 1010 |
| 1001 | 1100 |
| 1010 | XXXX |
| 1011 | XXXX |
| 1100 | XXXX |
| 1101 | XXXX |
| 1110 | XXXX |
| 1111 | X X X $\times$ |

9.29 (b)

$\mathrm{V}=\mathrm{ST}+\mathrm{R}$

$\mathrm{W}=\mathrm{S}^{\prime} \mathrm{T} \mathrm{U}+\underline{\mathrm{ST}^{\prime} \mathrm{U}}+\mathrm{RU}+\underline{\mathrm{ST}^{\prime} \mathrm{U}^{\prime}}$

9.29 (c)

| RS TU | VW Y Z |
| :---: | :---: |
| - 11 - | 1000 |
| 100 - | 1000 |
| 1--0 | 0010 |
| 1--1 | 0100 |
| -011 | 0100 |
| -101 | 0110 |
| -100 | 0101 |
| -010 | 0010 |
| 0001 | 0001 |
| -111 | 0001 |

9.30 (a)

| RS TU | VW Y Z |
| :---: | :---: |
| 0000 | 0000 |
| 0001 | 0001 |
| 0010 | 0100 |
| 0011 | 0010 |
| 0100 | X X X $\times$ |
| 0101 | X $\times \times \times$ |
| 0110 | 0101 |
| 0111 | X $\times \times \times$ |
| 1000 | 1100 |
| 1001 | 1010 |
| 1010 | 1000 |
| 1011 | 1001 |
| 1100 | X X X $\times$ |
| 1101 | X X X $\times$ |
| 1110 | 0110 |
| 1111 | X X X X |

Unit 9 Solutions
9.30 (b)

$\mathrm{V}=\mathrm{RS}^{\prime}$

$Y=R ' T U+R T^{\prime} U+R S$

$\mathrm{W}=\mathrm{R}^{\prime} \mathrm{T} \mathrm{U}^{\prime}+\mathrm{R} \mathrm{T}^{\prime} \mathrm{U}^{\prime}+\mathrm{S}$ T U

$Z=R^{\prime} T^{\prime} U+R^{\prime} S+R T U$

9.30 (c)

9.31 (a)


$$
\mathrm{F}_{1}=(\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D})\left(\mathrm{A}^{\prime}+\mathrm{C}+\mathrm{D}^{\prime}\right)\left(\mathrm{C}^{\prime}+\mathrm{D}^{\prime}\right)
$$

9.31 (a)
(contd)



Alternate solution:
$F_{1}=(a+b+c+d)\left(a+c^{\prime}+d^{\prime}\right)\left(a^{\prime}+d^{\prime}\right)$
$F_{2}=(a+b+c+d)\left(a+b^{\prime}+d^{\prime}\right)\left(c+d^{\prime}\right)$
9.31 (b)

|  | $a b c d$ | $F_{1} F_{2}$ |
| :---: | :---: | :---: |
| (cd') | --10 | 11 |
| (bd') | - 1-0 | 11 |
| (ad') | 1--0 | 11 |
| (ac) | 1-1- | 01 |
| ( $a^{\prime} c^{\prime} d$ ) | 0-01 | 10 |


9.32 (a)

| ABCD | WX Y Z |
| :---: | :---: |
| 0000 | 0111 |
| 0001 | 1000 |
| 0010 | 1001 |
| 0011 | 1100 |
| 0100 | 1110 |
| 0101 | 1010 |
| 0110 | 1101 |
| 0111 | 1111 |
| 1000 | 1011 |
| 1001 | 0101 |


$Y=\underline{A D^{\prime}}+B D+\underline{\underline{\underline{A^{\prime} C^{\prime} D^{\prime}}}}$

$Z=\underline{\underline{A D}}+\underline{\underline{\underline{B C}}}+B^{\prime} D^{\prime}$
Alt: $Z=A+\underline{\underline{\underline{B C}}}+B^{\prime} D^{\prime}$

$W=A^{\prime} D+C+B+\underline{A D^{\prime}}$
$X=\underline{\underline{\underline{\underline{A^{\prime}} C^{\prime} D^{\prime}}}}+C D+\underline{\underline{A D}}+\underline{\underline{\underline{B C}}}$


## Unit 9 Solutions

9.32 (b) $\left.\begin{array}{|l|l|lllll|}\hline a & b & c & d & W & W & Y\end{array}\right]$
9.32 (c)

9.33 (a) See solution for 7.10

| $a$ | $b$ | $c$ | $d$ | $f_{1}$ | $f_{2}$ | $f_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | - | 1 | 1 | 0 | 0 |
| -0 | 1 | 1 | 1 | 0 | 1 |  |
| 0 | 1 | - | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | - | 0 | 1 | 1 |
| - | 0 | 1 | 0 | 0 | 1 | 0 |
| -1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | - | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | - | 0 | 0 | 1 |


9.33 (b) See solution for 7.41

| $x$ | $y$ | $z$ | $f_{1}$ | $f_{2}$ | $f_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | - | 1 | 0 | 1 |
| -0 | 1 | 0 | 1 | 1 |  |
| 1 | 1 | 0 | 0 | 1 | 1 |

9.33 (c) Because a PLA works with a sum-of-products expression, see solution for 7.43(b), not (a).

| $a$ | $b$ | $c$ | $d$ | $f_{1}$ | $f_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | - | 0 | - | 1 | 0 |
| - | - | 1 | 1 | 0 |  |
| - | 0 | 1 | 1 | 1 | 1 |
| 0 | - | - | 1 | 0 | 1 |
| - | - | 1 | 0 | 0 | 1 |




$$
Z=I_{0} A^{\prime} B^{\prime} C^{\prime}+I_{1} A^{\prime} B^{\prime} C+I_{2} A^{\prime} B C^{\prime}+I_{3} A^{\prime} B C+I_{4} A B^{\prime} C^{\prime}+I_{5} A B^{\prime} C+I_{6} A B C^{\prime}+I_{7} A B C
$$

$=X_{1} A^{\prime}+X_{2} A$ where $X_{1}=I_{0} B^{\prime} C^{\prime}+I_{1} B^{\prime} C+I_{2} B C^{\prime}+I_{3} B C$ and $X_{2}=I_{4} B^{\prime} C^{\prime}+I_{5} B^{\prime} C+I_{6} B C^{\prime}+I_{7} B C$

Note: Unused inputs, outputs, and wires have been omitted from this diagram.
9.35 For an 8-to-3 encoder, using the truth table given in

FLD Figure 9-16, we get
$a=y_{4}+y_{5}+y_{6}+y_{7}$
$b=y_{2} y_{3}^{\prime} y_{4}^{\prime} y_{5}^{\prime} y_{6}^{\prime} y_{7}^{\prime}+y_{3} y_{4}^{\prime} y_{5}^{\prime} y_{6}^{\prime} y_{7}^{\prime}+y_{6} y_{7}^{\prime}+y_{7}$
$c=y_{1} y_{2}^{\prime} y_{3}^{\prime} y_{4}^{\prime} y_{5}^{\prime} y_{6}^{\prime} y_{7}^{\prime}+y_{3} y_{4}^{\prime} y_{5}^{\prime} y_{6}^{\prime} y_{7}^{\prime}+y_{5} y_{6}^{\prime} y_{7}^{\prime}+y_{7}$
$d=a+b+c+y_{0}$
Alternative solution for simplified expressions:
$b=y_{2} y_{4}^{\prime} y_{5}^{\prime}+y_{3} y_{4}^{\prime} y_{5}^{\prime}+y_{6}+y_{7}$
$c=y_{1} y_{2}^{\prime} y_{4}^{\prime} y_{6}^{\prime}+y_{3} y_{4}^{\prime} y_{6}^{\prime}+y_{5} y_{6}^{\prime}+y_{7}$


Note: Unused inputs, outputs, and wires have been omitted from this diagram.
9.36
$F=C D^{\prime} E+C D E+A^{\prime} D^{\prime} E+A^{\prime} B^{\prime} D E^{\prime}+B C D$
9.36 (a) $F=A^{\prime} B^{\prime}\left(C D^{\prime} E+C D E+D^{\prime} E+D E^{\prime}\right)+$

$$
A^{\prime} B\left(C D^{\prime} E+C D E+D^{\prime} E+C D\right)+
$$

$$
A B^{\prime}\left(C D^{\prime} E+C D E\right)+A B\left(C D^{\prime} E+C D E+C D\right)
$$

9.36 (b) $F=B^{\prime} C^{\prime}\left(A^{\prime} D^{\prime} E+A^{\prime} D E^{\prime}\right)+$

$$
B^{\prime} C\left(D^{\prime} E+D E+A^{\prime} D^{\prime} E+A^{\prime} D E^{\prime}\right)+
$$

$$
B C^{\prime}\left(A^{\prime} D^{\prime} E\right)+B C\left(D^{\prime} E+D E+A^{\prime} D^{\prime} E+D\right)
$$

9.36 (c) $F=A^{\prime} C^{\prime}\left(D^{\prime} E+B^{\prime} D E^{\prime}\right)+$
$A^{\prime} C\left(D^{\prime} E+D E+D^{\prime} E+B^{\prime} D E^{\prime}+B D\right)+$
$A C^{\prime}(0)+A C\left(D^{\prime} E+D E+B D\right)$
9.36 (d) Use the expansion about $A$ and $C$
$F=A^{\prime} C^{\prime}\left(F_{0}\right)+A^{\prime} C\left(F_{1}\right)+A C\left(F_{3}\right)$
where $F_{0}, F_{1}, F_{3}$ are implemented in lookup tables:

$F=B^{\prime} D^{\prime} E^{\prime}+A B^{\prime} C+C^{\prime} D E^{\prime}+A^{\prime} B C^{\prime} D$
9.37 (a) $F=A^{\prime} B^{\prime}\left(D^{\prime} E^{\prime}+C^{\prime} D E^{\prime}\right)+A^{\prime} B\left(C^{\prime} D E^{\prime}+C^{\prime} D\right)+A B^{\prime}\left(D^{\prime} E^{\prime}+C+C^{\prime} D\right)+A B\left(C^{\prime} D E^{\prime}\right)$
9.37 (b) $F=B^{\prime} C^{\prime}\left(D^{\prime} E^{\prime}+D E^{\prime}\right)+B^{\prime} C\left(D^{\prime} E^{\prime}+A\right)+B C^{\prime}\left(D E^{\prime}+A^{\prime} D\right)+B C(0)$
9.37 (c) $F=A^{\prime} C^{\prime}\left(B^{\prime} D^{\prime} E^{\prime}+D E^{\prime}+B D\right)+A^{\prime} C\left(B^{\prime} D^{\prime} E^{\prime}\right)+A C^{\prime}\left(B^{\prime} D^{\prime} E^{\prime}+D E^{\prime}\right)+A C\left(B^{\prime} D^{\prime} E^{\prime}+B^{\prime}\right)$

In this case, use the expansion about $B$ and $C$ to implement the function in 3 LUTs:
$F=B^{\prime} C^{\prime}\left(F_{0}\right)+B^{\prime} C\left(F_{1}\right)+B C^{\prime}\left(F_{2}\right)+B C(0)$
Here we use the LUTs to implement $F_{0}, F_{1}, F_{2}$ which are functions of $A, D, E$


| $A$ | $D$ | $E$ | $F_{0}$ | $F_{1}$ | $F_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 |

9.38

$$
\begin{aligned}
& \text { For a 4-to-1 MUX: } \\
& \begin{aligned}
Y= & A^{\prime} B^{\prime} I_{0}+A^{\prime} B I_{1}+A B^{\prime} I_{2}+A B I_{3} \\
& =A^{\prime}\left(B^{\prime} I_{0}+B I_{1}\right)+A\left(B^{\prime} I_{2}+B I_{3}\right) \\
& =A^{\prime} G+A F, \text { where } G=B^{\prime} I_{0}+B I_{1} ; F=B^{\prime} I_{2}+B I_{3}
\end{aligned}
\end{aligned}
$$

Set programmable MUX so that $Y$ is the output of MUX H.

9.39 (a)

9.39 (b)

9.39 (c)


Same answer as 9.39 except connect $E$ to the enable input in parts (a) and (c) and E'in part (b).

## Unit 9 Solutions

9.41 (a) $F=a^{\prime}+a c^{\prime} d^{\prime}+b^{\prime} c d^{\prime}+a d$

$$
\begin{aligned}
& =d^{\prime}\left(a^{\prime}+a c^{\prime}+b^{\prime} c\right)+d\left(a^{\prime}+a\right) \\
& =d^{\prime}\left(a \cdot+a c^{\prime}+b^{\prime} c\right)+d(1) \\
& =d^{\prime}(e)+d(g)
\end{aligned}
$$

9.41 (b)

9.41 (c)

| $a$ | $b$ | $c$ | $e$ | $g$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |

9.42 (a) $F=c d^{\prime}+a d^{\prime}+a^{\prime} b^{\prime} c d^{\prime}+b c^{\prime}$

$$
\begin{aligned}
& =d^{\prime}\left(c+a+b c^{\prime}\right)+d\left(a^{\prime} b^{\prime} c+b c^{\prime}\right) \\
& =d^{\prime}(e)+d(g)
\end{aligned}
$$

9.42 (b) Same as 9.41 (b).
9.42 (c)

| $a$ | $b$ | $c$ | $e$ | $g$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 |

9.43 (a) $F=b d+b c^{\prime}+a c^{\prime} d+a^{\prime} d^{\prime}$

$$
\begin{aligned}
& =d^{\prime}\left(b c^{\prime}+a^{\prime}\right)+d\left(b+b c^{\prime}+a c^{\prime}\right) \\
& =d^{\prime}\left(b c^{\prime}+a^{\prime}\right)+d\left(b+a c^{\prime}\right) \\
& =d^{\prime}(e)+d(g)
\end{aligned}
$$

9.43 (b) Same as 9.41 (b).
9.43 (c)

| $a$ | $b$ | $c$ | $e$ | $g$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |

## Unit 10 Problem Solutions

10.1 See FLD p. 709 for solution.
10.2 See FLD p. 707 for solution.
10.3 See FLD p. 710 for solution.
10.4 See FLD p. 710 for solution.
10.5 See FLD p. 710 for solution.

Notes: The function vec2int is found in bit_pack, which is in the library bitlib, so the following declarations are needed to use vec2int:
library bitlib;
use bitlib.bit_pack.all;

If std_logic is used instead of bits, then the index can be computed as:
index <= conv_integer(A\&B\&Cin); where A, B, and Cin are std_logic.
conv_integer is found in the std_logic_arith package.
10.6 See FLD p. 710 for solution.
10.7 See FLD p. 711 for solution.

Notes: In line 8 , " 00 "\&a converts a to a 18 -bit
std_logic_vector. The overloaded "+" operators automatically extend b, c, and d to 18 bits so that the sum is 18 bits. In line 9 , sum(17 downto 2 )
Add the following to the answer given on FLD p. 711:
Addout <= '0' \& E + Bus;
Sum <= Addout(3 downto 0);
Cout <= Addout(4);
drops the lower 2 bits of sum, which effectively divides by 4 to give the average. Adding sum(1) rounds up the value of $f$ if $\operatorname{sum}(1)=1$.
10.9 See FLD p. 711 for solution.
10.10 The network represented by the given code is:

(1) Statement (a) will execute as soon as either P or Q change. Hence, it will execute at 4 ns .
(2) Since the NAND gate has a delay of 10 ns , L will be updated at 14 ns .
(3) Statement (c) will execute when the value $M$ changes. It will execute at 19 ns .
(4) R will be updated at $19+\Delta \mathrm{ns}$, since $\Delta$ is the default delay time when no delay is explicitly specified.
10.11 (a) $H$ <= not $A$ nand $B$ nor not $D$ nand $E$;
(Note: not happens first, then it proceeds from left to right)

```
10.11(b)
AN <= not A after 5 ns;
    C <= AN nand B after 10ns;
    F <= not D after 5ns;
    G <= C nor F after 15ns;
    H <= G nand E after 10ns;
```

Unit 10 Solutions
10.12
10.13 $\mathrm{L}=\mathrm{X}$ (Since 1 and 0 in the resolution function yields X)
$\mathrm{M}=0$
$\mathrm{N}=1$ (1 overrides Z in the resolution function)
10.14 (a) The expression can be rewritten as:
F <= (((not E) \& "011") or "000100") and (not D);

Evaluating in this order, we get:

$$
F=000110
$$

10.15 library bitlib; use bitlib.bit_pack.all;
entity myrom is
port (A, B, C, D: in bit; W, X, Y: out bit);
end myrom;
architecture table of myrom is
type ROM16_3 is array(0 to 15)of bit_vector(0 to 2);
constant ROM1: ROM16_3 := ("010", "111", "100",
"110", "011", "110", "001", "000", "000", "111",
"100", "010", "001", "100", "101", "000");
signal index: integer range 0 to 15 ;
signal temp: bit_vector(0 to 2);
begin
index <= vec2int(A\&B\&C\&D);
temp <= ROM1(index);
W <= temp(0);
$X<=$ temp(1);
$\mathrm{Y}<=$ temp(2);
end table;
10.14 (b) LHS: not("101" \& "011") = "010100"

RHS: ("100" \& "101" and "010" \& "101") = "000101"
Since LHS > RHS, the expression evaluates to FALSE
10.16 (a)databus <= membus when mRead = ' 1 ' else "ZZZZZZZZ";
databus <= probus when $m W$ rite = '1' else "ZZZZZZZZ";
10.16 (b) The value will be determined by the std_logic resolution function. For example, if membus = "01010101" and probus = "00001111", then databus = "0X0XX1X1"
10.17 (a) with C\&D select
$\mathrm{F}<=$ not A after 15 ns when " 00 ",
$B$ after 15ns when "01",
not B after 15ns when " 10 ",
' 0 ' after 15 ns when " 11 ";
10.17 (b) $\mathrm{F}<=$ not A after 15 ns when $\mathrm{C} \& \mathrm{D}=$ " 00 "
else B after 15ns when C\&D = "01" else not $B$ after $15 n$ s when $C \& D=" 10 "$ else ' 0 ' after 15 ns ;
10.18 (b) entity main is
$\operatorname{port}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ : in bit; F: out bit);
end main;
architecture eqn of main is component mynand is
port( $\mathrm{X}, \mathrm{Y}$ : in bit; Z : out bit);
end component;
signal E, G: bit;
begin
n1: mynand port map(A, B, E);
n 2 : mynand port map(C, D, G);
n3: mynand port map(E, G, F);
end eqn;

```
10.19(a)
use IEEE.STD_LOGIC_1164.ALL;
entity hazard_circuit is
    port (a, b, c : in std_logic;
    f : out std_logic);
end hazard_circuit;
architecture hazarddf of hazard_circuit is
    signal d, e, g : std_logic;
    begin
    d <= not b after 10ns;
    e<= a and b after 10ns;
    g<= c and d after 10ns;
    f <= e or g after 10ns;
end hazarddf;
```

library IEEE;
10.19(c) change the assignment statement for d to d <= not b after 5 ns;
10.19(e) change the assignment statement for f to f <= transport e or g after 10 ns ;
10.19(g) When the output gate has a 10 ns inertial delay, the 5 ns glitch caused by the static- 1 hazard is not passed through the gate; however, with a transport delay the glitch does pass through. Note: The initial values of $d, e, f$ and $g$ are ' U ' becasue std_ logic type is used. These initial values are ' 0 ' when bit type is used.
10.19(b)

10.19(d)

10.19 (f)


## Unit 10 Solutions

| 10.20(a) | ```library IEEE; use IEEE.STD_LOGIC_1164.ALL; entity dynhaz_circuit is port ( \(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\) : in std_logic; f : out std_logic); end dynhaz_circuit; architecture hazarddf of dynhaz_circuit is signal e, g, h, i, j : std_logic; begin e <= not b after 10ns; \(\mathrm{g}<=\mathrm{a}\) and b after 10ns; \(\mathrm{h}<=\mathrm{c}\) and e after 10ns; i <= b or d after 40ns; j <= g or h after 10ns; \(\mathrm{f}<=\mathrm{i}\) and j after 10ns; end hazarddf;``` |
| :---: | :---: |

10.20(c) change the assignment statement for e to e <= not b after 5 ns ;
10.20(e) change the assignment statements for $j$ and $f$ to $\mathrm{j}<=$ transport g orh after 10 ns ; $\mathrm{f}<=$ transport i or jafter 10 ns ;
$\mathbf{1 0 . 2 0}(\mathbf{g}) \quad$ When the gate 4 has a 10 ns inertial delay, the 5 ns glitch caused by the static- 1 hazard for gate 4 is not passed through gate 4 ; however, with a transport delays for gates 4 and 5, the static- 1 hazard glitch at gate 4 does passes through gate 4 and gate 5 . In addition, the 5 ns glitch caused by the delay through gate 3 also passes through gate 5 . The three changes in f illustrate the dynamic hazard that exists in the circuit.
10.20(b)

10.20(d)



## Unit 10 Solutions

| 10.22(a) | ```library IEEE; use IEEE.STD_LOGIC_1164.ALL; entity c8421_to_excess3 is port (x : in std_logic_vector(3 downto 0); y : out std_logic_vector(3 downto 0)); end c8421_to_excess3; architecture behavioral 1 of c8421_to_excess 3 is begin y <= "0011" when \(\mathrm{x}=\) " 0000 " else "0100" when \(x=\) "0111" else "0101" when \(\mathrm{x}=\) " 0110 " else "0110" when \(\mathrm{x}=\) " 0101 " else "0111" when \(\mathrm{x}=\) " 0100 " else "1000" when \(x=\) "1011" else "1001" when \(x=\) " 1010 " else "1010" when \(x=\) "1001" else "1011" when \(x=\) "1000" else "1100" when \(\mathrm{x}=\) "1111" else "XXXX"; end behavioral1;``` |
| :---: | :---: |

10.22(b)

| Time | X | y |
| :--- | :--- | :--- |
| 0 ns | 0011 | XXXX |
| 5 ns | 0100 | 0111 |
| 10 ns | 1001 | 1010 |
| 15 ns | 1010 | 1001 |

```
10.22(c)
    library IEEE;
    use IEEE.STD_LOGIC_1164.ALL;
    entity c8421_t0_excess3 is
    port (x : in std_logic_vector(3 downt0 0);
    y : out std_logic_vector(3 downto 0));
end c8421_to_excess3;
    architecture behavioral2 of c8421_to_excess3 is
    begin
    with x select
    y <= "0011" when "0000",
        "0100" when "0111",
        "0101" when "0110",
        "0110" when "0101",
        "0111" when "0100",
        "1000" when "1011",
        "1001" when "1010",
        "1010" when "1001",
        "1011" when "1000",
        "1100" when "1111",
        "XXXX" when others;
    end behavioral2;
```

10.22(d)

| Time | x | y |
| :--- | :--- | :--- |
| 0 ns | 0100 | 0111 |
| 5 ns | 0101 | 0110 |
| 10 ns | 1001 | 1010 |
| 15 ns | 1010 | 1001 |

## Unit 11 Problem Solutions

11.1 $\quad Z$ responds to $X$ and to $Y$ after $10 \mathrm{~ns} ; ~ Y$ responds to
$Z$ after 5 ns . See FLD p. 713 for answer.
11.3 P and Q will oscillate. See FLD p. 713 for timing chart.
11.4 See FLD p. 714 for solution.
11.5 See FLD p. 714 for solution.
11.6 (a)

| $S$ | $R$ | $Q$ | $Q^{+}$ |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |


$\mathrm{Q}^{+}=\mathrm{R}^{\prime} \mathrm{Q}+\mathrm{S} \mathrm{R}^{\prime}$
11.2

See FLD p. 713 for solution. For part (b), also use the following Karnaugh map. Don't cares come from the restriction in part (a).

11.6 (b) See FLD p. 714 for solution.
11.7 See FLD p. 714 for solution.
11.8 See FLD p. 714 for solution.
11.9 See FLD p. 715 for solution.
11.10 See FLD p. 715 for solution.

### 11.11


11.13 (a)

| Present | ${\text { Next State } \mathrm{Q}^{+}}^{\text {State }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| AB | AB | AB | AB |  |
| Q | 00 | 01 | 11 | 10 |
| 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 |

$$
Q^{+}=A B+Q A+Q B
$$

11.13 (c) A change between $A B=01$ and 10 can cause Q to change depending on the inverter delays.
11.13 (b)


$$
Q^{+}=A B+Q(A+B)
$$

11.13 (d) $P=Q^{\prime}+A^{\prime} B^{\prime}$ equals $Q^{\prime}$ in all stable states.
11.13 (e)

11.14 (a)

| Present | Next State Q $^{+}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| State | A B |  |  |  |
| Q | 00 | 01 | 11 | 10 |
| 0 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | 1 |
| 1 | 0 | 0 | $\mathbf{1}$ | $\mathbf{1}$ |

11.13 (e) A change between $A B=01$ and 10 can cause Q to change depending on the inverter delays.
$P=Q^{\prime}\left(A^{\prime}+B^{\prime}\right)$ equals $Q^{\prime}$ in all stable states.
$11.14 \quad Q^{+}=A\left(B^{\prime}+Q\right)$
(b) \& This is a reset dominant latch where $A^{\prime}$ acts a reset
(c) and $B^{\prime}$ acts as a set.
11.15 (a) $Q^{+}=(M+N+G)\left[Q+(M+N+G) N^{\prime} G^{\prime}\right]$

$$
\begin{aligned}
& =(M+N+G)\left[Q+N^{\prime} G^{\prime}\right] \\
& =(M+N+G) Q+M N^{\prime} G^{\prime}
\end{aligned}
$$

11.15 (b)

|  | GMN |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |  |
| 0 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | 1 | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |  |
| 1 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |  |

The stable states are in bold.
11.15 (c) When $\mathrm{G}=1$, the circuit is always stable. When $\mathrm{G}=0$, M and N determine the state; $\mathrm{N}=1$ makes the state stable and with $\mathrm{N}=0$ the state becomes the value of M . There would be a restriction on M and N if they could cause both inputs to the output latch to be 1 when $G=0$. This is not possible so there is no restriction.
11.15 (d) $P=Q^{\prime}\left[N+G+M^{\prime} N^{\prime} G^{\prime}\right]=Q^{\prime}\left[N+G+M^{\prime}\right]$

For every stable state, $\mathrm{P}=\mathrm{Q}$ ' so P is usable as the complement of Q ..
11.16 (a) $Q^{+}=A B+Q B$
11.16 (c) $\mathrm{AB}=01$ is a hold input combination, $\mathrm{AB}=00$ and 10 are reset input combinations, and $A B=11$ is a set input combination. This is reset dominant latch

11.16 (b) | Present | Next State Q $^{+}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| State | A B |  |  |  |  |
| Q | 00 | 01 | 11 | 10 |  |
| 0 | $\mathbf{0}$ | $\mathbf{0}$ | 1 | $\mathbf{0}$ |  |
| 1 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | 0 |  |

The stable states are in bold.
(a) $Q^{+}=R^{\prime}(S+Q)$ if $\mathrm{SR}=0$
(b) $Q^{+}=(G+Q)\left(G^{\prime}+D\right)$
(c) $Q^{+}=D$
(d) $Q^{+}=(Q+C E)\left(C E^{\prime}+D\right)$
(e) $Q^{+}=(J+Q)\left(K^{\prime}+Q^{\prime}\right)$
(f) $Q^{+}=(T+Q)\left(T^{\prime}+Q^{\prime}\right)$
11.18

11.19

11.20 (b) A set-dominant FF from an S-R FF-The arrangement will ensure that when $S=R=1, S_{1}=1$, $R_{1}=0$, and $Q^{+}=1$.
11.20 (a)

| $S$ | $R$ | $Q$ | $Q^{+}$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |


11.21


## Unit 11 Solutions


11.24

11.25

11.26

11.27 (a)


When $D=0$, then $S=0$, and $R=1$, so $Q^{+}=0$.
When $D=1$, then $S=1$, and $R=0$, so $Q^{+}=1$.
11.27 (b) $R$ will not be ready until $D$ goes through the inverter, so we must add the delay of the inverter to the setup time:
Setup time $=1.5+1=2.5 \mathrm{~ns}$

Propagation delay for the DFF:
2.5 ns (same as for the S-R flip-flop, since the propagation delay is measured with respect to the clock)
11.28


## Unit 12 Problem Solutions

12.1 Consider $3 \times Y=Y+Y+Y$, that is, we need to add $Y$ to itself 3 times. First, clear the accumulator before the first rising clock edge so that the $X$-register is 000000 . Let the $A d$ pulse be 1 for 3 rising clock edges and let the $Y$ register contain the desired number $\left(y_{5} y_{4} y_{3} y_{2} y_{1} y_{0}\right)$ which is to be added three times. The timing diagram is on FLD p. 717. Note: ClrN should go to 0 and back to 1 before the first rising clock edge. Ad should be 1 before the same clock edge. However, it does not matter in what order, that is, $A d$ could go to 1 before $C l r N$ returns to 1 , or even before it goes to 0 .
12.2 Serial input connected to $D_{0}$ for left shift. Sh $=0, L=1$ causes a left shift.
Sh $=1, L=1$ or 0 causes a right shift

12.4 (a)

| $\begin{gathered} \text { Present } \\ \text { State } \\ D C B A \end{gathered}$ | $\begin{gathered} \text { Next State } \\ D^{+} C^{+} B^{+} A^{+} \end{gathered}$ | $\begin{aligned} & \text { Flip-Flop } \\ & \text { Inputs } \\ & T_{\mathrm{D}} T_{\mathrm{C}} T_{\mathrm{B}} T_{\mathrm{A}} \end{aligned}$ |
| :---: | :---: | :---: |
| 0000 | 0001 | 0001 |
| 0001 | 0010 | 0011 |
| 0010 | 0011 | 0001 |
| 0011 | 0100 | 0111 |
| 0100 | 0101 | 0001 |
| 0101 | 0110 | 0011 |
| 0110 | 0111 | 0001 |
| 0111 | 1000 | 1111 |
| 1000 | 1001 | 0001 |
| 1001 | 1010 | 0011 |
| 1010 | 1011 | 0001 |
| 1011 | 1100 | 0111 |
| 1100 | 1101 | 0001 |
| 1101 | 1110 | 0011 |
| 1110 | 1111 | 0001 |
| 1111 | 0000 | 1111 |

12.3 See FLD Appendix E for solution.


As explained in Section 12.3, it can be seen that $A$ changes on every rising clock edge: $T_{\mathrm{A}}=1$
$B$ changes only when $A=1: T_{B}=A$
$C$ changes only when both $B$ and $A=1: T_{\mathrm{C}}=A B$
$D$ changes only when $A, B$, and $C=1: T_{\mathrm{D}}=A B C$

## Unit 12 Solutions

12.4 (b) The binary counter using D flip-flops is obtained by converting each T flip-flop to a D flip-flop by adding an XOR gate.

See FLD p. 717 and Figure 12-15 on FLD p. 364.
12.5 Equations for $C, B$, and $A$ are from Equations (122) on FLD p. 364. Beginning with (b) of Problem 12.4 solutions,
$D^{+}=D \oplus C B A=D^{\prime} C B A+D(C B A)^{\prime}$
$=D^{\prime} C B A+D\left(C^{\prime}+B^{\prime}+A^{\prime}\right)$
$=D^{\prime} C B A+D C^{\prime}+D B^{\prime}+D A^{\prime}$

In the following state graph, the first flip-flop (C) takes on the required sequence $0,0,1,0,1,1$, (repeat).

B A

$\mathrm{A}^{+}=\mathrm{C}^{\prime} \mathrm{A}^{\prime}$

$B^{+}=C B^{\prime}$

$\mathrm{C}^{+}=\mathrm{A}+\mathrm{C}^{\prime} \mathrm{B}$
12.7 (a)

| CB A | $C^{+} B^{+} A^{+}$ |
| :---: | :---: |
| 000 | $\times \times \times$ |
| 001 | 011 |
| 010 | 110 |
| 011 | 010 |
| 100 | 001 |
| 101 | 100 |
| 110 | 111 |
| 111 | 101 |


$\mathrm{C}^{+}=\mathrm{CA}+\mathrm{BA}^{\prime}$

$\mathrm{B}^{+}=\mathrm{C}^{\prime}+\mathrm{BA}^{\prime}$


$T_{A}=C^{\prime} B A+C B^{\prime}+C A^{\prime}$
12.7 (b)


For T flip-flop: 000 goes to 110 because $T_{\mathrm{A}} T_{\mathrm{B}} T_{\mathrm{C}}=110$
12.8 (a)

| $C$ | $B$ | $A$ | $C^{+} B^{+}$ | $A^{+}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | $X$ | $X$ | $X$ |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 |

B A


B A




$\mathrm{J}_{\mathrm{A}}=\mathrm{C}$
$\mathrm{K}_{\mathrm{A}}=\mathrm{C}^{\prime} \mathrm{B}+\mathrm{CB} \mathrm{B}^{\prime}$

In state 000,
$J_{\mathrm{C}}=A^{\prime}=1, K_{\mathrm{C}}=B^{\prime} A^{\prime}=1, C^{+}=C^{\prime}=1$
$J_{\mathrm{B}}=C^{\prime}=1, K_{\mathrm{B}}=C A=0, B^{+}=1$
$J_{\mathrm{A}}=C=0, K_{\mathrm{A}}=C B^{\prime}+C^{\prime} B=0, A^{+}=A=0$
So the next state is $C^{+} B^{+} A^{+}=110$

## 12.8 (b)

B A

$S_{C}=B^{\prime}$

$\mathrm{R}_{\mathrm{C}}=\mathrm{B}^{\prime} \mathrm{A}^{\prime}$

$S_{B}=C^{\prime}$
$\mathrm{S}_{\mathrm{C}}=\mathrm{C}^{\prime} \mathrm{A}^{\prime}$
In state 000,
$S_{\mathrm{C}}=B A^{\prime}=0, R_{\mathrm{C}}=B^{\prime} A^{\prime}=1, C^{+}=0$
$S_{\mathrm{B}}=C^{\prime}=1, R_{\mathrm{B}}=C A=0, B^{+}=1$
$S_{\mathrm{A}}=C A^{\prime}=0, R_{\mathrm{A}}=C^{\prime} B+C^{\prime} B A=0, A^{+}=A=0$
So the next state is $C^{+} B^{+} A^{+}=010$

$R_{B}=C A$

$S_{A}=C^{\prime}$
12.9 (a)

| $Q Q^{+}$ | $M N$ |
| :---: | :---: |
| 00 | $\left.\begin{array}{ll} 0 & 0 \\ 0 & 1 \end{array}\right\} 0 x$ |
| 01 | $\left.\begin{array}{ll} 1 & 0 \\ 1 & 1 \end{array}\right\} 1 x$ |
| 10 | $\left.\begin{array}{ll} 1 & 0 \\ 0 & 0 \end{array}\right\} \times 0$ |
| 11 | $\left.\begin{array}{ll} 0 & 1 \\ 1 & 1 \end{array}\right\} \times 1$ |



$M_{C}=B$

$N_{C}=A$
$\mathrm{B}^{+}$
B A



$$
\mathrm{N}_{\mathrm{B}}=\mathrm{C}^{\prime}
$$



$\mathrm{M}_{\mathrm{A}}=\mathrm{C}^{\prime}$
12.10 See Lab Solutions for Unit 12 in this manual.
12.11 The flip-flops change state only when $L d$ or $S h=1$. So $C E=S h+L d$. Now only a 2-to-1 MUX is required to select the input to the D flip-flop.

12.12 (a) When $S h L d=00$, the MUX for flip-flop $i$ selects $Q_{i}$ to hold its state When $\operatorname{ShLd}=01$, the MUX for flip-flop $i$ selects $D_{\mathrm{i}}$ to load.
When $\operatorname{ShLd}=10$ or 11 , the MUX for flip-flop $i$ selects $Q_{i-1}$ to shift left.

12.12 (b) $Q_{3}^{+}=L d^{\prime} S h^{\prime} Q_{3}+L d S h^{\prime} D_{3}+S h Q_{2} ; Q_{2}^{+}=L d^{\prime} S h^{\prime} Q_{2}+L d S h^{\prime} D_{2}+S h Q_{1} ; Q_{1}^{+}=L d^{\prime} S h^{\prime} Q_{1}+L d S h^{\prime} D_{1}+S h Q_{0}$ $Q_{0}^{+}=L d^{\prime} S h^{\prime} Q_{0}+L d S h^{\prime} D_{0}+S h S I$
12.13 Notice that $S h$ overrides $L d$ when $S h=L d=1$

12.14 (a) Similar to problem 12.4 (a), $T_{\mathrm{E}}=\mathrm{ABCD} . T_{\mathrm{D}}, T_{\mathrm{C}}, T_{\mathrm{B}}$ and $T_{\mathrm{A}}$ remain unchanged.

12.14 (b) Similar to problem 12.4 (b),
$D_{\mathrm{E}}=E \oplus D B C A$.
$D_{\mathrm{D}}, D_{\mathrm{C}}, D_{\mathrm{B}}$ and $D_{\mathrm{A}}$ remain unchanged.


Unit 12 Solutions
12.15 4-bit Johnson counter using J-K flip-flops:


Starting in 0000: 0000, 1000, 1100, 1110, 1111, $0111,0011,0001$, (repeat) $0000, \ldots$
Starting in 0110: 0110, 1011, 0101, 0010, 1001, 0100, 1010, 1101, (repeat) $0110, \ldots$
12.16 When $U=1, D=0$, add 001 . When $U=0, D=1$, subtract 1: add 111 . When $U=0, D=0$, no change: add 000 .
$U=1, D=1$, can never occur.
So add the contents of the register to $X_{2} X_{1} X_{0}$, where $X_{2}=X_{1}=D$ and $X_{0}=D+U$. (Note: to save the OR gate, let $X_{0}=D$ and $C_{\text {in }}=U$.)


### 12.17 (a)

| $A B C D$ | $A^{+} B^{+} C^{+} D^{+}$ |
| :---: | :---: |
| 0000 | 0001 |
| 0001 | 0010 |
| 0010 | 0011 |
| 0011 | 0100 |
| 0100 | 0101 |
| 0101 | 0110 |
| 0110 | 0111 |
| 0111 | 1000 |
| 1000 | 1001 |
| 1001 | 0000 |
| 1010 | X $\times \times \times$ |
| 1011 | X $\times \times \times$ |
| 1100 | X $\times \times \times$ |
| 1101 | X $\times \times \times$ |
| 1110 | X $\times \times \times$ |
| 1111 | X $\times \times \times$ |


$\mathrm{D}_{\mathrm{A}}=\mathrm{BCD}+\mathrm{A} \mathrm{D}^{\prime}$

$D_{C}=A^{\prime} C^{\prime} D+C D^{\prime}$

$D_{B}=B^{\prime} C D+B C^{\prime}+B D^{\prime}$

$\mathrm{D}_{\mathrm{D}}=\mathrm{D}^{\prime}$
12.17 (b) See Table 12-7 (c) on FLD p. 374.

$\mathrm{J}_{\mathrm{A}}=\mathrm{BCD}$

$\mathrm{J}_{\mathrm{B}}=\mathrm{CD}$

$\mathrm{J}_{\mathrm{C}}=\mathrm{A}^{\prime} \mathrm{D}$
C

$\mathrm{J}_{\mathrm{D}}=1$
$C D A$

$\mathrm{K}_{\mathrm{A}}=\mathrm{D}$

$K_{B}=C D$

$\mathrm{K}_{\mathrm{C}}=\mathrm{D}$

$\mathrm{K}_{\mathrm{D}}=1$
12.17 (c) See Table 12-5 (c) on FLD p. 371.
$C_{D}^{S_{A}} A B$

$S_{A}=B C D$
$\mathrm{R}_{\mathrm{A}}$
C

|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | x | X | x |  |
| 01 | 区 | X | X | $1)$ |
| 11 | X |  | X | X |
| 10 | X | X | X | X |

$\mathrm{R}_{\mathrm{A}}=\mathrm{C}^{\prime} \mathrm{D}$
$\mathrm{R}_{\mathrm{A}}=\mathrm{AD}$
$R_{A}=B^{\prime} D$

$R_{B}=B C D$

$S_{B}=B^{\prime} C D$

$S_{C}=A^{\prime} C^{\prime} D$

$S_{D}=D^{\prime}$

$R_{C}=C D$

$R_{D}=D$

Unit 12 Solutions
12.17 (d) See Table 12-4 on FLD p. 368.

$T_{A}=B C D+A D$

$T_{B}=C D$

$T_{C}=A^{\prime} D$

$\mathrm{T}_{\mathrm{D}}=1$
12.17 (e) Use equations to find next states for unused states. State 1101:

$$
\begin{aligned}
& J_{\mathrm{A}}=B C D=0, K_{\mathrm{A}}=D=1, A^{+}=0 \\
& J_{\mathrm{B}}=C D=0, K_{\mathrm{B}}=C D=0, B^{+}=B=1 \\
& J_{\mathrm{C}}=A^{\prime} D=0, K_{\mathrm{C}}=D=1, C^{+}=0 \\
& J_{\mathrm{D}}=1, K_{\mathrm{D}}=1, D^{+}=D^{\prime}=0
\end{aligned}
$$

So the next state is 0100 . Other next states can be found in a similar way.

12.18

| $A B C D$ | $A^{+} B^{+} C^{+} D^{+}$ |
| :---: | :---: |
| 0000 | 1001 |
| 0001 | 0000 |
| 0010 | 0001 |
| 0011 | 0010 |
| 0100 | 0011 |
| 0101 | 0100 |
| 0110 | 0101 |
| 0111 | 0110 |
| 1000 | 0111 |
| 1001 | 1000 |
| 1010 | X $\times \times \times$ |
| 1011 | $X \times \times X$ |
| 1100 | X $\times \times \times$ |
| 1101 | $\times \times \times \times$ |
| 1110 | $X \times \times X$ |
| 1111 | X $\times \times \times$ |

12.18 (a) $D_{\mathrm{A}}=A^{\prime} \mathrm{B}^{\prime} C^{\prime} D^{\prime}+A D$;
$D_{\mathrm{B}}=B D+B C+A D^{\prime} ;$
$D_{\mathrm{C}}=C D+B C^{\prime} D^{\prime}+A D^{\prime} ;$
$D_{\mathrm{D}}=D^{\prime}$
12.18 (b) $J_{\mathrm{A}}=B^{\prime} C^{\prime} D^{\prime}, K_{\mathrm{A}}=D^{\prime}$;
$J_{\mathrm{B}}=A D^{\prime}, K_{\mathrm{B}}=C^{\prime} D^{\prime} ;$
$J_{\mathrm{C}}=B D^{\prime}+A D^{\prime}, K_{\mathrm{C}}=D^{\prime} ;$
$J_{\mathrm{D}}=1, K_{\mathrm{D}}=1$
12.18 (c) $S_{\mathrm{A}}=A^{\prime} \mathrm{B}^{\prime} C^{\prime} D^{\prime}, R_{\mathrm{A}}=A D^{\prime}$;
12.18 (d) $T_{\mathrm{A}}=B^{\prime} C^{\prime} D^{\prime}$;
$S_{\mathrm{B}}=A D^{\prime}, R_{\mathrm{B}}=B C^{\prime} D^{\prime}$ or $A^{\prime} C^{\prime} D^{\prime} ;$
$S_{\mathrm{C}}=B D^{\prime} C^{\prime}+A D^{\prime}, R_{\mathrm{C}}=C D^{\prime}$,
$T_{\mathrm{B}}=B C^{\prime} D^{\prime}+A C^{\prime} D^{\prime} ;$
$T_{\mathrm{C}}=C D^{\prime}+B D^{\prime}+A D^{\prime} ;$
$S_{\mathrm{D}}=D^{\prime}, R_{\mathrm{D}}=D$
$T_{\mathrm{D}}=1$
12.18 (e)

12.19

| $A$ | $B$ | $C$ | $A^{+} B^{+}$ | $C^{+}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | $X$ | $X$ | $X$ |
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 |

12.20(a)

| $A B C D$ | $D_{\mathrm{A}} D_{\mathrm{B}} D_{\mathrm{C}} D_{\mathrm{D}}$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0000 | 0 | 0 | 0 | 1 |
| 0001 | 0 | 0 | 1 | 0 |
| 0010 | 0 | 0 | 1 | 1 |
| 0011 | 0 | 1 | 0 | 0 |
| 0100 | 1 | 0 | 1 | 1 |
| 0101 | x | x | x | x |
| 0110 | x | x | x | x |
| 0111 | x | x | x | x |
| 1000 | x | x | x | x |
| 1001 | x | x | x | x |
| 1010 | x | x | x | x |
| 1011 | 1 | 1 | 0 | 0 |
| 1100 | 1 | 1 | 0 | 1 |
| 1101 | 1 | 1 | 1 | 0 |
| 1110 | 1 | 1 | 1 | 1 |
| 1111 | 0 | 0 | 0 | 0 |

12.20(c)

| $A B C D$ | $T_{\mathrm{A}} T_{\mathrm{B}} T_{\mathrm{C}} T_{\mathrm{D}}$ | $T_{\mathrm{A}}=A^{\prime} B+B C D$ |
| :---: | :---: | :---: |
| 0000 | $\begin{array}{lllll}0 & 0 & 0 & 1\end{array}$ | $T=D+A^{\prime} B$ |
| 0001 | $\begin{array}{lllll}0 & 0 & 1 & 1\end{array}$ | $T_{\mathrm{D}}=1$ |
| 0010 | $\begin{array}{lllll}0 & 0 & 0 & 1\end{array}$ |  |
| 0011 | $\begin{array}{lllll}0 & 1 & 1 & 1\end{array}$ |  |
| 0100 | $1 \begin{array}{llll}1 & 1 & 1\end{array}$ |  |
| 0101 | x X X x |  |
| 0110 | x X X x |  |
| 0111 | x X X x |  |
| 1000 | x X X x |  |
| 1001 | x X x x |  |
| 1010 | x X x x |  |
| 1011 | $\begin{array}{lllll}0 & 1 & 1 & 1\end{array}$ |  |
| 1100 | $\begin{array}{lllll}0 & 0 & 0 & 1\end{array}$ |  |
| 1101 | $\begin{array}{llll}0 & 0 & 1 & 1\end{array}$ |  |
| 1110 | $\begin{array}{llll}0 & 0 & 0 & 1\end{array}$ |  |
| 1111 | 1111 |  |

12.19 (a) $D_{\mathrm{A}}=B^{\prime}+A C ; D_{\mathrm{B}}=A C+B C^{\prime} ; D_{\mathrm{C}}=A^{\prime} B+A B^{\prime}$
12.19 (b) $J_{\mathrm{A}}=B^{\prime}, K_{\mathrm{A}}=B C^{\prime} ; J_{\mathrm{B}}=A C, K_{\mathrm{B}}=A^{\prime} C ; J_{\mathrm{C}}=A^{\prime}+B^{\prime}, K_{\mathrm{C}}=A^{\prime} B^{\prime}+A B$
12.19 (c) $T_{\mathrm{A}}=A^{\prime} B^{\prime}+A B C^{\prime} ; T_{\mathrm{B}}=A^{\prime} B C+A B^{\prime} C ; T_{\mathrm{C}}=A^{\prime} B^{\prime}+A^{\prime} C^{\prime}+B^{\prime} C^{\prime}+A B C$
12.19 (d) $S_{\mathrm{A}}=B^{\prime}, R_{\mathrm{A}}=B C^{\prime} ; S_{\mathrm{B}}=A C, R_{\mathrm{B}}=A^{\prime} C ; S_{\mathrm{C}}=A^{\prime} B+A B^{\prime}, R_{\mathrm{C}}=A^{\prime} \mathrm{B}^{\prime}+A B$
12.19 (e) State 000 goes to 100 , because $D_{\mathrm{A}} D_{\mathrm{B}} D_{\mathrm{C}}=100$.
$D_{\mathrm{A}}=A B^{\prime}+A D^{\prime}+B C^{\prime}$ or $\quad 12.20(\mathbf{b})$

12.20(d)


$$
\text { 12.21(a) } \begin{aligned}
D_{\mathrm{A}} & =\left(B^{\prime}+C^{\prime}+D^{\prime}\right)(A+B) \\
D_{\mathrm{B}} & =\left(B^{\prime}+C^{\prime}+D^{\prime}\right)(A+D)(A+C) \text { or } \\
& =\left(B^{\prime}+C^{\prime}+D^{\prime}\right)(B+D)(A+C) \text { or } \\
& =\left(B^{\prime}+C^{\prime}+D^{\prime}\right)(B+C)(A+D) \\
D_{\mathrm{C}} & =(B+C+D)\left(C^{\prime}+D^{\prime}\right)\left(A^{\prime}+C+D\right) \\
D_{\mathrm{D}} & =\left(D^{\prime}\right)
\end{aligned}
$$

$$
\text { 12.21(c) } \begin{aligned}
T_{\mathrm{A}} & =(B)\left(A^{\prime}+D\right)\left(A^{\prime}+C\right) \text { or } \\
& =(B)\left(A^{\prime}+C\right)\left(C^{\prime}+D\right) \text { or } \\
& =(B)\left(A^{\prime}+D\right)(C+D) \\
T_{\mathrm{B}} & =\left(A^{\prime}+D\right)(B+D)\left(C+D^{\prime}\right) \\
\text { or } & =(B+C)\left(A^{\prime}+C\right)\left(C^{\prime}+D\right) \\
T_{\mathrm{B}} & =(B+D)\left(A^{\prime}+D\right) \\
T_{\mathrm{B}} & =(1)
\end{aligned}
$$

12.21(b) $J_{\mathrm{A}}=(B)$
$K_{\mathrm{A}}^{\mathrm{A}}=(B)(C)(D)$
$J_{\mathrm{B}}=(C)(D)$
$K_{\mathrm{B}}=\left(A^{\prime}+D\right)\left(A^{\prime}+C\right)$ or

$$
=\left(A^{\prime}+C\right)\left(C^{\prime}+D\right) \text { or }
$$

$$
=\left(A^{\prime}+D\right)\left(C+D^{\prime}\right)
$$

$J_{\mathrm{C}}=(B+D)\left(A^{\prime}+D\right)$
$K_{\mathrm{C}}=(D)$
$J_{\mathrm{D}}=(1)$
$K_{\mathrm{D}}=(1)$

$$
\text { 12.21(d) } \begin{aligned}
S_{\mathrm{A}} & =(B)\left(D^{\prime}\right) \text { or } \\
& =(B)\left(C^{\prime}\right) \text { or } \\
& =(B)\left(A^{\prime}\right) \\
R_{\mathrm{A}} & =(B)(C)(D) \\
S_{\mathrm{B}} & =(C)(D)\left(B^{\prime}\right) \\
R_{\mathrm{B}} & =(B)\left(A^{\prime}+D\right)\left(A^{\prime}+C\right) \text { or } \\
& =(B)\left(A^{\prime}+C\right)\left(C^{\prime}+D\right) \text { or } \\
& =(B)\left(A^{\prime}+D\right)\left(C+D^{\prime}\right) \\
S_{\mathrm{C}} & =(B+D)(C)\left(A^{\prime}+D\right) \\
R_{\mathrm{C}} & =(C)(D) \\
S_{\mathrm{D}} & =(D) \\
R_{\mathrm{D}} & =(D)
\end{aligned}
$$

12.22(b)

| $A B C D$ | $J_{\mathrm{A}} K_{\mathrm{A}} J_{\mathrm{B}} K_{\mathrm{B}} J_{\mathrm{C}} K_{\mathrm{C}} J_{\mathrm{D}} K_{\mathrm{D}}$ |  |  |  | $\begin{aligned} & J_{\mathrm{A}}=B C D \\ & K_{\mathrm{A}}=B \\ & J_{\mathrm{B}}=C D \\ & K_{\mathrm{B}}=A+C D \\ & J_{\mathrm{C}}=D+A B \\ & K_{\mathrm{C}}=D \\ & J_{\mathrm{D}}=1 \\ & K_{\mathrm{D}}=1 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0000 |  | xx, | xx, | xx |  |  |  |  |  |
| 0001 | xx, | xx, | xx, | xx |  |  |  |  |  |
| 0010 | xx, | xx, | xx, | xx |  |  |  |  |  |
| 0011 | 0x, | 1x, | x1, | x1 |  |  |  |  |  |
| 0100 | 0x, | x 0 , | 0x, | 1x |  |  |  |  |  |
| 0101 | 0x, | x0, | 1x, | x1 |  |  |  |  |  |
| 0110 | 0x, | x0, | x 0 , | 1x |  |  |  |  |  |
| 0111 | 1x, | x1, | x 1 , | x1 |  |  |  |  |  |
| 1000 | x 0 , | 0x, | 0x, | 1x |  |  |  |  |  |
| 1001 | x 0 , | 0x, | 1 x , | x1 |  |  |  |  |  |
| 1010 |  | 0x, | x 0 , | 1x |  |  |  |  |  |
| 1011 | x 0 , | 1x, | x 1 , | x1 |  |  |  |  |  |
| 1100 | x1, | x1, | 1x, | 1x |  |  |  |  |  |
| 1101 | xx, | xx, | xx, | xx |  |  |  |  |  |
| 1110 | xx, | xx, | xx, | xx |  |  |  |  |  |
| 1111 |  | xx, | xx, | xx |  |  |  |  |  |

12.22(a)

| $A B C D$ | $\mathrm{D}_{\mathrm{A}} D_{\mathrm{B}} D_{\mathrm{C}} D_{\mathrm{D}}$ | $D_{\mathrm{A}}=B C D+A B^{\prime}$ |
| :---: | :---: | :---: |
| 0000 | X X X X | $D^{D}=C^{\prime} D+C D^{\prime}+A B$ |
| 0001 | x X x ( x | $D_{\mathrm{D}}=D^{\prime}$ |
| 0010 | x X x x |  |
| 0011 | $\begin{array}{lllll}0 & 1 & 0 & 0\end{array}$ |  |
| 0100 | $\begin{array}{llll}0 & 1 & 0 & 1\end{array}$ |  |
| 0101 | $\begin{array}{lllll}0 & 1 & 1 & 0\end{array}$ |  |
| 0110 | $\begin{array}{lllll}0 & 1 & 1 & 1\end{array}$ |  |
| 0111 | $1 \begin{array}{llll}1 & 0 & 0 & 0\end{array}$ |  |
| 1000 | $\begin{array}{llll}1 & 0 & 0 & 1\end{array}$ |  |
| 1001 | $1 \begin{array}{llll}1 & 0 & 1 & 0\end{array}$ |  |
| 1010 | $\begin{array}{lllll}1 & 0 & 1 & 1\end{array}$ |  |
| 1011 | $1 \begin{array}{llll}1 & 1 & 0 & 0\end{array}$ |  |
| 1100 | $\begin{array}{lllll}0 & 0 & 1 & 1\end{array}$ |  |
| 1101 | x X x x |  |
| 1110 | x X X X |  |
| 1111 | x X x x |  |

Unit 12 Solutions
12.22(c)

12.23(a) $D_{\mathrm{A}}=(A+B)\left(B^{\prime}+D\right)(A+C)$ or
$=(A+B)\left(B^{\prime}+C\right)(A+D)$ or
$=(A+B)\left(B^{\prime}+D\right)\left(B^{\prime}+C\right)$
$D_{\mathrm{B}}=\left(B^{\prime}+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+D\right)(B+C)$ or
$=\left(B^{\prime}+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+C\right)(B+D)$ or
$=\left(B^{\prime}+C^{\prime}+D^{\prime}\right)\left(A^{\prime}+D\right)\left(A^{\prime}+C\right)$
$D_{\mathrm{C}}=(A+C+D)\left(C^{\prime}+D^{\prime}\right)(B+C+D)$
$D_{\mathrm{D}}=\left(D^{\prime}\right)$
12.23(c) $T_{\mathrm{A}}=(B)\left(C^{\prime}+D\right)(A+C)$ or
$=(B)(A+D)(A+C)$ or
$=(B)\left(C+D^{\prime}\right)(A+D)$
$T_{\mathrm{B}}=\left(C+D^{\prime}\right)(B+D)(A+D)$ or
$=\left(C^{\prime}+D\right)(A+C)(B+C)$
$T_{\mathrm{B}}=(A+D)(B+D)$
$T_{\mathrm{B}}=$ (1)
12.22(d)

| ABCD | $S_{\text {A }} R$ | $S_{\text {B }} R_{\text {B }}$ | $S_{C} R_{C}$ | $S_{\mathrm{D}} R_{\mathrm{D}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0000 |  | Xx, | Xx, | xx | $=B D^{\prime} \text { or }$ |
| 0001 |  | xx, | xx, | xx | $=A B$ |
| 0010 | Xx, | xx, | xx, | XX | $S_{\mathrm{B}}=B^{\prime} C D$ |
| 0011 |  | 10, | 01, | 01 | $R_{\mathrm{B}}=B C D+A C^{\prime} \text { or }$ |
| 0100 | 0x, | x 0 , | 0x, | 10 | $=B C D+A B$ |
| 0101 | 0x, | x0, | 10, | 01 | $S_{\mathrm{C}}=C^{\prime} D+A B$ |
| 0110 | 0x, | x0, | x0, | 10 | $R_{\mathrm{C}}=C D$ |
| 0111 | 10, | 01, | 01, | 01 | $S_{\text {D }}=$ |
| 1000 |  | 0x, | 0x, | 10 |  |
| 1001 | x 0 , | 0x, | 10, | 01 |  |
| 1010 |  | 0x, | x0, | 10 |  |
| 1011 | x 0 , | 10, | 01, | 01 |  |
| 1100 | 01, | 01, | 10, | 10 |  |
| 1101 | xx, | xx, | xx, | xx |  |
| 1110 | xx, | xx, | xx, | xx |  |
| 1111 | xx, | xx, | xx, | xx |  |

12.23(b) $J_{\mathrm{A}}=(B)(C)(D)$
$K_{\mathrm{A}}=(B)$
$J_{\mathrm{B}}=(C)(D)$
$K_{\mathrm{B}}=\left(C^{\prime}+D\right)(A+C)$ or
$=(A+D)(A+C)$ or
$=\left(C+D^{\prime}\right)(A+D)$
$J_{\mathrm{C}}=(A+D)(B+D)$
$K_{\mathrm{C}}=(D)$
$J_{\mathrm{D}}=(1)$
$K_{\mathrm{D}}=(1)$
12.23(d) $S_{\mathrm{A}}=(B)(C)(D)$
$R_{\mathrm{A}}=(B)(A)$ or
$=(B)\left(D^{\prime}\right)$ or
$=(B)\left(C^{\prime}\right)$
$S_{\mathrm{B}}=\left(B^{\prime}\right)(C)(D)$
$R_{\mathrm{B}}=(B)\left(C^{\prime}+D\right)(A+C)$ or
$=(B)(A+D)(A+C)$ or
$=(B)\left(C+D^{\prime}\right)(A+D)$
$S_{\mathrm{C}}=(A+D)\left(C^{\prime}\right)(B+D)$
$R_{\mathrm{C}}=(C)(D)$
$S_{\mathrm{D}}=\left(D^{\prime}\right)$
$R_{\mathrm{D}}=(D)$
12.24 (a) The counter must clear on the next clock edge when the count is 1011 so $\operatorname{Clr} N=($ Q3Q1Q0)'.
(b) The counter must clear when the count reaches 1100 so $C l r N=(Q 3 Q 2)^{\prime}$.

Unit 12 Solutions
12.25

| $Q_{3} Q_{2} Q_{1} Q_{0}$ | ClrN Ld |
| :---: | :---: |
| 0000 | 10 |
| 0001 | 10 |
| 0010 | 10 |
| 0011 | 10 |
| 0100 | 111 |
| 0101 | x x |
| 0110 | x x |
| 0111 | x x |
| 1000 | x x |
| 1001 | x x |
| 1010 | x x |
| 1011 | 10 |
| 1100 | 10 |
| 1101 | 10 |
| 1110 | 10 |
| 1111 | 10 |

The transition from state 1111 to state 0000 can be effected using Clear, Parallel Load or increment. The latter gives the simplest equations. Then $\operatorname{Clr} N$ $=1, L d=Q_{3}{ }^{\prime} Q_{2}$, and $P_{3} P_{2} P_{1} P_{0}=1011$.
12.27 (a) All stages toggle the same as for a binary counter except when the count becomes 1001, in which case stages $Q_{0}, Q_{1}$ and $Q_{2}$ respond the same as for a binary counter, but $\mathrm{Q}_{3}$ must toggle (reset). Taking into account the don't cares, the equations become
$J_{0}=K_{0}=1$
$J_{1}=K_{1}=Q_{0}$
$J_{2}=K_{2}=Q_{0} Q_{1}$
$J_{3}=Q_{0} Q_{1} Q_{2}$
$K_{3}=Q_{0} Q_{1} Q_{2}+Q_{0} Q_{3}$
12.27 (c) To create a design that can be cascaded, we need to add a count enable input, CE, which is ANDed with the above equations, and terminal count output, TE, such as $T E=C E\left(Q_{2} Q_{3}\right)$. TE would be connected to CE of the next counter.
(a) $\begin{aligned} & \text { (b) There are two } \\ & S_{\text {in }}=Q_{2} \oplus Q_{3} \text { or } \\ & S_{\text {in }}=Q_{0} \oplus Q_{3} .\end{aligned}$
(c) The state 0000 can only occur between states 0001 and 1000. The resulting Karnaugh map for the $S_{\text {in }}=Q_{2} \oplus Q_{3}$ case is shown below.


If the circuit for $S_{\text {in }}=Q_{0} \oplus Q_{3}$ is modified, then $S_{\text {in }}=Q_{0} Q_{3}{ }^{\prime}+Q_{0}{ }^{\prime} Q_{1} Q_{3}+Q_{0}{ }^{\prime} Q_{2} Q_{3}+Q_{1}{ }^{\prime} Q_{2}{ }^{\prime} Q_{3}{ }^{\prime}$.
12.27 (b) All stages toggle the same as for a binary counter for counts 0011 through 1011. For count 1100 stages 3 and 2 must reset and stage 1 must set while stage 0 toggles as it does it does for a binary counter. Taking into account the don't cares, the equations become

$$
\begin{aligned}
& J_{0}=K_{0}=1 \\
& J_{1}=Q_{0}+Q_{2} Q_{3} \\
& K_{1}=Q_{0} \\
& J_{2}=Q_{0} Q_{1} \\
& K_{2}=Q_{0} Q_{1}+Q_{2} Q_{3} \\
& J_{3}=Q_{0} Q_{1} Q_{2} \\
& K_{3}=Q_{0} Q_{1} Q_{2}+Q_{2} Q_{3} \\
& K_{3} \text { can be further simplified to } K_{3}=Q_{2} Q_{3} .
\end{aligned}
$$

12.28 (a)

| UABC | $S_{A} R_{A}$ | $S_{B} R_{B}$ | $S_{C} R_{C}$ |
| :--- | :---: | :---: | :---: |
| 0000 | 10 | 10 | 10 |
| 0001 | $0 x, x 1$ | $0 x, x 1$ | $x 1$ |
| 0010 | $0 x, x 1$ | $x 1$ | 10 |
| 0011 | $0 x, x 1$ | $x 0$ | $x 1$ |
| 0100 | $x 1$ | 10 | 10 |
| 0101 | $x 0$ | $0 x, x 1$ | $x 1$ |
| 0110 | $x 0$ | $x 1$ | 10 |
| 0111 | $x 0$ | $x 0$ | $x 1$ |
| 1000 | $0 x, x 1$ | $0 x, x 1$ | 10 |
| 1001 | $0 x, x 1$ | 10 | $x 1$ |
| 1010 | $0 x, x 1$ | $x 0$ | 10 |
| 1011 | 10 | $x 1$ | $x 1$ |
| 1100 | $x 0$ | $0 x, x 1$ | 10 |
| 1101 | $x 0$ | 10 | $x 1$ |
| 1110 | $x 0$ | $x 0$ | 10 |
| 1111 | $x 1$ | $x 1$ | $x 1$ |




Another solution for the A FF:
$S_{\mathrm{A}}=B+C^{\prime}$
$R_{\mathrm{A}}=U A B C+U^{\prime} A B^{\prime} C^{\prime}+U^{\prime} A^{\prime} B+U A^{\prime} C^{\prime}$



Unit 12 Solutions

12.28 (b) |  | UABC | $\mathrm{CE}_{\mathrm{A}} \mathrm{D}_{\mathrm{A}}$ | $\mathrm{CE}_{\mathrm{B}} \mathrm{D}_{\mathrm{B}}$ |
| :--- | :---: | :---: | :---: |
| $\mathrm{CE}_{\mathrm{C}} \mathrm{D}_{\mathrm{C}}$ |  |  |  |
| 0000 | 11 | 11 | 11 |
| 0001 | $0 x, 10$ | $0 x, 10$ | 10 |
| 0010 | $0 x, 10$ | 10 | 11 |
| 0011 | $0 x, 10$ | $0 x, 11$ | 10 |
| 0100 | 10 | 11 | 11 |
| 0101 | $0 x, 11$ | $0 x, 10$ | 10 |
| 0110 | $0 x, 11$ | 10 | 11 |
| 0111 | $0 x, 11$ | $0 x, 11$ | 10 |
| 1000 | $0 x, 10$ | $0 x, 10$ | 11 |
| 1001 | $0 x, 10$ | 11 | 10 |
| 1010 | $0 x, 10$ | $0 x, 11$ | 11 |
| 1011 | 11 | 10 | 10 |
| 1100 | $0 x, 11$ | $0 x, 10$ | 11 |
| 1101 | $0 x, 11$ | 11 | 10 |
| 1110 | $0 x, 11$ | $0 x, 11$ | 11 |
| 1111 | 10 | 10 | 10 |



$C E_{\mathrm{B}}=U^{\prime} C^{\prime}+U C$
$\mathrm{CE}_{\mathrm{C}}$

| U A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 1 | 1 | 1 | 1 |
|  | 01 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 1 | 1 |  |
|  | 10 | 1 | 1 | 1 | 1 |


$\mathrm{D}_{\mathrm{C}}$


Unit 12 Solutions
12.29 (a)

| Present <br> State | Next State |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{MB}=$ | 00 | 01 | 11 | 10 |
| 00 | 01 | 01 | xx | 10 |
| 01 | 10 | 00 | 10 | xx |
| 11 | 00 | xx | xx | xx |
| 10 | 11 | xx | 01 | 00 |


| Present <br> State | MN $=$ <br> AB |  |  |  |  | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 0 x | 0 x | xx | 1 x |  |  |  |  |  |
| 01 | 1 x | 0 x | 1 x | xx |  |  |  |  |  |
| 11 | x 1 | xx | xx | xx |  |  |  |  |  |
| 10 | x 0 | xx | x 1 | x 1 |  |  |  |  |  |


| $J_{\mathrm{A}}=M+N^{\prime} B \quad K_{\mathrm{A}}=M+B$ |
| :--- |


| Present <br> State | $\mathrm{MN}=$ <br> MB |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| AB |  |  |  |  |
| 00 | 01 | 11 | 10 |  |
| 00 | 1 x | 1 x | xx | 0 x |
| 01 | x 1 | x 1 | x 1 | xx |
| 11 | x 1 | xx | xx | xx |
| 10 | 1 x | xx | 1 x | 0 x |


| 12.29 (b) | Present | $\mathrm{O}_{0} \mathrm{O}_{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | AB | 00 | 01 | 11 | 10 |
|  | 00 | 00 | 00 | 01 | 10 |
|  | 01 | 01 | 01 | 10 | 00 |
|  | 11 | 11 | 01 | 10 | 00 |
|  | 10 | 10 | 00 | 01 | 10 |
|  | $\begin{aligned} & l_{1}=M^{\prime} N \\ & 0_{0}^{\prime}=M^{\prime} E \end{aligned}$ | $\begin{aligned} & A+M N \\ & +M N B^{\prime} \end{aligned}$ | MN |  |  |

12.30 Since the FFs are changing on the negative edge of the clock, the pulses must be concident with positive portions of the clock. Assuming the clock is symmetrical, the FFs' propagation delay must be less than half of the clock period.
(a) The ring counter requires 8 stages: $\mathrm{Q}_{0}, \mathrm{Q}_{1}, \ldots, \mathrm{Q}_{7}$ and $T_{\mathrm{i}}=(C l k) Q_{\mathrm{i}}$ for $i=0,1, \ldots, 7$.
(b) The Johnson counter requires 4 stages: $\mathrm{Q}_{0}, \mathrm{Q}_{1}, \mathrm{Q}_{2}, \mathrm{Q}_{3}$. The count sequence will be $0000,1000,1100,1110$, 1111, 0111, 0011, 0001. Then $T_{0}=(C l k) Q_{0}{ }^{\prime} Q_{3}{ }^{\prime}, T_{1}=(C l k) Q_{0} Q_{1}{ }^{\prime}, T_{2}=(C l k) Q_{1} Q_{2}{ }^{\prime}, T_{3}=(C l k) Q_{2} Q_{3}{ }^{\prime}$, $T_{4}=(C l k) Q_{0} Q_{3}, T_{5}=(C l k) Q_{0}{ }^{\prime} Q_{3}, T_{6}=(C l k) Q_{1}{ }^{\prime} Q_{3}, T_{7}=(C l k) Q_{2}{ }^{\prime} Q_{3}$.
(c) The binary counter requires 3 stages: $\mathrm{Q}_{0}, \mathrm{Q}_{1}, \mathrm{Q}_{2}$. The count sequence will be $000,001,010,011,100,101$, 110, 111. Then $T_{0}=(C l k) Q_{0}{ }^{\prime} Q_{1}{ }^{\prime} Q_{2}{ }^{\prime}, T_{1}=(C l k) Q_{0}{ }^{\prime} \mathrm{Q}_{1}{ }^{\prime} \mathrm{Q}_{2}, T_{2}=(C l k) Q_{0}{ }^{\prime} Q_{1} Q_{2}{ }^{\prime}, T_{3}=(C l k) Q_{0}{ }^{\prime} \mathrm{Q}_{1} Q_{2}$, $T_{4}=(C l k) Q_{0} Q_{1}{ }^{\prime} Q_{2}{ }^{\prime}, T_{5}=(C l k) Q_{0} Q_{1}{ }^{\prime} Q_{2}, T_{6}=(C l k) Q_{0} Q_{1} Q_{2}, T_{7}=(C l k) Q_{0} Q_{1} Q_{2}$.

\subsection*{12.31 (a) <br> | Q UV <br> $=00$  UV <br> $=01$    |
| :--- |
| 0 |}

12.31 (b)

| $\mathrm{QQ}^{+}$ | $\mathrm{U} V$ |
| :---: | :---: |
| 00 | x 0 |
| 01 | 11 |
| 10 | 1 x |
| 11 | 00 |

12.31 (c)

| Q | $\mathrm{Q}^{+}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | AB <br> $=00$ | AB <br> $=01$ | AB <br> $=11$ | AB <br> $=10$ |  |
| 0 | 0 | 0 | 1 | 1 |  |
| 1 | 0 | 1 | 1 | 1 |  |


| Q | U V |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { A B } \\ & =00 \end{aligned}$ | $\begin{aligned} & \text { A B } \\ & =01 \end{aligned}$ | $\begin{aligned} & \text { A B } \\ & =11 \end{aligned}$ | $\begin{aligned} & \text { A B } \\ & =10 \end{aligned}$ |
| 0 | x0 | x0 | 11 | 11 |
| 1 | 1x | 00 | 00 | 00 |

Unit 12 Solutions
12.32 (a)

| $Q$ $\mathrm{Q}^{+}$    <br> M F     <br> $=00$     |
| :--- |
| M F <br> $=01$ |
| 0 |
| 1 |

12.32 (c)

|  | $\mathrm{Q}^{+}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Q | CD | CD | C D | C D |
|  | $=00$ | $=01$ | $=11$ | $=10$ |
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |

12.32 (b)

| $\mathrm{Q} \mathrm{Q}^{+}$ | M F |
| :---: | :---: |
| 00 | 11 |
| 01 | 0 x |
| 10 | x 1 |
| 11 | 00 |


| Q | M F |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { C D } \\ & =00 \end{aligned}$ | $\begin{aligned} & \text { C D } \\ & =01 \end{aligned}$ | $\begin{aligned} & \text { C D } \\ & =11 \end{aligned}$ | $\begin{aligned} & \text { C D } \\ & =10 \end{aligned}$ |
| 0 | 11 | 0x | 0x | 11 |
| 1 | x1 | x1 | 00 | 00 |

12.33 (a)

| $Q Q^{+}$ | LM |
| :---: | :---: |
| 00 | $\left.\begin{array}{ll} 0 & 1 \\ 1 & 1 \end{array}\right\} \times 1$ |
| 01 | $\left.\begin{array}{ll} 0 & 0 \\ 1 & 0 \end{array}\right\} \times 0$ |
| 10 | $\left.\begin{array}{ll} 1 & 0 \\ 1 & 1 \end{array}\right\} 1 X$ |
| 11 | $\left.\begin{array}{ll} 0 & 0 \\ 0 & 1 \end{array}\right\} 0 X$ |

12.33 (b)

| $A$ | $B$ | $C$ | $A^{+} B^{+}$ | $C^{+}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | $X$ | $X$ | $X$ |
| 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | $X$ | $X$ | $X$ |
| 1 | 1 | 1 | 0 | 1 | 1 |


B C


12.33 (b) $L_{\mathrm{A}}=B, M_{\mathrm{A}}=C$; $L_{\mathrm{B}}=A^{\prime}, M_{\mathrm{B}}=A^{\prime}+C^{\prime} ; L_{\mathrm{C}}=A^{\prime} \mathrm{B}^{\prime}$,
(contd) $M_{\mathrm{C}}=A^{\prime}$
B C

$L_{A}=B$
B C

$\mathrm{M}_{\mathrm{A}}=\mathrm{C}$
B C

$L_{B}=A^{\prime}$

$\mathrm{M}_{\mathrm{B}}=\mathrm{A}^{\prime}+\mathrm{C}^{\prime}$

$\mathrm{M}_{\mathrm{C}}=\mathrm{A}^{\prime}$

| A B C D | $A^{+} B^{+} C^{+} D^{+}$ | $J_{\mathrm{A}} K_{\mathrm{A}} J_{\mathrm{B}} K_{\mathrm{B}} J_{\mathrm{C}} K_{\mathrm{C}} J_{\mathrm{D}} K_{\mathrm{D}}$ |
| :---: | :---: | :---: |
| 0000 | 0011 | $0 \times 0 \times 1 \times 1 \times$ |
| 0001 | 0100 | $0 \times 1 \times 0 \times \times 1$ |
| 0010 | 0101 |  |
| 0011 | 0110 | $0 \times 1 \times \times 0 \times 1$ |
| 0100 | 0111 | $0 \times \mathrm{X} 01 \times 1 \mathrm{X}$ |
| 0101 | 1000 | $1 \times \times 10 \times 1$ |
| 0110 | 1001 | $1 \times \mathrm{X} 1 \times 11 \times$ |
| 0111 | 1010 | $1 \times \times 1 \times 0 \times 1$ |
| 1000 | 1011 | X 0 O X 1 X $1 \times$ |
| 1001 | 1100 | X $011 \times 0 \times \times 1$ |
| 1010 | 1101 | X $011 \times \mathrm{X} 111 \mathrm{X}$ |
| 1011 | 1110 | X $011 \times \mathrm{X}$ |
| 1100 | 1111 | X $0 \times 018 \mathrm{X}$ ( X |
| 1101 | X $\times \times \times$ | X $\times$ X $\times$ X $\mathrm{X} \times \mathrm{X}$ |
| 1110 | X $\times \times \times$ | X $\times$ X $\times$ X $\mathrm{X} \times \mathrm{X}$ |
| 1111 | X $\times \times \times$ | X $\times$ X $\times$ X $\times$ X $\times$ |

Using Karnaugh maps:
$J_{\mathrm{A}}=A+B D+B C, K_{\mathrm{A}}=0 ; J_{\mathrm{B}}=C+D, K_{\mathrm{B}}=C+D ;$ $J_{\mathrm{C}}=D^{\prime}, K_{\mathrm{C}}=D^{\prime} ; J_{\mathrm{D}}=1, K_{\mathrm{D}}=1$
12.35

| Clock <br> Cycle | Input <br> Data | EnIn | EnAd | LdAc | LdAd | Accumulator <br> Register | Addend <br> Register | Bus | Description |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 0 | 18 | 1 | 0 | 1 | 0 | 0 | 0 | 18 | Input to accumulator |
| 1 | 13 | 1 | 0 | 0 | 1 | 18 | 0 | 13 | Input to addend |
| 2 | 15 | 0 | 1 | 1 | 0 | 18 | 13 | 31 | Sum to accumulator |
| 3 | 93 | 1 | 0 | 0 | 1 | 31 | 13 | 93 | Input to addend |
| 4 | 47 | 0 | 1 | 1 | 0 | 31 | 93 | 124 | Sum to accumulator |
| 5 | 22 | 1 | 0 | 0 | 1 | 124 | 93 | 22 | Input to addend |
| 6 | 0 | 0 | 1 | 0 | 0 | 124 | 22 | 146 | Sum on bus |

Note: Register values change after the clock edge. So a value loaded from the bus appears in the register on the next clock cycle after the load signal and bus value are present.

## Unit 12 Solutions

12.36
(a), (b)

12.36 (c) Call the values beginning in the $A \& D$ registers $X$ and $Y$, respectively. We want $C=X+Y=\left(X^{\prime} Y^{\prime}\right)^{\prime}$. Invert using $M^{\prime}=1$ NAND $M$. To invert a value on the right side, in register $C$ or $D$, we will need a 1 on the left side, in register $A$ or $B$. This can be accomplished using $1=0$ NAND (anything.)

There are several solutions using different registers. Here is an example:

| Clock <br> Cycle | $\mathrm{G}_{0} \mathrm{G}_{1}$ | $\mathrm{E}_{0}$ | $\mathrm{E}_{1} \mathrm{E}_{2}$ | Description |
| :---: | :---: | :---: | :---: | :--- |
| 1 | 00 | 0 | 11 | 1 NAND $A=A^{\prime}=X^{\prime} \rightarrow A$ |
| 2 | 01 | 1 | 10 | 0 NAND $B=1 \rightarrow B$ |
| 3 | 11 | 1 | 01 | $B$ NAND $D=1$ NAND $Y=Y^{\prime} \rightarrow D$ |
| 4 | 10 | 0 | 01 | $A$ NAND $D=X^{\prime}$ NAND $Y^{\prime}=X+Y \rightarrow C$ |

Alternate three-cycle solution:
Use $X+Y=X+X^{\prime} Y=\left(X^{\prime}\left(X^{\prime} Y\right)^{\prime}\right)^{\prime}$

| Clock <br> Cycle | $\mathrm{G}_{0} \mathrm{G}_{1}$ | $\mathrm{E}_{0}$ | $\mathrm{E}_{1} \mathrm{E}_{2}$ | Description |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 00 | 0 | 11 | 1 NAND $A=A^{\prime}=X^{\prime} \rightarrow A$ |
| 2 | 11 | 0 | 01 | A NAND $D=\left(X^{\prime} Y\right)^{\prime} \rightarrow D$ |
| 3 | 10 | 0 | 01 | A NAND $D=\left(X^{\prime}\left(X^{\prime} Y\right)^{\prime}\right)^{\prime}=X+Y \rightarrow C$ |

12.37 (a) For bit reversal using the D inputs of the shift register: $S h=0, L d=1$

12.37 (b) Same as Figure 12-10 (b) on FLD p. 360, except that for the "11" input of each MUX, instead of SI, $Q_{3}, Q_{2}$, or $Q_{1}$, use $Q_{0}, Q_{1}, Q_{2}$, or $Q_{3}$, respectively. Also, replace $S h$ with $A$ and $L d$ with $B$.

## Unit 13 Problem Solutions

13.2 Notice that this is a shift register. At each falling clock edge, $Q_{3}$ takes on the value $Q_{2}$ had right before the clock edge, $Q_{2}$ takes on the value $Q_{1}$ had right before the clock edge, and $Q_{1}$ takes on the value $X$ had right before the clock edge. For example, if the initial state is 000 and the input sequence is $X=1100$, the state sequence is $=100,110,011,001$, and the output sequence is $Z=(0) 0011 . Z$ is always $Q_{3}$, which does not depend on the present value of $X$. So it's a Moore machine. See FLD p. 720 for the state graph.
13.3 (a) $A^{+}=A K_{\mathrm{A}}^{\prime}+A^{\prime} J_{\mathrm{A}}=A\left(B^{\prime}+X\right)+A^{\prime}\left(B X^{\prime}+B^{\prime} X\right)$
$B^{+}=B^{\prime} J_{\mathrm{B}}+B K_{\mathrm{B}}^{\prime}=A B^{\prime} X+B\left(A^{\prime}+X^{\prime}\right)$
$Z=A B$

13.3 (b) $X=\begin{array}{lllll}0 & 1 & 1 & 0 & 0\end{array}$
$A B=\begin{array}{llllll}00 & 00 & 10 & 11 & 01 & 11\end{array}$
$Z=(0) \quad 0 \quad 0 \quad 1 \quad 0 \quad 1$
13.4 (a)

$\mathrm{Q}_{1}{ }^{+}=\mathrm{D}_{1}$



Z
$Z$ depends on the input $X$, so this is a Mealy machine. Because there are more than 2 state variables, we cannot put the state table in Karnaugh Map order (i.e. 00, 01, 11, 10), but we can still read the next state and output from the Karnaugh map. For example, when the input is $X=1$ and the state is $Q_{1} Q_{2} Q_{3}=110$, we can read the next state and output from the $X Q_{1} Q_{2} Q_{3}=1110$ position in the Karnaugh maps for $Q_{1}^{+}, Q_{2}^{+}, Q_{2}^{+}$, and $Z$. So in this case, the next state is $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}=101$ and the output is $Z=0$. The entire table can be derived from the Karnaugh maps in this manner. Note: We can also fill in the state table directly from the equations, without using Karnaugh maps. See FLD p. 720 for the state table and state graph.
13.4 (b-d) See FLD p. 721 for solutions.

## Unit 13 Solutions

13.5 (a) Mealy machine, because the output, $Z$, depends on the input $X$ as well as the present state.
13.5 (c - d) See FLD p. 721 for solutions.
13.5 (b)


Note: Not all Karnaugh map entries are needed. See FLD p. 721 for the state table.
13.6 (a) After a rising clock edge, it takes 4 ns for the flip-flop outputs to change. Then the ROM will take 8 ns to respond to the new flip-flop outputs. The ROM outputs must be correct at the flip-flop inputs for at least the setup time of 2 ns before the next rising clock edge. So the minimum clock period is $(4+8+2) \mathrm{ns}=14 \mathrm{~ns}$.
13.6 (b) The correct output sequence is 0101 . See FLD p. 722 for the timing diagram.
13.6 (c) Read the state transition table from ROM truth table. See FLD p. 722 for the state graph and table.

| Present State | Next State$Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| 00 | 10 | 10 | 0 | 0 |
| 01 | 00 | 11 | 0 | 0 |
| 10 |  | 01 | 0 | 1 |
| 11 | 01 | 11 | 1 | 1 |

Alternate solution: Using Karnaugh map order, swap states $S_{2}$ and $S_{3}$ in the graph and table.
13.7 (a)
$Q_{1}{ }^{+}=J_{1} Q_{1}{ }^{\prime}+K_{1}{ }^{\prime} Q_{1}=X Q_{1}{ }^{\prime}+X Q_{2}{ }^{\prime} Q_{1}$
$Q_{2}{ }^{+}=J_{2} Q_{2}{ }^{\prime}+K_{2}{ }^{\prime} Q_{2}=X Q_{2}{ }^{\prime}+X Q_{1} Q_{2}$
$Z=X^{\prime} Q_{2}{ }^{\prime}+X Q_{2}$

| Present <br> State | Next State <br> $Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| 00 | 00 | 11 | 1 | 0 |
| 01 | 00 | 10 | 0 | 1 |
| 11 | 00 | 01 | 0 | 1 |
| 10 | 00 | 11 | 1 | 0 |

13.7 (c) $\mathrm{Z}=00011$

13.7 (b)

13.8 (a)
$Q_{1}{ }^{+}=J_{1} Q_{1}{ }^{\prime}+K_{1}{ }^{\prime} Q_{1}=X Q_{2}{ }^{\prime} Q^{\prime}{ }^{\prime}+X^{\prime} Q_{1}$
$Q_{2}{ }^{+}=J_{2} Q_{2}{ }^{\prime}+K_{2}{ }^{\prime} Q_{2}=X Q_{1} Q_{2}{ }^{\prime}+X^{\prime} Q_{2}$
$Z=X Q_{2}{ }^{\prime}+X^{\prime} Q_{2}$

| Present <br> State | Next State <br> $Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| 00 | 00 | 10 | 1 | 0 |
| 01 | 01 | 00 | 0 | 1 |
| 11 | 11 | 00 | 0 | 1 |
| 10 | 10 | 01 | 1 | 0 |

13.8 (c) $\mathrm{Z}=10110$

13.9 (a) $Q_{1}{ }^{+}=D_{1}=\left(X_{1}{ }^{\prime}+X_{2}{ }^{\prime}+Q_{1}\right)\left(Q_{1}+Q_{2}\right)\left(X_{1}{ }^{\prime}+Q_{2}\right)$
$Q_{2}{ }^{+}=D_{2}=\left(X_{1}{ }^{\prime} X_{2}{ }^{\prime}+Q_{1}{ }^{\prime}\right)\left(X_{1} X_{2}+Q_{2}\right)$
$Z=Q_{1} Q_{2}{ }^{\prime}$

| State | Present <br> State | Next State <br> $X_{1} X_{2}$ |  |  |  | Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 |  |
| $S_{0}$ | 00 | 00 | 01 | 01 | 00 | 0 |
| $S_{1}$ | 01 | 11 | 11 | 01 | 11 | 0 |
| $S_{3}$ | 11 | 11 | 10 | 10 | 10 | 0 |
| $S_{2}$ | 10 | 10 | 10 | 00 | 00 | 1 |


13.9 (b)

13.9 (c) $\mathrm{Z}=(0) 000110$
13.10(a) $Q_{1}{ }^{+}=D_{1}=X_{1} X_{2} Q_{1}+Q_{1} Q_{2}+X_{2} Q_{2}$
$Q_{2}{ }^{+}=D_{2}=\left(X_{1}{ }^{\prime}+X_{2}{ }^{\prime}\right) Q_{2}+\left(X_{1}+X_{2}\right) Q_{1}{ }^{\prime}$
$Z=Q_{1} Q_{2}$

|  | Present | Next State |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | State | $X_{1}=$ |  |  |  | Z |
|  | $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 |  |
| $S_{0}$ | 00 | 00 | 01 | 01 | 01 | 0 |
| $S_{1}$ | 01 | 01 | 11 | 11 | 01 | 0 |
| $S_{3}$ | 11 | 11 | 11 | 10 | 11 | 0 |
| $S_{2}$ | 10 | 00 | 00 | 10 | 00 | 1 |



Unit 13 Solutions

13.10(c)
$Z=(0) 000110$
13.11 (a) Notice that $Z$ depends on the input $X$, so this is a Mealy machine.
$Q_{1}^{+}=J_{1} Q_{1}^{\prime}+K_{1}^{\prime} Q_{1}=X Q_{1}^{\prime} Q_{2}+X^{\prime} Q_{1}$
$Q_{2}^{+}=J_{2} Q_{2}^{\prime}+K_{2}^{\prime} Q_{2}=X Q_{1}^{\prime} Q_{2}^{\prime}+X^{\prime} Q_{2}$
$Z=Q_{2} \oplus X=X Q_{2}^{\prime}+X^{\prime} Q_{2}$

| State | Present <br> State | Next State <br> $Q_{1}^{+} Q_{2}^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | 00 | 00 | 01 | 0 | 1 |
| $S_{1}$ | 01 | 01 | 10 | 1 | 0 |
| $S_{2}$ | 11 | 11 | 00 | 1 | 0 |
| $S_{3}$ | 10 | 10 | 00 | 0 | 1 |



Alternate solution: Swap states $S_{2}$ and $S_{3}$.

13.12(a) Notice that $Z$ does not depend on either input, so this is a Moore machine.
$Q_{1}^{+}=X_{1} X_{2} Q_{1}+Q_{1} Q_{2}+X_{1} Q_{2}$
$Q_{2}^{+}=Q_{1}^{\prime}\left(X_{1}+X_{2}\right)+Q_{2}\left(X_{1}^{\prime}+X_{2}^{\prime}\right)$
$=X_{1} Q_{1}^{\prime}+X_{2} Q_{1}^{\prime}+X_{1}^{\prime} Q_{2}+X_{2}^{\prime} Q_{2}$
$Z=Q_{1} Q_{2}^{\prime}$

13.11
(b)


$\mathrm{Q}_{2}{ }^{+}$

| 13.12(a) <br> (contd) | State | Present <br> State | Next State <br> $X_{1} X_{2}=$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 | Z |
| $S_{0}$ | 00 | 00 | 01 | 01 | 01 | 0 |
| $S_{1}$ | 01 | 01 | 01 | 11 | 11 | 0 |
| $S_{2}$ | 11 | 11 | 11 | 10 | 11 | 0 |
| $S_{3}$ | 10 | 00 | 00 | 10 | 00 | 1 |


13.13


Correct output: $Z=1011$

Correct output: $Z=(0) 00110$
13.14


Correct output: $Z=0011$

### 13.15 (a)


$\mathrm{D}_{1}=\mathrm{Q}_{1}^{+}$



| $\mathrm{Q}_{2} \mathrm{Q}_{3}$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 0 | 1 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 0 | 0 |

Unit 13 Solutions
13.15 (a) (contd)

| State | Present <br> State | Next State <br> $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Q_{1} Q_{2} Q_{3}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | 000 | 001 | 101 | 0 | 1 |
| $S_{1}$ | 001 | 011 | 111 | 0 | 1 |
| $S_{2}$ | 010 | 110 | 101 | 1 | 0 |
| $S_{3}$ | 011 | 010 | 111 | 1 | 0 |
| $S_{4}$ | 100 | 001 | 001 | 0 | 1 |
| $S_{5}$ | 101 | 011 | 011 | 0 | 1 |
| $S_{6}$ | 110 | 110 | 101 | 1 | 0 |
| $S_{7}$ | 111 | 010 | 011 | 1 | 0 |


13.15 (b)

13.15 (c) From diagram: $0,1,(0), 1,0,1$

From graph: $\quad 0,1,1,0,1$
(they are the same, except for the false output)
13.15 (d) Change the input on the falling edge of the clock (assuming negligible circuit delays).
13.16 (a)


| State | $\begin{gathered} \text { Present } \\ \text { State } \\ Q_{1} Q_{2} Q_{3} \\ \hline \end{gathered}$ | Next State$Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | 000 | 100 | 011 | 1 | 0 |
| $S_{1}$ | 001 | 000 | 011 | 0 | 1 |
| $S_{2}$ | 010 | 100 | 000 | 1 | 0 |
| $S_{3}$ | 011 | 000 | 000 | 0 | 1 |
| $S_{4}$ | 100 | 110 | 011 | 1 | 0 |
| $S_{5}$ | 101 | 000 | 011 | 0 | 1 |
| $S_{6}$ | 110 | 111 | 011 | 1 | 0 |
| $S_{7}$ | 111 | 001 | 001 | 0 | 1 |


13.16 (b)

13.17 (a) $Q_{1}+=J_{1} Q_{1}{ }^{\prime}+K_{1}{ }^{\prime} Q_{1}$

$$
\begin{aligned}
& =\left(X Q_{2}^{\prime}+X Q_{2}^{\prime}\right) Q_{1}^{\prime}+\left(X+Q_{2}^{\prime}\right) Q_{1} \\
& =X Q_{1}^{\prime} Q_{2}^{\prime}+X^{\prime} Q_{1}^{\prime} Q_{2}+X Q_{1}+Q_{1} Q_{2}^{\prime} \\
& =X Q_{2}^{\prime}+X^{\prime} Q_{1}^{\prime} Q_{2}+X Q_{1}+Q_{1} Q_{2}^{\prime} \\
Q_{2}+ & =J_{2} Q_{2}^{\prime}+K_{2}^{\prime} Q_{2} \\
& =X Q_{1}^{\prime} Q_{2}^{\prime}+\left(X^{\prime}+Q_{1}\right) Q_{2} \\
& =X Q_{1}^{\prime} Q_{2}^{\prime}+X^{\prime} Q_{2}+Q_{1} Q_{2} \\
Z= & Q_{1} Q_{2}
\end{aligned}
$$

| Present <br> State | Next State <br> $Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  | $Z$ |
| :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ |  |
| 00 | 00 | 11 | 0 |
| 01 | 11 | 00 | 0 |
| 10 | 10 | 10 | 0 |
| 11 | 01 | 11 | 1 |

13.17 (c) $\mathrm{Z}=(0) 01101$

### 13.16 (c)

From diagram: 101 (0) 11
From graph: 10111
(they are the same, except for the false output)

### 13.16 (d)

Change the input on the falling edge of the clock (assuming negligible circuit delays).


The circuit is a Moore circuit. State 2 is unused.
13.17 (b)


### 13.18

| Clock Cycle | Information Gathered |
| :---: | :--- |
| 1 | $Q_{1} Q_{2}=00, X=0 \Rightarrow Z=1, Q_{1}^{+} Q_{2}^{+}=01$ |
| 2 | $Q_{1} Q_{2}=01, X=0 \Rightarrow Z=0 ; X=1 \Rightarrow Z=1, Q_{1}^{+} Q_{2}^{+}=11$ |
| 3 | $Q_{1} Q_{2}=11, X=1 \Rightarrow Z=1 ; X=0 \Rightarrow Z=0, Q_{1}^{+} Q_{2}^{+}=10$ |
| 4 | $Q_{1} Q_{2}=10, X=0 \Rightarrow Z=1 ; X=1 \Rightarrow Z=0, Q_{1}^{+} Q_{2}^{+}=00$ |
| 5 | $Q_{1} Q_{2}=00, X=1 \Rightarrow Z=0, Q_{1}^{+} Q_{2}^{+}=10$ |
| 6 | $Q_{1} Q_{2}=10, X=1 \Rightarrow(Z=0) ; X=0 \Rightarrow(Z=1), Q_{1}^{+} Q_{2}^{+}=11$ |
| 7 | $Q_{1} Q_{2}=11, X=0 \Rightarrow(Z=0) ; X=1 \Rightarrow(Z=1), Q_{1}^{+} Q_{2}^{+}=01$ |
| 8 | $Q_{1} Q_{2}=01, X=1 \Rightarrow(Z=1) ; X=0 \Rightarrow(Z=0), Q_{1}^{+} Q_{2}^{+}=00$ |
| 9 | $Q_{1} Q_{2}=00, X=0 \Rightarrow(Z=1)$ |
| Note: Information inside parentheses was already obtained in a previous <br> clock cycle. |  |


| Present <br> State | Next State <br> $Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| 00 | 01 | 10 | 1 | 0 |
| 01 | 00 | 11 | 0 | 1 |
| 10 | 11 | 00 | 1 | 0 |
| 11 | 10 | 01 | 0 | 1 |



Unit 13 Solutions
13.19

| Clock Cycle | Information Gathered |
| :---: | :--- |
| 1 | $Q_{1} Q_{2}=00, X=0 \Rightarrow Z=0, Q_{1}^{+} Q_{2}^{+}=10$ |
| 2 | $Q_{1} Q_{2}=10, X=0 \Rightarrow Z=1 ; X=1 \Rightarrow Z=0, Q_{1}^{+} Q_{2}^{+}=01$ |
| 3 | $Q_{1} Q_{2}=01, X=1 \Rightarrow Z=0 ; X=0 \Rightarrow Z=1, Q_{1}^{+} Q_{2}^{+}=10$ |
| 4 | $Q_{1} Q_{2}=10, X=0 \Rightarrow(Z=1), Q_{1}^{+} Q_{2}^{+}=11$ |
| 5 | $Q_{1} Q_{2}=11, X=0 \Rightarrow Z=0, Q_{1}^{+} Q_{2}^{+}=11$ |
| 6 | $Q_{1} Q_{2}=11, X=0 \Rightarrow(Z=0) ; X=1 \Rightarrow Z=1, Q_{1}^{+} Q_{2}^{+}=01$ |
| 7 | $Q_{1} Q_{2}=01, X=1 \Rightarrow(Z=0), Q_{1}^{+} Q_{2}^{+}=00$ |
| 8 | $Q_{1} Q_{2}=00, X=1 \Rightarrow Z=1, Q_{1}^{+} Q_{2}^{+}=11$ |
| 9 | $Q_{1} Q_{2}=11, X=1 \Rightarrow(Z=1)$ |
| Note: Information inside parentheses was already obtained in a previous <br> clock cycle. |  |


| Present <br> State | Next State <br> $Q_{1}^{+} Q_{2}^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $\mathrm{X}=0$ | $\mathrm{X}=1$ |
| 00 | 10 | 11 | 0 | 1 |
| 01 | 10 | 00 | 1 | 0 |
| 10 | 11 | 01 | 1 | 0 |
| 11 | 11 | 01 | 0 | 1 |


13.20

| Clock Cycle | Information Gathered |
| :---: | :---: |
| 1 | $Q_{1} Q_{2}=00, X_{1} X_{2}=01 \Rightarrow Z_{1} Z_{2}=10, Q_{1}^{+} Q_{2}^{+}=01$ |
| 2 | $Q_{1} Q_{2}=01, X_{1} X_{2}=01 \Rightarrow Z_{1} Z_{2}=01 ; X_{1} X_{2}=10 \Rightarrow Z_{1} Z_{2}=10, Q_{1}^{+} Q_{2}^{+}=10$ |
| 3 | $Q_{1} Q_{2}=10, X_{1} X_{2}=10 \Rightarrow Z_{1} Z_{2}=00 ; X_{1} X_{2}=11 \Rightarrow Z_{1} Z_{2}=00, Q_{1}^{+} Q_{2}^{+}=01$ |
| 4 | $Q_{1} Q_{2}=01, X_{1} X_{2}=11 \Rightarrow Z_{1} Z_{2}=11 ; X_{1} X_{2}=01 \Rightarrow\left(Z_{1} Z_{2}=01\right), Q_{1}^{+} Q_{2}^{+}=11$ |
| 5 | $Q_{1} Q_{2}=11, X_{1} X_{2}=01 \Rightarrow Z_{1} Z_{2}=01$ |

Note: When $Q_{1} Q_{2}=01$, the outputs $Z_{1} Z_{2}$ vary depending on the inputs $X_{1} X_{2}$, so this is a Mealy machine.

| Present |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | $Q_{1}^{+} Q_{2}^{+}$ |  |  |  | $Z_{1} Z_{2}$ |  |  |  |  |
| $X_{1} X_{2}=$ |  | $X_{1} X_{2}=$ |  |  |  |  |  |  |  |
| $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |  |
| 00 | $?$ | 01 | $?$ | $?$ | $?$ | 10 | $?$ | $?$ |  |
| 01 | $?$ | 11 | $?$ | 10 | $?$ | 01 | 11 | 10 |  |
| 11 | $?$ | $?$ | $?$ | $?$ | $?$ | 01 | $?$ | $?$ |  |
| 10 | $?$ | $?$ | 01 | $?$ | $?$ | $?$ | 00 | 00 |  |

13.21

| Present | $Q_{1}^{+} Q_{2}^{+}$ |  |  |  |  | $Z_{1} Z_{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | $X_{1} X_{2}=$ |  |  |  | $X_{1} X_{2}=$ |  |  |  |  |  |
| $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |  |  |
| 00 | $?$ | 01 | $?$ | $?$ | $?$ | 10 | $?$ | $?$ |  |  |
| 01 | $?$ | 11 | $?$ | 10 | $?$ | 01 | 11 | 10 |  |  |
| 11 | $?$ | $?$ | $?$ | $?$ | $?$ | 01 | $?$ | $?$ |  |  |
| 10 | $?$ | $?$ | 01 | $?$ | $?$ | $?$ | 00 | 00 |  |  |

? indicates next state or output values that cannot be determined from the timing chart
13.22(a) $Q_{1}{ }^{+}=D_{1}=X^{\prime} Q_{1}+X Q_{1}{ }^{\prime} Q_{2}$
$Q_{2}{ }^{+}=D_{2}=X^{\prime} Q_{2}+X Q_{1}{ }^{\prime} Q_{2}^{\prime}$

$$
Z=X^{\prime} Q_{2}+X Q_{1}^{\prime} Q_{2}^{\prime}+X Q_{1} Q_{2}^{\prime}
$$

| Present <br> State | Next State <br> $Q_{1}{ }^{+}{ }^{+}$ <br> $Q^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $\mathrm{~S}_{0}=00$ | 00 | 01 | 0 | 1 |
| $\mathrm{~S}_{1}=01$ | 01 | 10 | 1 | 0 |
| $\mathrm{~S}_{3}=11$ | 11 | 00 | 1 | 0 |
| $\mathrm{~S}_{2}=10$ | 10 | 00 | 0 | 1 |

13.22(c) $Z=11101$
13.23 (a)


Correct output: $Z_{1} Z_{2}=10,10,00,01$
13.23 (b)

| Present <br> State | $Q_{1}^{+} Q_{2}^{+}$ |  |  |  | $Z_{1} Z_{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X_{1} X_{2}=$ |  |  | $X_{1} X_{2}=$ |  |  |  |  |  |  |
| $Q_{1} Q_{2}$ | 00 | 01 | 10 | 11 | 00 | 01 | 10 | 11 |  |
| 00 | 00 | 00 | 01 | 01 | 10 | 10 | 10 | 10 |  |
| 01 | 11 | 11 | 10 | 10 | 00 | 10 | 01 | 11 |  |
| 10 | 11 | 00 | 11 | 00 | 00 | 00 | 01 | 01 |  |
| 11 | 10 | 01 | 10 | 01 | 00 | 00 | 00 | 00 |  |


13.22(b)



## Unit 13 Solutions

Transition table using a straight binary state assignment:

|  | Present | Next State |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| State | State |  |  |  |  |
|  | $Q_{1} Q_{2} Q_{3}$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}^{+} Q_{2}^{+} Q_{3}^{+}$ | 000 | 001 | 011 | 0 | 0 |
| $S_{1}$ | 001 | 010 | 011 | 0 | 0 |
| $S_{2}$ | 010 | 001 | 011 | 0 | 1 |
| $S_{3}$ | 011 | 100 | 000 | 0 | 0 |
| $S_{4}$ | 100 | 011 | 000 | 0 | 1 |


13.25 (a)

13.26


All flip-flop inputs are stable for more than the setup time before each falling clock edge. So the circuit is operating properly.
13.25(b) If $X$ is changed early enough:

Minimum clock period $=$ Flip-flop propagation delay + Two NAND-gate delays + Setup time $=4+(3+3)+2=12 \mathrm{~ns}$
$X$ can change as late as 8 ns (two NAND-gate delays plus the setup time) before the next falling edge without causing improper operation.

Correct output: Z = 10101
Deriving the State Table:
JK flip-flop equation:
$Q^{+}=J Q^{\prime}+K^{\prime} Q$
$\therefore A^{+}=\left(X^{\prime} C+X C^{\prime}\right) A^{\prime}+X^{\prime} A$
[As, $\left.J_{\mathrm{A}}=X^{\prime} C+X C^{\prime}, K_{\mathrm{A}}=X, Q=A\right]$
$=A^{\prime} X^{\prime} C+A^{\prime} X C^{\prime}+X^{\prime} A$
Similarly, $B^{+}=X C^{\prime}+X A+X^{\prime} A^{\prime} C$
$C^{+}=0^{\prime} \cdot C+\left(X B^{\prime} A\right) \cdot C^{\prime}=C+X B^{\prime} A$
$Z=X B+X^{\prime} C^{\prime}+X^{\prime} B^{\prime} A$
13.26
(contd)

| $\mathrm{A}^{+}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| B C | 00 | 01 | 11 | 10 |
|  |  |  |  |  |
|  | 0 | 1 | 0 | 1 |
| 01 | 1 | 1 | 0 | 0 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 0 | 1 | 0 | 1 |

From the Karnaugh maps, we can get the state table that follows:

| State | Present <br> State | Next State <br> $A^{+} B^{+} C^{+}$ |  | Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $A B C$ | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | 000 | 000 | 110 | 1 | 0 |
| $S_{1}$ | 001 | 111 | 001 | 0 | 0 |
| $S_{2}$ | 010 | 000 | 110 | 1 | 1 |
| $S_{3}$ | 011 | 111 | 001 | 0 | 1 |
| $S_{4}$ | 100 | 100 | 011 | 1 | 0 |
| $S_{5}$ | 101 | 101 | 011 | 1 | 0 |
| $S_{6}$ | 110 | 100 | 010 | 1 | 1 |
| $S_{7}$ | 111 | 101 | 011 | 0 | 1 |

13.27 (contd)

| State | Present State $A B$ | $A^{+} B^{+}$ |  |  |  | $Z_{1} Z_{2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $X_{1} X_{2}=$ |  |  |  | $X_{1} X_{2}=$ |  |  |  |
|  |  | 00 | 01 | 10 | 11 | 00 | 01 | 10 | 11 |
| $S_{0}$ | 00 | 11 | 01 | 10 | 00 | 10 | 10 | 00 | 00 |
| $S_{1}$ | 01 | 11 | 01 | 01 | 01 | 11 | 00 | 11 | 00 |
| $S_{2}$ | 10 | 10 | 00 | 10 | 10 | 01 | 00 | 01 | 00 |
| $S_{3}$ | 11 | 10 | 00 | 11 | 01 | 00 | 00 | 00 | 00 |


13.27

$$
\begin{aligned}
R= & X_{2}\left(X_{1}^{\prime}+B\right) \\
S= & X_{2}^{\prime}\left(X_{1}^{\prime}+B^{\prime}\right) \\
A^{+} & =A\left[\left(X_{2}\right)\left(X_{1}^{\prime}+B\right)\right]^{\prime}+X_{2}^{\prime}\left(X_{1}^{\prime}+B^{\prime}\right) \\
& =A\left(X_{2}^{\prime}+X_{1} B^{\prime}\right)+X_{2}^{\prime} X_{1}^{\prime}+X_{2}^{\prime} B^{\prime} \\
A^{+} & =A X_{2}^{\prime}+A X_{1} B^{\prime}+X_{2}^{\prime} X_{1}^{\prime}+X_{2}^{\prime} B^{\prime} \\
T= & X_{1}^{\prime} B A+X_{1}^{\prime} B^{\prime} A^{\prime} \\
B^{+} & =B T^{\prime}+B^{\prime} T \\
& =B\left(X_{1}^{\prime} B A+X_{1}^{\prime} B^{\prime} A^{\prime}\right)^{\prime} B^{\prime}\left(X_{1}^{\prime} B A+X_{1}^{\prime} B^{\prime} A^{\prime}\right)^{\prime} \\
& =B\left[\left(X_{1}^{\prime} B A\right)^{\prime}\left(X_{1}^{\prime} B^{\prime} A^{\prime}\right)^{\prime}\right]+X_{1}^{\prime} B^{\prime} A^{\prime} \\
& =B\left[\left(X_{1}+B^{\prime}+A^{\prime}\right)\left(X_{1}+B+A\right)\right]+X_{1}^{\prime} B^{\prime} A^{\prime} \\
& =\left(B X_{1}+B A^{\prime}\right)\left(X_{1}+B+A\right)+X_{1}^{\prime} B^{\prime} A^{\prime} \\
& =B X_{1}+B X_{1}+B X_{1} A+B A^{\prime} X_{1}+B A^{\prime}+X_{1}^{\prime} B^{\prime} A^{\prime} \\
& =X_{1} B\left(1+1+A+A^{\prime}\right)+A^{\prime}\left(B+X_{1}^{\prime} B\right) \\
& =X_{1} B+A^{\prime} B+X_{1}^{\prime} A^{\prime}
\end{aligned}
$$



Unit 13 Solutions

## Unit 14 Problem Solutions

14.4 Typical input and output sequences:
$X=0100000101011 \ldots$
$Z=(0) 0000000111 \ldots$ (output remains 1$)$
$X=111110111111000101 \ldots$
$Z=(0) 000000000000111111 \ldots$ (output remains 1)
$X=010101 \ldots$
$Z=(0) 000111 \ldots$ (output remains 1)
See FLD p. 723 for state graph.

The state meanings are given in the following table:

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | One 0, no 1's |
| $S_{2}$ | $\geq$ Two 0's, no 1's |
| $S_{3}$ | $\geq$ Two 0's and one 1 |
| $S_{4}$ | $\geq$ Two 0's and $\geq$ Two 1's |
| $S_{5}$ | $\geq$ One 1, no 0's |
| $S_{6}$ | $\geq$ Two 1's, no 0's |
| $S_{7}$ | $\geq$ Two 1's and one 0 |
| $S_{8}$ | One 0 and one 1 |

14.5 Typical input and output sequence:
$X=001010110010100 \ldots$
$Z_{1}=000101000000000 \ldots$ (output remains 0 after 100 received)
$Z_{2}=000000000100001 \ldots$ (at this point, the sequence 01 has occurred, so $Z_{1}=0$ from now on)
The graph needs two distinct parts. The first checks for 010 and 100. If 100 is received, we proceed to the second part of the graph, which checks only for 100 . The two parts are joined by a one-way arc, so once in the second part it is impossible to go back to the first.
See FLD p. 723 for state table and graph.
The state meanings are given in the following table:

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | Last input was 0, 100 has never occurred |
| $S_{2}$ | Last input was 01, 100 has never occurred |
| $S_{3}$ | Last input was 1, 100 has never occurred |
| $S_{4}$ | Last input was 10, 100 has never occurred |
| $S_{5}$ | Last input was 0,- 100 has occurred at least once- |
| $S_{6}$ | Last input was 1, 100 has occurred at least once |
| $S_{7}$ | Last input was10, 100 has occurred at least once |

## Unit 14 Solutions

14.6 This should be solved in the same way as Example 3 on FLD p. 443. Assign a state to each possible input (00, 01, 11, 10) with an output of 0 , and another state to each input with an output of 1 . This gives eight states.

See FLD p. 724 for the state table.

| State | $\mathrm{Z}=0$ |
| :---: | :--- |
| $S_{0}$ | Last input was 00 |
| $S_{1}$ | Last input was 01 |
| $S_{2}$ | Last input was 11 |
| $S_{3}$ | Last input was 10 |


| State | $\mathrm{Z}=1$ |
| :---: | :--- |
| $S_{4}$ | Last input was 00 |
| $S_{5}$ | Last input was 01 |
| $S_{6}$ | Last input was 11 |
| $S_{7}$ | Last input was 10 |

Each input takes you to the state defined by that input (e.g. an input of 01 takes you to either $S_{1}$ or $S_{5}$ ). The only thing in question is whether the output is 0 or 1 . Determine the output by checking whether the last two inputs correspond to the three input sequences.

Alternate Solution: Notice that when $Z=0$, "causes the output to become 0 " is the same as remaining constant, and "causes the output to become 1 " is the same as toggling the output. The situation is similar when $Z=1$. So we can use only four states, as follows:

| State | Meaning |
| :---: | :---: |
| $S_{0}$ | $Z=0$ and last input was either 00 or 01 |
| $S_{1}$ | $Z=0$ and last input was either 10 or 11 |
| $S_{2}$ | $Z=1$ and last input was either 00 or 11 |
| $S_{3}$ | $Z=1$ and last input was either 01 or 10 |


|  | Next State |  |  |  |  |
| :---: | ---: | :--- | :--- | :--- | :--- |
| State | $X_{1} X_{2}=00$ | 01 | 10 | 11 | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{2}$ | $S_{0}$ | $S_{1}$ | $S_{1}$ | 0 |
| $S_{2}$ | $S_{2}$ | $S_{3}$ | $S_{3}$ | $S_{2}$ | 1 |
| $S_{3}$ | $S_{0}$ | $S_{3}$ | $S_{3}$ | $S_{2}$ | 1 |

Note: The state table with 8 states reduces to this 4 -state table using methods in Unit 15.

14.7 (a) Typical input and output sequence:
$X=00100110001101001 \ldots$
$\mathrm{Z}=11000011110001110 \ldots$

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Number of 1's is divisible by three |
| $S_{1}$ | Number of 1's is one more than divisible by 3 |
| $S_{2}$ | Number of 1's is two more than divisible by 3 |

See FLD p. 724 for state graph.
14.7 (b) Typical input and output sequence:
$X=000011110001101111 \ldots$
$Z=010100100000010010 \ldots$

See FLD p. 724 for state graph.
14.7 (b)
(contd)

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Number of 1's is divisible by three, no 0's |
| $S_{1}$ | Number of 1's is one more than divisible by 3, no 0's |
| $S_{2}$ | Number of 1's is two more than divisible by 3, no 0's |
| $S_{3}$ | Number of 1's is divisible by three, number of 0's is odd |
| $S_{4}$ | Number of 1's is one more than divisible by 3, number of 0's is odd |
| $S_{5}$ | Number of 1's is two more than divisible by 3, number of 0's is odd |
| $S_{6}$ | Number of 1's is divisible by three, number of 0's is even and < 0 |
| $S_{7}$ | Number of 1's is one more than divisible by 3, number of 0's is even and <0 |
| $S_{8}$ | Number of 1's is two more than divisible by 3, number of 0's is even and <0 |

14.8 (a) Typical input and output sequence:
$X_{1}=1001001110 \ldots$
$X_{2}=1000110011 \ldots$
$Z_{1}=0^{*} 001001010 \ldots$
$Z_{2}=0^{*} 100100001 \ldots$

* Regardless of any value of $N$.

See FLD p. 724 for state table.

14.8 (b) Similar to part (a), but we need a separate state for each possible output and previous input.

See FLD p. 725 for state table.

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset state / current output is $=00$ |
| $S_{1}$ | Previous input was $00 /$ current output is $=00$ |
| $S_{2}$ | Previous input was $00 /$ current output is $=01$ |
| $S_{3}$ | Previous input was $01 /$ current output is $=10$ |
| $S_{4}$ | Previous input was $01 /$ current output is $=00$ |
| $S_{5}$ | Previous input was $01 /$ current output is $=01$ |
| $S_{6}$ | Previous input was $10 /$ current output is $=10$ |
| $S_{7}$ | Previous input was $10 /$ current output is $=00$ |
| $S_{8}$ | Previous input was $10 /$ current output is $=01$ |
| $S_{9}$ | Previous input was $11 /$ current output is $=10$ |
| $S_{10}$ | Previous input was $11 /$ current output is $=00$ |

14.9 (a)

| State | Meaning |
| :---: | :---: |
| $S_{0}$ | Previous output bit was 0 |
| $S_{1}$ | Previous output bit was 1 |



See FLD p. 725 for state table.
14.9 (b)

| State | Meaning |
| :--- | :--- |
| $S_{0}$ | Output bit is 0 |
| $S_{1}$ | Output bit is 1 |

14.9 (c) A false output occurs in NRZI just before the input NRZ goes from 1 to 0 .
14.10 See FLD p. 725 for solution.
14.12 (a)


### 14.13 (a)

|  | Next State |  | $Z$ |  | State <br> State |
| :---: | :---: | :---: | :---: | :---: | :--- |
| $x=0$ | $x=1$ | $x=0 \quad x=1$ | Meaning |  |  |

14.13 (c) The 'Mealy' circuit of Part (a) is such a Moore circuit. This is possible since the output does not depend upon the fourth (least significant) bit.


See FLD p. 725 for state table.
14.9 (d) Notice that the Moore output is delayed to the next clock cycle.
14.11 See FLD p. 726 for state graph.

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | Button pressed. First full clock cycle with <br> $Z=1$. |
| $S_{2}$ | Second full clock cycle with $Z=1$. |
| $S_{3}$ | Third full clock cycle with $Z=1$. |
| $S_{4}$ | Fourth full clock cycle with $Z=1$. |
| $S_{5}$ | $X$ has not yet returned to 0. |

14.12 (b)

|  | Next State |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $x=0$ | $x=1$ | $x=0 \quad x=1$ |  |
| A | B | A | 0 | 0 |
| B | C | A | 0 | 0 |
| C | E | A | 0 | 1 |
| E | E | A | 0 | 0 |


| State | Meaning |
| :---: | :--- |
| A | Sequence ending in 1 |
| B | Sequence ending in 10 |
| C | Sequence ending in 100 |
| E | Sequence ending in 000 |

14.13 (b)

| State | Next State |  | z | State Meaning |
| :---: | :---: | :---: | :---: | :---: |
|  | $x=0$ | $x=1$ |  |  |
| 1 | 2 | 3 | 0 | Initial State, Valid BCD digit |
| 2 | 4 | 4 | 0 | 1st bit was 0 |
| 3 | 5 | 6 | 0 | 1st bit was 1 |
| 4 | 7 | 7 | 0 | 1 st 2 bits were 0- |
| 5 | 7 | 8 | 0 | 1st 2 bits were 10 |
| 6 | 8 | 8 | 0 | 1st 2 bits were 11 |
| 7 | 1 | 1 | 0 | 1 st 3 bits were 0-- or -00 |
| 8 | 9 | 9 | 0 | 1 st 3 bits were 1-1 or 11- |
| 9 | 2 | 3 | 1 | Invalid BCD digit |

### 14.14 (a)

|  | Next State |  | $z$ |  | State <br> State |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x=0 \quad x=1$ | $x=0 \quad x=1$ | Meaning |  |  |  |
| 1 | 1 | 2 | 0 | 0 | Previous 3 bits were -00 |
| 2 | 3 | 4 | 0 | 0 | Previous 3 bits were 001 |
| 3 | 1 | 5 | 0 | 0 | Previous 3 bits were 010 |
| 4 | 6 | 7 | 0 | 0 | Previous 3 bits were 011 |
| 5 | 3 | 4 | 1 | 1 | Previous 3 bits were 101 |
| 6 | 1 | 5 | 1 | 1 | Previous 3 bits were 110 |
| 7 | 6 | 7 | 1 | 1 | Previous 3 bits were 111 |

14.14 (c) The 'Mealy' circuit of Part (a) is such a Moore circuit. This is possible since the output does not depend upon the fourth (least significant) bit.

### 14.15 (a)

|  | Next State |  | $Z$ |  | State <br> Meaning |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 1 | 2 | 2 | 0 | 0 | Initial State |
| 2 | 3 | 4 | 0 | 0 | 1 st bit was - |
| 3 | 5 | 6 | 0 | 0 | 1 st 2 bits were -0 |
| 4 | 6 | 6 | 0 | 0 | 1 st 2 bits were -1 |
| 5 | 1 | 1 | 0 | 0 | 1 st 3 bits were -00 |
| 6 | 1 | 1 | 0 | 1 | 1 st 3 bits were -1 or $-1-$ |

14.15 (c) It is not possible in this case since the output does depend upon the fourth (most significant) bit.

### 14.16 (a)

| State | Next State |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $x=0 \quad x=1$ | $z=0$ |  | State <br> Meaning |  |
| 1 | 1 | 2 | 0 | 0 | Previous 3 bits were -00 |
| 2 | 3 | 2 | 0 | 1 | Previous 3 bits were --1 |
| 3 | 1 | 2 | 0 | 1 | Previous 3 bits were -10 |

14.16 (c) It is not possible in this case since the output does depend upon the fourth (most significant) bit.
14.14 (b)

| State | Next State |  |  |  |
| :---: | :---: | :---: | :---: | :--- |
|  | $x=0 \quad x=1$ | $z$ |  |  |
| 1 | 1 | 2 | 0 | Previous 4 bits: $-000,0-00$ |
| 2 | 3 | 4 | 0 | Previous 4 bits:-001 |
| 3 | 1 | 5 | 0 | Previous 4 bits: 0010 |
| 4 | 6 | 7 | 0 | Previous 4 bits: 0011 |
| 5 | 8 | 9 | 0 | Previous 4 bits: 0101 |
| 6 | 10 | 11 | 0 | Previous 4 bits: 0110 |
| 7 | 12 | 13 | 0 | Previous 4 bits: 0111 |
| 8 | 1 | 5 | 1 | Previous 4 bits: 1010 |
| 9 | 6 | 7 | 1 | Previous 4 bits: 1011 |
| 10 | 1 | 2 | 1 | Previous 4 bits: 1100 |
| 11 | 8 | 9 | 1 | Previous 4 bits: 1101 |
| 12 | 10 | 11 | 1 | Previous 4 bits: 1110 |
| 13 | 12 | 13 | 1 | Previous 4 bits: 1111 |

Note: A more obvious solution uses 16 states; it can be reduced to the 13 states above using the method described in Section 15.1.

### 14.15 (b)

| State | Next State |  | Z | State Meaning |
| :---: | :---: | :---: | :---: | :---: |
|  | $x=0$ | $x=1$ |  |  |
| 1 | 2 | 2 | 0 | Valid digit |
| 2 | 3 | 4 | 0 | 1st bit was - |
| 3 | 5 | 6 | 0 | 1st 2 bits were -0 |
| 4 | 6 | 6 | 0 | 1st 2 bits were -1 |
| 5 | 1 | 1 | 0 | 1st 3 bits were -00 |
| 6 | 1 | 7 | 0 | 1st 3 bits were --1 or -1- |
| 7 | 2 | 2 | 1 | Invalid digit |

14.16 (b)

| State | Next State |  | Z | State Meaning |
| :---: | :---: | :---: | :---: | :---: |
|  | $x=0$ | $x=1$ |  |  |
| 1 | 1 | 2 | 0 | Previous 4 bits were --00 |
| 2 | 3 | 4 | 0 | Previous 4 bits were -001 |
| 3 | 1 | 4 | 0 | Previous 4 bits were --10 |
| 4 | 3 | 4 | 1 | Previous 4 bits were --11 or - 101 (invalid digit) |

Unit 14 Solutions
14.17 (a)

| State | Next State |  | z |  | State <br> Meaning |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $x=0$ | $x=1$ | $x=0$ | $=1$ |  |
| 1 | 2 | 3 | 0 | 0 | Initial State |
| 2 | 4 | 5 | 0 | 0 | 1st bit was 0 |
| 3 | 5 | 6 | 0 | 0 | 1st bit was 1 |
| 4 | 7 | 8 | 0 | 0 | 1st 2 bits were 00 |
| 5 | 9 | 9 | 0 | 0 | 1 st 2 bits were 01 or 10 |
| 6 | 10 | 7 | 0 | 0 | 1st 2 bits were 11 |
| 7 | 1 | 1 | 1 | 1 | 1st 3 bits were 000 or 111 |
| 8 | 1 | 1 | 1 | 0 | 1st 3 bits were 001 |
| 9 | 1 | 1 | 0 | 0 | 1 st 3 bits were 01- or 10- |
| 10 | 1 | 1 | 0 | 1 | 1st 3 bits were 110 |

$\mathbf{1 4 . 1 7}$ (c) It is not possible because the output depends on the value of the fourth bit, e.g., see state 8 in Part (a).

### 14.18 (a)

|  | Next State |  | $Z$ |  | State <br> Meaning |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | $x=0 \quad x=1$ | $x=0 \quad x=1$ |  | 1 |  |  |
| 1 | 1 | 2 | 1 | 1 | Previous 3 bits were 000 |  |
| 2 | 3 | 4 | 1 | 0 | Previous 3 bits were 001 |  |
| 3 | 5 | 6 | 0 | 0 | Previous 3 bits were 010 |  |
| 4 | 7 | 8 | 0 | 0 | Previous 3 bits were 011 |  |
| 5 | 1 | 2 | 0 | 0 | Previous 3 bits were 100 |  |
| 6 | 3 | 4 | 0 | 0 | Previous 3 bits were 101 |  |
| 7 | 5 | 6 | 0 | 1 | Previous 3 bits were 110 |  |
| 8 | 7 | 8 | 1 | 1 | Previous 3 bits were 111 |  |

14.18 (c) It is not possible because the output depends on the value of the fourth bit, e.g., see state 2 in Part (a).
14.17 (b)

|  |  | Next State |  |  |
| :---: | :---: | :---: | :---: | :--- |
| State | $x=0 \quad x=1$ | $z$ | State <br> Meaning |  |
| 1 | 2 | 3 | 0 | Valid digit |
| 2 | 4 | 5 | 0 | 1 st bit was 0 |
| 3 | 5 | 6 | 0 | 1 st bit was 1 |
| 4 | 7 | 8 | 0 | 1 st 2 bits were 00 |
| 5 | 9 | 9 | 0 | 1 st 2 bits were 01 or 10 |
| 6 | 10 | 7 | 0 | 1 st 2 bits were 11 |
| 7 | 11 | 11 | 0 | 1 st 3 bits were 000 or 111 |
| 8 | 11 | 1 | 0 | 1 st 3 bits were 001 |
| 9 | 1 | 1 | 0 | 1 st 3 bits were $01-$ or $10-$ |
| 10 | 1 | 11 | 0 | 1 st 3 bits were 110 |
| 11 | 2 | 3 | 1 | Invalid digit |

14.18 (b)

| State | Next State |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $x=0$ | $x=1$ | $z$ | State |
| 1 | 1 | 2 | 1 | Previous 4 bits were 0000 |
| 2 | 3 | 4 | 1 | Previous 4 bits were 0001 |
| 3 | 5 | 6 | 1 | Previous 4 bits were 0010 |
| 4 | 7 | 8 | 0 | Previous 4 bits were -011 |
| 5 | 9 | 10 | 0 | Previous 4 bits were -100 |
| 6 | 11 | 4 | 0 | Previous 4 bits were 0101 |
| 7 | 5 | 12 | 0 | Previous 4 bits were 0110 |
| 8 | 13 | 14 | 0 | Previous 4 bits were 0111 |
| 9 | 1 | 2 | 0 | Previous 4 bits were 1000 |
| 10 | 3 | 4 | 0 | Previous 4 bits were 1001 |
| 11 | 5 | 6 | 0 | Previous 4 bits were 1010 |
| 12 | 11 | 4 | 1 | Previous 4 bits were 1101 |
| 13 | 5 | 12 | 1 | Previous 4 bits were 1110 |
| 14 | 13 | 14 | 1 | Previous 4 bits were 1111 |

14.19 (a)

|  | Next State |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :--- |
| State | $Z$ |  | State <br> Meaning |  |  |
| 1 | 2 | 3 | 0 | 0 | Initial State |
| 2 | 4 | 5 | 0 | 0 | 1 st bit was 0 |
| 3 | 5 | 6 | 0 | 0 | 1 st bit was 1 |
| 4 | 7 | 8 | 0 | 0 | 1 st 2 bits were 00 |
| 5 | 7 | 9 | 0 | 0 | 1 st 2 bits were 01 or 10 |
| 6 | 8 | 9 | 0 | 0 | 1 st 2 bits were 11 |
| 7 | 1 | 1 | 1 | 0 | 1 st 3 bits were -00 or $0-0$ |
| 8 | 1 | 1 | 0 | 0 | 1 st 3 bits were 001 or 110 |
| 9 | 1 | 1 | 0 | 1 | 1 st 3 bits were -11 or $1-1$ |

14.19 (c) It is not possible because the output depends on the value of the fourth bit, e.g., see state 7 in Part (a).
14.20 (a)

|  | Next State |  | $Z$ |  | State <br> State |
| :---: | :---: | :---: | :---: | :---: | :--- |
| $x=0$ | $x=1$ | $x=0 \quad x=1$ | Meaning |  |  |
| 1 | 1 | 2 | 1 | 0 | Previous 3 bits were -00 |
| 2 | 3 | 4 | 0 | 0 | Previous 3 bits were 001 |
| 3 | 1 | 5 | 1 | 0 | Previous 3 bits were 010 |
| 4 | 6 | 4 | 0 | 1 | Previous 3 bits were -11 |
| 5 | 3 | 4 | 0 | 1 | Previous 3 bits were 101 |
| 6 | 1 | 5 | 0 | 0 | Previous 3 bits were 110 |

14.20 (c) It is not possible because the output depends on the value of the most significant (fourth) bit.
14.21 Plot 0's horizontally. Plot 1's vertically. Receiving a 0 takes us one state to the right. Receiving a 1 takes us one state down. The output is a 1 only in the "three 0 's or more, one 1 or more" state:

14.19 (b)

| State | Next State |  |  | State |
| :---: | :---: | :---: | :---: | :--- |
|  | $x=0$ | $x=1$ | $z$ | Meaning |
| 1 | 2 | 3 | 0 | Initial State, Valid digit |
| 2 | 4 | 5 | 0 | 1 st bit was 0 |
| 3 | 5 | 6 | 0 | 1 st bit was 1 |
| 4 | 7 | 8 | 0 | 1 st 2 bits were 00 |
| 5 | 7 | 9 | 0 | 1 st 2 bits were 01 or 10 |
| 6 | 8 | 9 | 0 | 1st 2 bits were 11 |
| 7 | 10 | 1 | 0 | 1 st 3 bits were -00 or $0-0$ |
| 8 | 1 | 1 | 0 | 1 st 3 bits were 001 or 110 |
| 9 | 1 | 10 | 0 | 1 st 3 bits were -11 or $1-1$ |
| 10 | 2 | 3 | 1 | Initial State, Invalid digit |

14.20 (b)

|  | Next State |  |  |  |
| :---: | :---: | :---: | :---: | :--- |
| State |  |  |  |  |
|  | $x=0$ | $x=1$ | $z$ |  |
| 1 | 7 | 2 | 0 | Previous 4 bits:1100 |
| 2 | 3 | 4 | 0 | Previous 4 bits: -001 |
| 3 | 7 | 5 | 0 | Previous 4 bits: -010 |
| 4 | 6 | 8 | 0 | Previous 4 bits: 0011 |
| 5 | 3 | 8 | 0 | Previous 4 bits: -101 |
| 6 | 1 | 5 | 0 | Previous 4 bits: -110 |
| 7 | 7 | 2 | 1 | Previous 4 bits: $-000,0-00$ |
| 8 | 6 | 8 | 1 | Previous 4 bits: -111 |

## Unit 14 Solutions

14.22


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | Previous input was 0 / 011 has not occurred |
| $S_{2}$ | Previous input was 01 / 011 has not occurred |
| $S_{3}$ | (No sequence) / 011 has occurred |
| $S_{4}$ | Previous input was $0 / 011$ has occurred |
| $S_{5}$ | Previous input was 01 / 011 has occurred |
| $S_{6}$ | Previous input was $1 / 011$ has not occurred |
| $S_{7}$ | Previous input was $10 / 011$ has not occurred |

* When this point in the graph is reached, 011 has been received, and we are only looking for 011 to occur again.

|  | Next State |  | $Z_{1} Z_{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0 \quad X=1$ |  |
| $S_{0}$ | $S_{1}$ | $S_{6}$ | 00 | 00 |
| $S_{1}$ | $S_{1}$ | $S_{2}$ | 00 | 00 |
| $S_{2}$ | $S_{7}$ | $S_{3}$ | 00 | 01 |
| $S_{3}$ | $S_{4}$ | $S_{3}$ | 00 | 00 |
| $S_{4}$ | $S_{4}$ | $S_{5}$ | 00 | 00 |
| $S_{5}$ | $S_{4}$ | $S_{3}$ | 00 | 01 |
| $S_{6}$ | $S_{7}$ | $S_{6}$ | 00 | 00 |
| $S_{7}$ | $S_{1}$ | $S_{2}$ | 10 | 00 |

14.23


| State | Next State |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | :--- |
|  | $X_{1} X_{2}=00$ | 01 | 10 | 11 | $Z$ |
| $S_{0}$ | $S_{1}$ | $S_{1}$ | $S_{0}$ | $S_{0}$ | 0 |
| $S_{1}$ | $S_{1}$ | $S_{1}$ | $S_{2}$ | $S_{0}$ | 0 |
| $S_{2}$ | $S_{2}$ | $S_{3}$ | $S_{2}$ | $S_{3}$ | 1 |
| $S_{3}$ | $S_{2}$ | $S_{3}$ | $S_{0}$ | $S_{3}$ | 1 |


| State | Meaning |
| :---: | :---: |
| $S_{0}$ | $Z=0$, last input was 10 or 11 |
| $S_{1}$ | $Z=0$, last input was 00 or 01 |
| $S_{2}$ | $Z=1$, last input was 00 or 10 |
| $S_{3}$ | $Z=1$, last input was 01 or 11 |

Alternate solution has 8 states, similar to problem 14.6:

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | $Z=0$, last input was 10 (reset) |
| $S_{1}$ | $Z=0$, last input was 00 |
| $S_{2}$ | $Z=0$, last input was 01 |
| $S_{3}$ | $Z=0$, last input was 11 |
| $S_{4}$ | $Z=1$, last input was 10 |
| $S_{5}$ | $Z=1$, last input was 00 |
| $S_{6}$ | $Z=1$, last input was 01 |
| $S_{7}$ | $Z=1$, last input was 11 |


|  | Next State |  |  |  |  |
| :---: | ---: | :--- | :--- | :--- | :--- | :--- |
| State | $X_{1} X_{2}=00$ | 01 | 10 | 11 | $Z$ |
| $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{0}$ | $S_{3}$ | 0 |
| $S_{1}$ | $S_{1}$ | $S_{2}$ | $S_{4}$ | $S_{3}$ | 0 |
| $S_{2}$ | $S_{1}$ | $S_{2}$ | $S_{4}$ | $S_{3}$ | 0 |
| $S_{3}$ | $S_{1}$ | $S_{2}$ | $S_{0}$ | $S_{3}$ | 0 |
| $S_{4}$ | $S_{5}$ | $S_{6}$ | $S_{4}$ | $S_{7}$ | 1 |
| $S_{5}$ | $S_{5}$ | $S_{6}$ | $S_{4}$ | $S_{7}$ | 1 |
| $S_{6}$ | $S_{5}$ | $S_{6}$ | $S_{0}$ | $S_{7}$ | 1 |
| $S_{7}$ | $S_{5}$ | $S_{6}$ | $S_{0}$ | $S_{7}$ | 1 |

14.24 (a) We need four states to describe the 1's received, as there are four possible remainders when dividing by four. An input of 1 takes us to the next state in cyclic fashion. An input of zero leaves us in the same state.

14.24 (b) Now, expand the state graph into two dimensions: one for 1 's and the other for 0 's. We need two states to describe the zeros, odd and even.


Left column: odd zeros
Right column: even zeros
First row: Remainder $=0$
Second row: Remainder $=1$
Second row: Remainder $=1$
Third row: Remainder $=2$
Fourth row: Remainder = 3
As part (a)
$\qquad$
Fouth row: Remaider $=3$ l As $\qquad$

## Unit 14 Solutions

14.26 There are two identical parts: one with an output of 0 and one with an output of 1 .

| State | Meaning |
| :---: | :--- |
| $S_{1}, S_{4}$ | Previous input was 0 |
| $S_{2}, S_{5}$ | Previous inputs were 01 |
| $S_{3}, S_{0}$ | Previous input was $1 / \operatorname{Reset}\left(S_{0}\right)$ |


14.27 There are two identical parts: one with an output of 0 and one with an output of 1 .

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | Previous input was 1 |
| $S_{2}$ | Previous inputs were 10 |
| $S_{3}$ | Previous inputs were 101 (first 101) |
| $S_{4}$ | Previous inputs were 10 (start of second 101) |
| $S_{5}$ | Previous inputs were 00 |


14.28 This is another problem similar to 14.10. Plot the number of 0 's horizontally and the number of pairs vertically:

14.28 (contd)

| Pairs | 0 0's | Present <br> State | Next State |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 01 | 10 | 11 | $Z_{1} Z_{2}$ |  |  |  |  |  |  |
| 00 | 01 | 10 | 11 |  |  |  |  |  |  |  |
| 0 | 0 | $S_{0}$ | $S_{3}$ | $S_{2}$ | $S_{2}$ | $S_{1}$ | 0 | 0 | 0 | 0 |
| 1 | 0 | $S_{1}$ | $S_{6}$ | $S_{5}$ | $S_{5}$ | $S_{4}$ | 0 | 0 | 0 | 0 |
| 1 | 1 | $S_{2}$ | $S_{7}$ | $S_{6}$ | $S_{6}$ | $S_{5}$ | 0 | 0 | 0 | 0 |
| 1 | 2 | $S_{3}$ | $S_{8}$ | $S_{7}$ | $S_{7}$ | $S_{6}$ | 0 | 0 | 0 | 0 |
| 2 | 0 | $S_{4}$ | $S_{6}$ | $S_{5}$ | $S_{5}$ | $S_{4}$ | 0 | 0 | 0 | 0 |
| 2 | 1 | $S_{5}$ | $S_{7}$ | $S_{6}$ | $S_{6}$ | $S_{5}$ | 0 | 0 | 0 | 0 |
| 2 | 2 | $S_{6}$ | $S_{0}$ | $S_{7}$ | $S_{7}$ | $S_{6}$ | 1 | 0 | 0 | 0 |
| 2 | 3 | $S_{7}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{7}$ | 1 | 1 | 1 | 0 |
| 2 | 4 | $S_{8}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | 1 | 1 | 1 | 1 |

Note: There is a seven-state solution.
14.29 0 's are plotted horizontally. 1's are plotted vertically.

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Z$ |
| $S_{0}$ | $S_{2}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{3}$ | $S_{0}$ | 1 |
| $S_{2}$ | $S_{4}$ | $S_{3}$ | 0 |
| $S_{3}$ | $S_{5}$ | $S_{2}$ | 1 |
| $S_{4}$ | $S_{2}$ | $S_{5}$ | 0 |
| $S_{5}$ | $S_{3}$ | $S_{4}$ | 1 |


14.30

|  | Next State |  | $Z_{1} Z_{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{1}$ | $S_{0}$ | 00 | 00 |
| $S_{1}$ | $S_{1}$ | $S_{2}$ | 00 | 00 |
| $S_{2}$ | $S_{1}$ | $S_{3}$ | 00 | 10 |
| $S_{3}$ | $S_{1}$ | $S_{0}$ | 00 | 01 |


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset, 0111 |
| $S_{1}$ | 0 |
| $S_{2}$ | 01 |
| $S_{3}$ | 011 |



Unit 14 Solutions
14.31

|  | $X_{1} X_{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | 00 | 01 | 10 | 11 | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 |
| $S_{1}$ | $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 |
| $S_{2}$ | $S_{0}$ | $S_{3}$ | $S_{2}$ | $S_{3}$ | 0 |
| $S_{3}$ | $S_{0}$ | $S_{3}$ | $S_{3}$ | $S_{3}$ | 1 |


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | Previous input was $01, Z=0$ |
| $S_{2}$ | Previous input was $10, Z=0$ |
| $S_{3}$ | $Z=1$ (Until input 00 ) |


14.32

Example: $\begin{aligned} X & =001100110101 \\ Z= & 011110111101\end{aligned}$
Note: Overlapping sequences are allowed.

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | No sequence |
| $S_{1}$ | 0 |
| $S_{2}$ | 00 |
| $S_{3}$ | 001 |
| $S_{4}$ | 0011 |


|  | Next State |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{1}$ | $S_{0}$ | 0 | 1 |
| $S_{1}$ | $S_{2}$ | $S_{0}$ | 0 | 1 |
| $S_{2}$ | $S_{2}$ | $S_{3}$ | 0 | 1 |
| $S_{3}$ | $S_{1}$ | $S_{4}$ | 0 | 1 |
| $S_{4}$ | $S_{1}$ | $S_{0}$ | 1 | 1 |


14.33

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{6}$ | $S_{2}$ | 0 |
| $S_{2}$ | $S_{3}$ | $S_{6}$ | 0 |
| $S_{3}$ | $S_{3}$ | $S_{4}$ | 0 |
| $S_{4}$ | $S_{6}$ | $S_{5}$ | 0 |
| $S_{5}$ | $S_{5}$ | $S_{6}$ | 1 |
| $S_{6}$ | $S_{6}$ | $S_{6}$ | 0 |


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | No 1's |
| $S_{1}$ | One 1 in first group |
| $S_{2}$ | Two 1's in first group |
| $S_{3}$ | First group 11 complete, had exactly two 1's |
| $S_{4}$ | One 1 in second group |
| $S_{5}$ | Two 1's in second group $(Z=1)$ |
| $S_{6}$ | "Disqualified" state $(Z=0)$ |


14.34 To delay by two clock periods, we need to remember the previous two inputs. So we have four states, one for each combination of two inputs:

|  | Next State |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 | 0 |
| $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| $S_{2}$ | $S_{0}$ | $S_{1}$ | 1 | 1 |
| $S_{3}$ | $S_{2}$ | $S_{3}$ | 1 | 1 |


| State | Meaning |
| :---: | :---: |
| $S_{0}$ | Previous two inputs were 00 |
| $S_{1}$ | Previous two inputs were 01 |
| $S_{2}$ | Previous two inputs were 10 |
| $S_{3}$ | Previous two inputs were 11 |

Note: Just go to the state that represents the last two inputs.
14.35 This is the same as 14.34 , except that we need to remember the last three inputs. So we have eight states:

|  | Next State |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 | 0 |
| $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 | 0 |
| $S_{3}$ | $S_{6}$ | $S_{7}$ | 0 | 0 |
| $S_{4}$ | $S_{0}$ | $S_{1}$ | 1 | 1 |
| $S_{5}$ | $S_{2}$ | $S_{3}$ | 1 | 1 |
| $S_{6}$ | $S_{4}$ | $S_{5}$ | 1 | 1 |
| $S_{7}$ | $S_{6}$ | $S_{7}$ | 1 | 1 |

Note: The state number expressed in binary gives the last 3 inputs.
14.36 (a)

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 |
| $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 |
| $S_{3}$ | $S_{6}$ | $S_{7}$ | 0 |
| $S_{4}$ | $S_{0}$ | $S_{1}$ | 1 |
| $S_{5}$ | $S_{2}$ | $S_{3}$ | 1 |
| $S_{6}$ | $S_{4}$ | $S_{5}$ | 1 |
| $S_{6}$ | $S_{6}$ | $S_{7}$ | 1 |


14.36 (b) 16 states are required since the last four inputs must be remembered.

## Unit 14 Solutions

14.37

|  | Next State |  | $S V$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{1}$ | $S_{1}$ | 00 | 10 |
| $S_{1}$ | $S_{2}$ | $S_{4}$ | 10 | 00 |
| $S_{2}$ | $S_{3}$ | $S_{3}$ | 00 | 10 |
| $S_{3}$ | $S_{0}$ | $S_{0}$ | 00 | 10 |
| $S_{4}$ | $S_{3}$ | $S_{5}$ | 10 | 00 |
| $S_{5}$ | $S_{0}$ | $S_{0}$ | 10 | 01 |


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | No bits received |
| $S_{1}$ | One bit received |
| $S_{2}$ | Two bits received; Carry-in $=0$ |
| $S_{4}$ | Two bits received; Carry-in $=1$ |
| $S_{3}$ | Three bits received; Carry-in $=0$ |
| $S_{5}$ | Three bits received; Carry-in $=1$ |

14.38

|  | Next State |  | $D B$ |  |
| :---: | :---: | :---: | ---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{1}$ | $S_{1}$ | 00 | 10 |
| $S_{1}$ | $S_{2}$ | $S_{3}$ | 10 | 00 |
| $S_{2}$ | $S_{4}$ | $S_{5}$ | 10 | 00 |
| $S_{3}$ | $S_{5}$ | $S_{5}$ | 00 | 10 |
| $S_{4}$ | $S_{0}$ | $S_{0}$ | 11 | 00 |
| $S_{5}$ | $S_{0}$ | $S_{0}$ | 00 | 10 |


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | No bits received |
| $S_{1}$ | One bit received |
| $S_{2}$ | Two bits received; Borrow-in $=1$ |
| $S_{4}$ | Two bits received; Borrow-in $=0$ |
| $S_{3}$ | Three bits received; Borrow-in $=1$ |
| $S_{5}$ | Three bits received; Borrow-in $=0$ |


14.39 This is similar to 14-15, and should be answered in the same way. See the solution to 14-15 for more information.

Horizontally: Number of 1's modulo 3 Vertically: Number of 0's modulo 3.

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Y Z$ |
| $S_{0}$ | $S_{3}$ | $S_{1}$ | 00 |
| $S_{1}$ | $S_{4}$ | $S_{2}$ | 01 |
| $S_{2}$ | $S_{5}$ | $S_{0}$ | 10 |
| $S_{3}$ | $S_{6}$ | $S_{4}$ | 00 |
| $S_{4}$ | $S_{7}$ | $S_{5}$ | 01 |
| $S_{5}$ | $S_{8}$ | $S_{0}$ | 10 |
| $S_{6}$ | $S_{0}$ | $S_{7}$ | 00 |
| $S_{7}$ | $S_{0}$ | $S_{8}$ | 01 |
| $S_{8}$ | $S_{0}$ | $S_{0}$ | 10 |

14.40 This problem is essentially a circular counting exercise. Pairs of 1's take you further around the state graph. Pairs can overlap, so if the last input was a 1 , and the present input is a 1 , you move on. If the sequence is interrupted, you branch off while you wait for the next 1. Then, you go back to the cycle of counting.

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Y Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 00 |
| $S_{1}$ | $S_{0}$ | $S_{2}$ | 00 |
| $S_{2}$ | $S_{3}$ | $S_{4}$ | 01 |
| $S_{3}$ | $S_{3}$ | $S_{2}$ | 01 |
| $S_{4}$ | $S_{5}$ | $S_{6}$ | 10 |
| $S_{5}$ | $S_{5}$ | $S_{4}$ | 10 |
| $S_{6}$ | $S_{7}$ | $S_{0}$ | 11 |
| $S_{7}$ | $S_{7}$ | $S_{6}$ | 11 |



## Unit 14 Solutions

14.41 We notice that input $A B X X$ becomes output $A A B B$. It can be seen that it is not necessary to remember both $A$ and $B$ at once. We remember $A$ for the first two clocks and $B$ for the next two. Notice that if the output were, say, $A B A B$, we could not do this.

|  | Next State |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{1}$ | $S_{5}$ | 0 | 1 |
| $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| $S_{2}$ | $S_{4}$ | $S_{4}$ | 0 | 0 |
| $S_{3}$ | $S_{6}$ | $S_{6}$ | 1 | 1 |
| $S_{4}$ | $S_{0}$ | $S_{0}$ | 0 | 0 |
| $S_{5}$ | $S_{2}$ | $S_{3}$ | 1 | 1 |
| $S_{6}$ | $S_{0}$ | $S_{0}$ | 1 | 1 |



| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | $A=0$ |
| $S_{5}$ | $A=1$ |
| $S_{2}, S_{4}$ | $B=0$ |
| $S_{3}, S_{6}$ | $B=1$ |

This problem is simply addition. We need a state to describe every possible sum of money entered, i.e., 0థ to 45థ in 5\$ intervals.

Just go to the state with the correct sum. The $25 \$$ state dispenses the product $(R=1)$ and resets. States above this in value cascade down to $S_{5}$ by giving out a nickel. When they get to $S_{5}$, the product is dispensed.

|  | Present <br> State | $N D Q$ |  |  |  |  | 000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 100 | 010 | 001 | $R C$ |  |  |  |
| .00 | $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{5}$ | 00 |  |
| .05 | $S_{1}$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | $S_{6}$ | 00 |  |
| .10 | $S_{2}$ | $S_{2}$ | $S_{3}$ | $S_{4}$ | $S_{7}$ | 00 |  |
| .15 | $S_{3}$ | $S_{3}$ | $S_{4}$ | $S_{5}$ | $S_{8}$ | 00 |  |
| .20 | $S_{4}$ | $S_{4}$ | $S_{5}$ | $S_{6}$ | $S_{9}$ | 00 |  |
| .25 | $S_{5}$ | $S_{0}$ | - | - | - | 10 |  |
| .30 | $S_{6}$ | $S_{5}$ | - | - | - | 01 |  |
| .35 | $S_{7}$ | $S_{6}$ | - | - | - | 01 |  |
| .40 | $S_{8}$ | $S_{7}$ | - | - | - | 01 |  |
| .45 | $S_{9}$ | $S_{8}$ | - | - | - | 01 |  |

14.43 (a) Look at Figure 14-19, FLD p. 445, to see that Manchester 01 gives NRZ 00
Manchester 10 gives NRZ 11
Other Manchester inputs are presumed not to occur.


|  | Next State |  | $Z$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |  |  |
| $S_{0}$ | $S_{1}$ | $S_{2}$ | 0 | 1 |  |  |
| $S_{1}$ | - | $S_{0}$ | $0^{*}$ | 0 |  |  |
| $S_{2}$ | $S_{0}$ | - | 1 | $1^{*}$ |  |  |
| * Filled in to prevent False outputs. |  |  |  |  |  |  |

14.43 (b) This is the same as the Mealy, except that we need two reset states, one with an output of zero, the other with an output of 1 . Invalid inputs never occur.

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Z$ |
| $S_{0}$ | $S_{1}$ | $S_{2}$ | 0 |
| $S_{1}$ | - | $S_{0}$ | 0 |
| $S_{2}$ | $S_{3}$ | - | 1 |
| $S_{3}$ | $S_{1}$ | $S_{2}$ | 1 |


14.43
(c), (d)


Note: Moore output is delayed one clock cycle of CLOCK2.

Unit 14 Solutions
14.44

| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | One ring, waiting for two (or answer) |
| $S_{3}, S_{4}, S_{5}$ | One, two, or three rings, respectively; <br> waiting for four (or answer) |
| $S_{2}$ | Activate answering machine; wait for <br> it to answer |

14.45

| Present State | Next State | $Z_{1} Z_{2}$ |
| :---: | :---: | :---: |
|  | 00011011 | 00011011 |
| $S_{0}$ | $S_{1} S_{0} S_{0} S_{2} S_{2}$ | 01100101 |
| $S_{1}$ | $S_{1} S_{2} S_{0} S_{0}$ | 00110000 |
| $S_{2}$ | $S_{2} S_{1} \quad S_{0} S_{0}$ | 00000000 |

14.46 In state $S_{0}$ there is no specification for $\mathrm{X}_{1} \mathrm{X}_{2}{ }^{\prime}$. This can be corrected by adding an arc for $\mathrm{X}_{1} \mathrm{X}_{2}^{\prime}$ or changing $X_{1} X_{2}$ to $X_{1}$ or changing $X_{1}{ }^{\prime} \mathrm{X}_{2}^{\prime}$ to $\mathrm{X}_{2}{ }^{\prime}$.

In state $S_{1}$ there is a conflict for $X_{1} X_{2}$. This can be corrected by changing $X_{1}$ to $X_{1} X_{2}^{\prime}$ or changing $X_{2}$ to $X_{1}{ }^{\prime} \mathrm{X}_{2}$.
-



## Unit 15 Problem Solutions

15.1 (a) Implication chart after one pass:


Reduced state table:

|  | Next State |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0 \quad X=1$ |  |
| $A$ | $A$ | $C$ | 1 | 0 |
| $B$ | $C$ | $F$ | 0 | 0 |
| $C$ | $B$ | $A$ | 0 | 0 |
| $F$ | $B$ | $F$ | 1 | 0 |

15.2


Unit 15 Solutions
15.3

15.3 (a) $a \equiv S_{0}, S_{5}$
$b \equiv S_{1}$
$c \equiv S_{3}, S_{6}$
Since $S_{2}$ and $S_{4}$ do not have corresponding states, the circuits are not equivalent.
15.3 (b) Starting from $S_{0}$, it is not possible to reach $S_{2}$ or $S_{4}$. So then the circuits would perform the same.
15.4 (a)

$D=X_{2}{ }^{\prime} X_{3} Q+X_{1}{ }^{\prime} X_{2} Q^{\prime}+X_{1} X_{2}{ }^{\prime} Q^{\prime}+X_{2} X_{3}{ }^{\prime} Q$

$$
Z=Q
$$

15.4 (b)

| $x_{1} x_{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | X | 0 | X | 0 |
| 01 | $1)$ | 0 | 0 | 1 |
| 11 | 0 | 1 | $1)$ | 0 |
| 10 | X | 0 | 0 | 0 |

$R=X_{2} X_{3} Q+X_{2}{ }^{\prime} X_{3}{ }^{\prime} Q$
$Z=Q$

$S=X_{1}{ }^{\prime} X_{2} Q^{\prime}+X_{1} X_{2}{ }^{\prime} Q^{\prime}$
15.5 (a) The first row may be all 0 's, because if a column has a 1 in the first row, we can invert it so that it has a 0 in first row without changing the number of gates. No column should be all 0 's, because that is the same as the two flipflop case. There are only 3 columns which fit these criteria: 001,010 , and 011 . No column may be used twice, because again that is the same as the two flip-flop case. So we need only check one assignment (which consists of the three columns in any order) to see whether a three flip-flop solution is better than a two flip-flop solution. One such assignment is:
$0 \quad 0 \quad 0$
011
101
15.5 (b) Excluding 0000, there are 7 possible columns. All possible non-repeating combinations are given below. Those with repeating rows are crossed out; 29 assignments remain to try.

| 15.5 (b) (contd) |  | boo | 000 | 000 | 000 | 000 | 000 | 000 | 000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 000 | 001 | 001 | 001 | 001 | 001 | 001 | 001 |
|  |  | $0 \times 1$ | 010 | 010 | 011 | 011 | 010 | 010 | 011 |
|  |  | 101 | 100 | 101 | 100 | 101 | 110 | 111 | 110 |
|  |  | (123) | (12 4) | (125) | (126) | (12 7) | (13 4) | (135) | (136) |
|  | 000 | 000 | 000 | 000 | 000 | 000 | 000 | 000 | 000 |
|  | 001 | $01 / 1$ | 011 | 011 | 011 | 011 | 0111 | 001 | 001 |
|  | 011 | 000 | 001 | 001 | 001 | 001 | $0 \times 1$ | 110 | 110 |
|  | 111 | $1 / 01$ | 100 | 101 | 110 | 111 | 101 | 010 | 011 |
|  | (137) | (145) | (146) | (147) | (156) | (157) | (167) | (234) | (235) |
|  | 000 | 000 | 000 | 000 | 000 | 000 | ¢00 | 000 | 000 |
|  | 001 | 001 | 011 | 0111 | 011 | 011 | 011 | 011 | 011 |
|  | 111 | 111 | 100 | 101 | 101 | 101 | 101 | 111 | 100 |
|  | 010 | 011 | 001 | 000 | 001 | 010 | 011 | 001 | 101 |
|  | (236) | (237) | (245) | (246) | (247) | (256) | (257) | (267) | (345) |
|  | 000 | ¢0¢ | 000 | 000 | 000 | 000 | 000 | 000 | 000 |
|  | 011 | 01/1 | 011 | 011 | 011 | 111 | 111 | 111 | 111 |
|  | 101 | $1 \propto 1$ | 101 | 101 | 111 | 001 | 001 | 011 | 011 |
|  | 100 | $1 / 01$ | 110 | 111 | 101 | 010 | 011 | 001 | 101 |
|  | (346) | (347) | (346) | (357) | (367) | (45 6) | (457) | (467) | (56 7) |

15.6 (a) Group $\left(S_{1}, S_{4}, S_{6}, S_{7}\right)$ and $\left(S_{2}, S_{3}, S_{5}, S_{8}\right)$.

One possible assignment:
$S_{1}=000 \quad S_{5}=111$
$S_{2}=100 \quad S_{6}=011$
$S_{3}=101 \quad S_{7}=010$
$S_{4}=001 \quad S_{8}=110$

15.6 (b) I: $\left(S_{3}, S_{4}\right) \vee\left(S_{1}, S_{8}\right) \vee\left(S_{3}, S_{7}\right) \vee\left(S_{5}, S_{8}\right) \vee$

II: $\left(S_{4}, S_{5}\right) \vee\left(S_{1}, S_{6}\right)\left(S_{7}, S_{8}\right)\left(S_{1}, S_{7}\right) \vee\left(S_{2}, S_{3}\right) \vee$

$$
\left(S_{2}, S_{4}\right)\left(S_{6}, S_{8}\right) \vee\left(S_{3}, S_{5}\right)
$$

Adjacencies that are satisfied are checked $(\checkmark)$
One possible assignment:
$S_{1}=000 \quad S_{5}=101$
$S_{2}=010 \quad S_{6}=110$
$S_{3}=011 \quad S_{7}=001$
$S_{4}=111 \quad S_{8}=100$


$$
D_{A}=A^{\prime} B^{\prime}+X^{\prime} A C^{\prime}+X A^{\prime}
$$

|  |  | $A^{+} B^{+} C^{+}$ |  |
| :---: | :---: | :---: | :---: |
| State | $A B C$ | $X=0$ | $X=1$ |
| $S_{1}$ | 000 | 101 | 111 |
| $S_{7}$ | 001 | 110 | 100 |
| $S_{2}$ | 010 | 000 | 110 |
| $S_{3}$ | 011 | 001 | 100 |
| $S_{8}$ | 100 | 101 | 011 |
| $S_{5}$ | 101 | 010 | 011 |
| $S_{6}$ | 110 | 111 | 010 |
| $S_{4}$ | 111 | 001 | 000 |

1. $(A, D, F)(C, E)(A, D)(C, E)(B, F)$
2. $(F, D) \times 2(D, B)(A, C) \times 2(B, F)$
3. $(A, B, D, F)(C, E)$

See FLD p. 728 for one good solution.
15.8 (a) Guidelines:

1. $(B, D) \times 2(C, D) \times 2(A, B)$
2. $(B, D)(A, C)(A, C, B)(A, B, C, D)$
3. $(A, B) \times 2(B, D) \times 2(C, D) \times 2$

Best assignment: $A=00, B=01, C=10, D=11$

15.8 (b)

15.9 See FLD p. 728 for solution using $\mathrm{Q}_{1}, \mathrm{Q}_{2}$, and $\mathrm{Q}_{3}$.

Alternate solution using $Q_{0}, Q_{1}$ and $Q_{2}$ :

$$
\begin{aligned}
& D_{0}=X^{\prime} Q_{0}+X Y^{\prime} Q_{2} \\
& D_{1}=X Q_{0}+Y Q_{2}+X^{\prime} Q_{1} \\
& D_{2}=Y Q_{1}+X^{\prime} Y^{\prime} Q_{2} \\
& P=X Q_{0}+Y^{\prime} Q_{2}+X Q_{1} \\
& S=X^{\prime} Q_{0}+X Y^{\prime} Q_{2}
\end{aligned}
$$

15.10 (a)


|  | Next State |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0 \quad X=1$ |  |
| $a$ | $a$ | $c$ | 1 | 0 |
| $b$ | $c$ | $d$ | 0 | 1 |
| $c$ | $a$ | $b$ | 0 | 0 |
| $d$ | $d$ | $a$ | 0 | 0 |

15.10 (b) Input: 00

Output starting in state $c$ :

Output starting in state $d$ :
00 (state $d \xrightarrow{\boldsymbol{\rightarrow}}$ state $d \xrightarrow{\boldsymbol{\rightarrow}}$ state $d$ )
15.11 (a)

15.12 (a) Equivalent States: $\mathrm{S}_{0} \equiv \mathrm{~S}_{8}, \mathrm{~S}_{2} \equiv \mathrm{~S}_{10}, \mathrm{~S}_{3} \equiv \mathrm{~S}_{11}$, $\mathrm{S}_{4} \equiv \mathrm{~S}_{12}, \mathrm{~S}_{7} \equiv \mathrm{~S}_{15}$.

| Input <br> Pattern | Present <br> State | Next State <br>  <br> -000 |  | $S_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $S_{0}$ | $S_{1}$ | 0 |  |
| -0001 | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 |
| -010 | $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 |
| -011 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 1 |
| -100 | $S_{4}$ | $S_{0}$ | $S_{9}$ | 1 |
| 0101 | $S_{5}$ | $S_{2}$ | $S_{3}$ | 0 |
| 0110 | $S_{6}$ | $S_{4}$ | $S_{13}$ | 1 |
| -111 | $S_{7}$ | $S_{14}$ | $S_{7}$ | 0 |
| 1001 | $S_{9}$ | $S_{2}$ | $S_{3}$ | 1 |
| 1101 | $S_{13}$ | $S_{2}$ | $S_{3}$ | 1 |
| 1110 | $S_{14}$ | $S_{4}$ | $S_{13}$ | 0 |


| State | Next State |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $a$ | $e$ | c | 0 | 1 |
| $b$ | $b$ | $f$ | 0 | 1 |
| $c$ | $e$ | c | 1 | 0 |
| $e$ | c | $f$ | 0 | 1 |
| $f$ | $b$ | $b$ | 1 | 0 |

15.11 (b) Input: 000

Output starting in state $a$ :
001 (state $a^{0}$ state $e^{-0}$ state $g^{0}$ state $e$ )
Output starting in state $b$ :
000 (state $b \xrightarrow{\longrightarrow}$ state $d \xrightarrow{0}$ state $b \xrightarrow{\rightarrow}$ state $d$ )
15.12 (b) New Equivalent States: $S_{1} \equiv S_{5}, S_{9} \equiv S_{13}$.

| Input <br> Pattern | Present <br> State | Next State |  | Output |
| :---: | :---: | :---: | :---: | :---: |
| -000 | $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 |
| $0-01$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 |
| -010 | $S_{2}$ | $S_{4}$ | $S_{1}$ | 0 |
| -011 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 1 |
| -100 | $S_{4}$ | $S_{0}$ | $S_{9}$ | 1 |
| 0110 | $S_{6}$ | $S_{4}$ | $S_{9}$ | 1 |
| -111 | $S_{7}$ | $S_{14}$ | $S_{7}$ | 0 |
| $1-01$ | $S_{9}$ | $S_{2}$ | $S_{3}$ | 1 |
| 1110 | $S_{14}$ | $S_{4}$ | $S_{9}$ | 0 |

Unit 15 Solutions
15.12 (c)

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| -000 | $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 | 0 |
| $0-01$ | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 1 |
| -010 | $S_{2}$ | $S_{4}$ | $S_{1}$ | 1 | 0 |
| -011 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 1 | 0 |
| -100 | $S_{4}$ | $S_{0}$ | $S_{9}$ | 0 | 1 |
| 0110 | $S_{6}$ | $S_{4}$ | $S_{9}$ | 1 | 1 |
| -111 | $S_{7}$ | $S_{14}$ | $S_{7}$ | 0 | 0 |
| $1-01$ | $S_{9}$ | $S_{2}$ | $S_{3}$ | 0 | 1 |
| 1110 | $S_{14}$ | $S_{4}$ | $S_{9}$ | 1 | 1 |

15.13 (a) Moore circuit.
$15.13 \quad \mathrm{~S}_{8} \equiv \mathrm{~S}_{9} \equiv \mathrm{~S}_{10} \equiv \mathrm{~S}_{11} \equiv \mathrm{~S}_{12}$
(b), (c) and $\mathrm{S}_{13} \equiv \mathrm{~S}_{14} \equiv \mathrm{~S}_{15}$.

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| - | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| 0 | $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 | 0 |
| 1 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 0 | 0 |
| 00 | $S_{4}$ | $S_{8}$ | $S_{8}$ | 0 | 0 |
| 01 | $S_{5}$ | $S_{8}$ | $S_{8}$ | 0 | 0 |
| 10 | $S_{6}$ | $S_{8}$ | $S_{13}$ | 0 | 0 |
| 11 | $S_{7}$ | $S_{13}$ | $S_{13}$ | 0 | 0 |
| $-00,0--$ | $S_{8}$ | $S_{1}$ | $S_{1}$ | 0 | 0 |
| $1-1,11-$ | $S_{13}$ | $S_{1}$ | $S_{1}$ | 1 | 1 |

15.14 (a)

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| - | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| 0 | $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 | 0 |
| 1 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 0 | 0 |
| 00 | $S_{4}$ | $S_{8}$ | $S_{9}$ | 0 | 0 |
| 01 | $S_{5}$ | $S_{10}$ | $S_{11}$ | 0 | 0 |
| 10 | $S_{6}$ | $S_{12}$ | $S_{13}$ | 0 | 0 |
| 11 | $S_{7}$ | $S_{14}$ | $S_{15}$ | 0 | 0 |
| 000 | $S_{8}$ | $S_{1}$ | $S_{1}$ | 0 | 0 |
| 001 | $S_{9}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |
| 010 | $S_{10}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |
| 011 | $S_{11}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |
| 100 | $S_{12}$ | $S_{1}$ | $S_{1}$ | 0 | 0 |
| 101 | $S_{13}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |
| 110 | $S_{14}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |
| 111 | $S_{15}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |

15.12 (d) $\mathrm{S}_{1} \equiv \mathrm{~S}_{9}$ and $\mathrm{S}_{6} \equiv \mathrm{~S}_{14}$.

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| -000 | $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 | 0 |
| --01 | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 1 |
| -010 | $S_{2}$ | $S_{4}$ | $S_{1}$ | 1 | 0 |
| -011 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 1 | 0 |
| -100 | $S_{4}$ | $S_{0}$ | $S_{1}$ | 0 | 1 |
| -110 | $S_{6}$ | $S_{4}$ | $S_{1}$ | 1 | 1 |
| -111 | $S_{7}$ | $S_{6}$ | $S_{7}$ | 0 | 0 |

$S_{4} \equiv S_{5}$.

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| - | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| 0 | $S_{2}$ | $S_{4}$ | $S_{4}$ | 0 | 0 |
| 1 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 0 | 0 |
| $0-$ | $S_{4}$ | $S_{8}$ | $S_{8}$ | 0 | 0 |
| 10 | $S_{6}$ | $S_{8}$ | $S_{13}$ | 0 | 0 |
| 11 | $S_{7}$ | $S_{13}$ | $S_{13}$ | 0 | 0 |
| $-00,0--$ | $S_{8}$ | $S_{1}$ | $S_{1}$ | 0 | 0 |
| $1-1,11-$ | $S_{13}$ | $S_{1}$ | $S_{1}$ | 1 | 1 |

$15.14 \quad \mathrm{~S}_{9} \equiv \mathrm{~S}_{10} \equiv \mathrm{~S}_{11} \equiv \mathrm{~S}_{13} \equiv \mathrm{~S}_{14} \equiv \mathrm{~S}_{15}$
(b), (c) and $\mathrm{S}_{8} \equiv \mathrm{~S}_{12}$.

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| - | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| 0 | $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 | 0 |
| 1 | $S_{3}$ | $S_{6}$ | $S_{7}$ | 0 | 0 |
| 00 | $S_{4}$ | $S_{8}$ | $S_{9}$ | 0 | 0 |
| 01 | $S_{5}$ | $S_{9}$ | $S_{9}$ | 0 | 0 |
| 10 | $S_{6}$ | $S_{8}$ | $S_{9}$ | 0 | 0 |
| 11 | $S_{7}$ | $S_{9}$ | $S_{9}$ | 0 | 0 |
| -00 | $S_{8}$ | $S_{1}$ | $S_{1}$ | 0 | 0 |
| $-01,-1-$ | $S_{9}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |

15.14(b), Equivalent States: $S_{4} \equiv S_{6}$ and $S_{5} \equiv S_{7}$.
(c)

| Input | Present | Next State |  | Output Z |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pattern | State | $X=0 \quad X=1$ | $X=0 \quad X=1$ |  |  |
| - | $S_{1}$ | $S_{2}$ | $S_{3}$ | 0 | 0 |
| 0 | $S_{2}$ | $S_{4}$ | $S_{5}$ | 0 | 0 |
| 1 | $S_{3}$ | $S_{4}$ | $S_{5}$ | 0 | 0 |
| -0 | $S_{4}$ | $S_{8}$ | $S_{9}$ | 0 | 0 |
| -1 | $S_{5}$ | $S_{9}$ | $S_{9}$ | 0 | 0 |
| -00 | $S_{8}$ | $S_{1}$ | $S_{1}$ | 0 | 0 |
| $-01,-1-$ | $S_{9}$ | $S_{1}$ | $S_{1}$ | 0 | 1 |

(contd)


| Present <br> State | Next State |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 01 | 11 | 10 | $Z$ |  |  |
| $a$ | $a$ | $c$ | $c$ | $a$ | 0 |  |
| $c$ | $c$ | $a$ | $f$ | $a$ | 1 |  |
| $f$ | $f$ | $a$ | $a$ | $a$ | 1 |  |

15.15 (b)


|  | Next State |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0$ | $X=1$ |
| $a$ | $b$ | $c$ | 1 | 0 |
| $b$ | $e$ | $b$ | 1 | 0 |
| $c$ | $g$ | $b$ | 1 | 1 |
| $e$ | $c$ | $g$ | 1 | 0 |
| $g$ | $g$ | $i$ | 0 | 1 |
| $i$ | $a$ | $a$ | 0 | 1 |

Unit 15 Solutions
15.16 (a)

15.16 (b)

15.17 (a) $S_{0} \equiv e \equiv f, S_{1} \equiv c \equiv d, S_{2} \equiv S_{3} \equiv a \equiv b$

Since every state in $N$ has an equivalent state in $M$, and vice versa, $N$ and $M$ are equivalent.

15.17 (b)

| M |  |  |  |
| :---: | ---: | ---: | ---: |
|  | $\mathrm{X}=0$ | 1 |  |
| $S_{0}$ | $S_{2}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{0}$ | $S_{1}$ | 0 |
| $S_{2}$ | $S_{0}$ | $S_{2}$ | 1 |
| $S_{2}=S_{3}$ |  |  |  |

Note: $\begin{aligned} & S_{2} \equiv A \\ & S_{1} \equiv C \\ & S_{0} \equiv E\end{aligned}$

|  | $\mathrm{X}=0$ | 1 |  |
| ---: | ---: | ---: | ---: |
| $A$ | $E$ | $A$ | 1 |
| $C$ | $E$ | $C$ | 0 |
| $E$ | $A$ | $C$ | 0 |

15.18 (b)

$S_{2}$ and $S_{2}^{1}$ have no corresponding states,
$\therefore N$ and $N$ ' are not equivalent.
15.18 (c) $X=011$
$Z=(0) 011$
$Z^{1}=(0) 010$

Set don't care to 0 so $S_{1} \equiv S_{3} \equiv S_{4}$ :

| Present | Next State |  | Output |  |
| :---: | ---: | ---: | :---: | :---: |
| State | X $=0$ | 1 | $X=0$ | $X=1$ |
| $S_{0}^{1}$ | $S_{1}^{1}$ | $S_{5}^{1}$ | 0 | 0 |
| $S_{1}^{1}$ | $S_{1}^{1}$ | $S_{2}^{1}$ | 1 | 1 |
| $S_{2}^{1}$ | $S_{2}^{1}$ | $S_{1}^{1}$ | 0 | 1 |
| $S_{5}^{1}$ | $S_{5}^{1}$ | $S_{2}^{1}$ | 0 | 1 |

15.19 (b)

15.19 (c) $X=10$
$Z=01$
$Z^{1}=00$

## Unit 15 Solutions

15.20 (a) Invert all three columns of assignment (iv), and then swap the first and last columns. Then (iii) and (iv) are the same, therefore, Assignment $(i i i) \equiv$ Assignment (iv).
15.20 (c) Many state assignments are not equivalent to (i) through (v), for example:

| 101 | or | 011 |
| :--- | :--- | :--- |
| 000 |  | 101 |
| 011 |  | 000 |
| 100 |  | 100 |
| 010 |  | 010 |
| 110 |  | 110 |

15.20 (b) Equivalent assignments to each column having 000 as the starting state. Invert any column with 1 in the first row.

|  | (ii) - $\left(c_{2}^{\prime}\right)$ | iii $-c_{1}^{\prime}$ | iv $-c_{1}^{\prime} c_{2}^{\prime}$ | $v-c_{3}^{\prime}$ |
| :--- | :---: | :---: | :---: | :---: |
| $S_{0}$ | 000 | 000 | 000 | 000 |
| $S_{1}$ | 101 | 001 | 100 | 110 |
| $S_{2}$ | 011 | 100 | 001 | 100 |
| $S_{3}$ | 100 | 101 | 101 | 010 |
| $S_{4}$ | 010 | 011 | 110 | 001 |
| $S_{5}$ | 110 | 010 | 010 | 011 |

15.21 (a)

| Straight <br> Binary | $c_{2} \leftrightarrow c_{3}$ | $c_{1} \leftrightarrow c_{3}$ | $c_{1} \leftrightarrow c_{2}$ | $c_{1} \rightarrow c_{3} \rightarrow c_{2} \rightarrow c_{1}$ | $c_{1} \rightarrow c_{2} \rightarrow c_{3} \rightarrow c_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Assignment | $c_{2}$ |  |  |  |  |
| 000 | 000 | 000 | 000 | 000 | 000 |
| 001 | 001 | 100 | 010 | 010 | 100 |
| 010 | 100 | 010 | 001 | 100 | 001 |
| 011 | 101 | 110 | 011 | 110 | 101 |
| 100 | 010 | 001 | 100 | 001 | 010 |
| 101 | 011 | 101 | 110 | 011 | 110 |
| 110 | 110 | 011 | 101 | 101 | 011 |
| 111 | 111 | 111 | 111 | 111 | 111 |

15.21 (b) Many state assignments are not equivalent to the straight binary assignment, for example:

| 111 | 111 | etc. |
| :--- | :--- | :--- |
| 101 | 001 |  |
| 110 | 010 |  |
| 100 | 011 |  |
| 011 | 100 |  |
| 010 | 101 |  |
| 001 | 000 |  |
| 000 | 110 |  |

15.22 (a) 1. $(A, H)(B, G)(A, D)(E, G)$
2. $(D, G)(E, H)(B, F)(F, G)(C, A)(H, C)(E, A)$ ( $D, B$ )
3. $(A, C, E, G)(B, D, F, H)$

Consider Guideline \#3 only:

15.22 (b) Consider Guidelines \#1, 2:
$A=000, B=111, C=110, D=001, E=010$,
$\mathrm{F}=101, \mathrm{G}=011, \mathrm{H}=100$

$\mathrm{D}_{1}=\mathrm{X}^{\prime} \mathrm{Q}_{2}^{\prime} \mathrm{Q}_{3}+\mathrm{X}^{\prime} \mathrm{Q}_{2} \mathrm{Q}_{3}^{\prime}+\mathrm{XQ}_{1}$
15.23 (a) 1. $(A, C) \times 2 \checkmark(B, C) \times 2 \checkmark(A, D) \checkmark$
2. $(A, C) \vee(B, D)^{\vee}(A, B, D)^{\vee}$
$(A, B, C, D)^{\vee}$
3. $(A, D)^{r}$

Adjacencies that are satisfied are checked $(\checkmark)$


|  | $Q_{1}^{+} Q_{2}^{+}$ |  |  | $Z_{1} Z_{2}$ |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
| 00 | 00 | 00 | 10 | 10 | 01 | 01 | 01 | 01 |
| 11 | 11 | 0 | 11 | 01 | 11 | 11 | 11 | 11 |
| 10 | 00 | 00 | 11 | 01 | 11 | 11 | 00 | 00 |
| 01 | 01 | 11 | 0 | 10 | 01 | 01 | 01 | 01 |



$$
\mathrm{D}_{2}=\mathrm{X}^{\prime} \mathrm{Q}_{2}+\mathrm{XQ}_{2}{ }^{\prime}
$$


$\mathrm{D}_{3}=\mathrm{Q}_{1} \mathrm{Q}_{2}{ }^{\prime}+\mathrm{Q}_{1} \mathrm{Q}_{3}{ }^{\prime}$
15.23 (b)


15.23 (b)
(contd)


## Unit 15 Solutions

15.24 (a) Equations for one-hot state assignment:
$D_{\mathrm{A}}=X(A+B+D+E), D_{\mathrm{B}}=X^{\prime}(A+D)$,
$D_{\mathrm{C}}=X^{\prime} \mathrm{B}, D_{\mathrm{D}}=X C, D_{\mathrm{E}}=X^{\prime}(C+E), z=D$
15.24 (b) Guidelines:

1. $(A, D) \times 2(C, E)(A, B, D, E)$
2. $(A, B) \times 2(A, C)(D, E)(A, E)$

The following assignment satisfies all but (A, E),
$(\mathrm{A}, \mathrm{C})$ and (B, D):


|  | $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  |  |
| :---: | ---: | ---: | :---: |
| $Q_{1} Q_{2} Q_{3}$ | $X=0$ | 1 | $z$ |
| 000 | 001 | 000 | 0 |
| 010 | 111 | 000 | 0 |
| 011 | 011 | 000 | 0 |
| 010 | 001 | 000 | 1 |
| 110 | --- | --- | - |
| 111 | 011 | 010 | 0 |
| 101 | --- | --- | - |
| 100 | --- | --- | - |


| $D_{1}=X^{\prime} Q_{2}{ }^{\prime} Q_{3}, D_{2}=X^{\prime} Q_{3}+Q_{1}, D_{3}=X^{\prime}$, |
| :--- |

$z=Q_{2} Q_{3}{ }^{\prime}$
15.25 (a)

15.25 (b) 1. $(A, C)^{\checkmark}(B, D)^{\checkmark}(C, E)^{\checkmark}$
2. $(A, B)^{\checkmark}(C, E)^{\checkmark}(A, D)(A, C)^{\checkmark}(B, D)^{\checkmark}$
3. $(A, C, E)^{\checkmark}(B, D)^{\checkmark}$

Adjacencies that are satisfied are checked $(\checkmark)$
$A=000, B=100, C=001, D=101, E=011$
All are satisfied except $(A, D)$


Alternate:

15.25 (c)

| $Q_{1} Q_{2} Q_{3}$ | $\begin{gathered} Q_{1}^{+} Q_{2}^{+} Q_{3}^{+} \\ X=0 \quad 1 \end{gathered}$ | Z |
| :---: | :---: | :---: |
| 000 | 000100 | 1 |
| 100 | 001011 | 0 |
| 001 | 000101 | 1 |
| 101 | 001000 | 0 |
| 011 | 100101 | 1 |

15.25 (c) (contd)








15.25 (d)


Output $Z$ equation is the same for D and J -K flip-flops. (Actually, it is the same for any flip-flop.)
15.26 (a)


| Present <br> State | Next State |  |  |
| :---: | ---: | :---: | :---: |
| $A$ | X | 1 | Output |
| $A$ | $A$ | $C$ | 1 |
| $B$ | $B$ | $A$ | 1 |
| $C$ | $C$ | $G$ | 1 |
| $D$ | $A$ | $C$ | 0 |
| $E$ | $D$ | $E$ | 0 |
| $G$ | $E$ | $D$ | 0 |

## Unit 15 Solutions

15.26 (b) 1. $(A, D) \times 2$
2. $(A, C) \times 2(A, B)(C, G)(D, E) \times 2$
3. $(A, B, C)(D, E, G)$

There are several solutions. Here is one satisfying all guidelines:
$A=000, B=010, C=001, D=100, E=110$, $G=101$

15.26 (c)

|  | $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  |
| :---: | :---: | :---: |
| $Q_{1} Q_{2} Q_{3}$ | $X=0 \quad 1$ | $Z$ |
| 000 | 000001 | 1 |
| 010 | 010000 | 1 |
| 001 | 001101 | 1 |
| 100 | 000001 | 0 |
| 110 | 100110 | 0 |
| 101 | 110100 | 0 |



| $\mathrm{Q}_{2} \mathrm{Q}_{3}$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 1 | 0 | 0 |
| 11 | ( | X | Х | X |
| 10 | 1 | 0 | 1 | 0 |
| $D_{2}$ |  |  |  |  |



15.26 (d) Again, $Z=Q_{1}^{\prime}$ :

15.27 (a)

| Present State | Next State | Output |  |
| :---: | :---: | :---: | :---: |
|  | X = 0 1 | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{1} S_{4}$ | 0 | 0 |
| $S_{1}$ | $S_{1} \quad S_{2}$ | 0 | 0 |
| $S_{2}$ | $S_{3} S_{4}$ | 1 | 0 |
| $S_{3}$ | $S_{5} S_{2}$ | 0 | 0 |
| $S_{4}$ | $S_{3} S_{4}$ | 0 | 0 |
| $S_{5}$ | $S_{1} \quad S_{2}$ | 0 | 1 |


| $Q_{2} Q_{3} Q_{1}$ | 0 | 1 |
| :---: | :---: | :---: |
| 00 | $\mathrm{S}_{0}$ |  |
| 01 | $\mathrm{S}_{4}$ | $\mathrm{S}_{3}$ |
| 11 | $\mathrm{S}_{2}$ |  |
| 10 | $\mathrm{S}_{1}$ | $\mathrm{S}_{5}$ |

1. $\left(S_{0}, S_{1}, S_{5}\right)\left(S_{0}, S_{2}, S_{4}\right)\left(S_{1}, S_{3}, S_{5}\right)$
2. $\left(S_{1}, S_{4}\right)\left(S_{1}, S_{2}\right) \times 2\left(S_{3}, S_{4}\right) \times 2\left(S_{2}, S_{5}\right)$
3. $\left(S_{0}, S_{1}, S_{3}, S_{4}\right)$
$S_{0}=000 . S_{1}=010, S_{2}=011, S_{3}=101, S_{4}=001$,
$S_{5}=110$

|  | $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2} Q_{3}$ | $X=0$ | 1 | $X=0$ | $X=1$ |
| 000 | 010001 | 0 | 0 |  |
| 010 | 010011 | 0 | 0 |  |
| 011 | 101001 | 1 | 0 |  |
| 101 | 110011 | 0 | 0 |  |
| 001 | 101001 | 0 | 0 |  |
| 110 | 010011 | 0 | 1 |  |


15.27 (b)


$S_{1}=X^{\prime} \mathrm{Q}_{3}$
One alternative assignment:

(a) $D_{1}=X Q_{1}^{\prime} Q_{2}^{\prime}+Q_{2}+X^{\prime} Q_{1} Q_{3}^{\prime} ; D_{2}=X ; D_{3}=X^{\prime} Q_{2}{ }^{\prime}$ $Z=X^{\prime} Q_{1}^{\prime} Q_{2}+X Q_{1} Q_{3}$
(b) $S_{1}=X Q_{1}{ }^{\prime} Q_{3}{ }^{\prime}+Q_{2} ; R_{1}=X Q_{1} Q_{2}{ }^{\prime}+Q_{3} ; S_{2}=X$; $R_{2}=X^{\prime} ; S_{3}=X^{\prime} Q_{2}{ }^{\prime} ; R_{3}=X ; Z=X^{\prime} Q_{1}{ }^{\prime} Q_{2}+X Q_{1} Q_{3}$

| Present | Next State |  |  |
| :---: | ---: | ---: | :---: |
| State | $X=0$ | 1 | $Z$ |
| $S_{0}$ | $S_{2}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{5}$ | $S_{0}$ | 0 |
| $S_{2}$ | $S_{3}$ | $S_{1}$ | 0 |
| $S_{3}$ | $S_{3}$ | $S_{4}$ | 0 |
| $S_{4}$ | $S_{4}$ | $S_{3}$ | 1 |
| $S_{5}$ | $S_{4}$ | $S_{0}$ | 0 |


| $Q_{1} Q_{2} Q_{3}$ | $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ <br> $X=0$ <br> 1 | Z |
| :---: | :---: | :---: |
| 000 | 010001 | 0 |
| 001 | 011000 | 0 |
| 010 | 110001 | 0 |
| 110 | 110111 | 0 |
| 111 | 111110 | 1 |
| 011 | 111000 | 0 |


| $Q_{1} Q_{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 2 | 3 |  |
| 1 | 1 | 5 | 4 |  |

1. $\left(S_{2}, S_{3}\right)\left(S_{4}, S_{5}\right)\left(S_{0}, S_{2}\right)\left(S_{1}, S_{5}\right)$
2. $\left(S_{1}, S_{2}\right)\left(S_{0}, S_{5}\right)\left(S_{1}, S_{3}\right)\left(S_{3}, S_{4}\right) \times 2\left(S_{0}, S_{4}\right)$
$S_{0}=000, S_{1}=001, S_{2}=010, S_{3}=110, S_{4}=111$,
$S_{5}=011$
Guideline 3 is of no use for this state table.

| $\mathrm{Q}_{2} \mathrm{Q}_{3}$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | X | X | 0 |
| 01 | 0 | X | X | 0 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 1 | 1 | 1 | 0 |
| $\mathrm{Q}_{1}{ }^{+}$ |  |  |  |  |

15.28 (a)

15.28 (b)


Unit 15 Solutions
15.29 See solutions to 14.22 for the state table.
I. $\left(S_{0}, S_{1}, S_{7}\right)\left(S_{2}, S_{6}\right)\left(S_{3}, S_{4}, S_{5}\right)\left(S_{0}, S_{6}\right)\left(S_{1}, S_{7}\right)$
$\left(S_{2}, S_{3}, S_{5}\right)$
II. $\left(S_{1}, S_{6}\right)\left(S_{6}, S_{7}\right)\left(S_{1}, S_{2}\right) \times 2\left(S_{3}, S_{7}\right)\left(S_{3}, S_{4}\right) \times 2$ $\left(S_{4}, S_{5}\right)$
III. $\left(S_{0}, S_{1}, S_{3}, S_{4}, S_{6}\right)\left(S_{2}, S_{5}\right)$

| $Q_{2} Q_{3} Q_{1}$ | 0 | 1 |
| :---: | :---: | :---: |
| 00 | $\mathrm{S}_{0}$ | $\mathrm{S}_{6}$ |
| 01 | $\mathrm{S}_{1}$ | $\mathrm{S}_{5}$ |
| 11 | $\mathrm{S}_{3}$ | $\mathrm{S}_{4}$ |
| 10 | $\mathrm{S}_{7}$ | $\mathrm{S}_{2}$ |


|  | $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2} Q_{3}$ | $X=0$ | 1 | $X=0$ | $X=1$ |
| 000 | 001 | 100 | 00 | 00 |
| 001 | 001 | 110 | 00 | 00 |
| 110 | 010 | 011 | 00 | 01 |
| 011 | 111 | 011 | 00 | 00 |
| 111 | 111 | 101 | 00 | 00 |
| 101 | 111 | 011 | 00 | 01 |
| 100 | 010 | 100 | 00 | 00 |
| 010 | 001 | 110 | 10 | 00 |



$$
\begin{aligned}
Q_{1}^{+}= & X^{\prime} Q_{2} Q_{3}+X^{\prime} Q_{1} Q_{3} \\
& +X Q_{1}^{\prime} Q_{2}^{\prime}+X Q_{1}^{\prime} Q_{3}^{\prime} \\
& +X Q_{2}^{\prime} Q_{3}^{\prime}+Q_{1} Q_{2} Q_{3}
\end{aligned}
$$

$$
\begin{aligned}
Q_{2}^{+}= & X^{\prime} Q_{1}+Q_{1}^{\prime} Q_{2} Q_{3} \\
& +X Q_{2}^{\prime} Q_{3}+X Q_{2} Q_{3}^{\prime}
\end{aligned}
$$

$$
Q_{3}^{+}=X^{\prime} Q_{1}^{\prime}+Q_{2} Q_{3}+Q_{1} Q_{3}
$$

$$
+X Q_{1} Q_{2}
$$


$\mathrm{Z}_{1}=\mathrm{X}^{\prime} \mathrm{Q}_{1} \mathrm{Q}_{2} \mathrm{Q}_{3}{ }^{\prime}$


$$
\mathrm{Z}_{2}=\mathrm{XQ}_{1} \mathrm{Q}_{2}^{\prime} \mathrm{Q}_{3}+\mathrm{X}_{1} \mathrm{Q}_{2} \mathrm{Q}_{3}^{\prime}
$$

15.30 See FLD p. 723 for the state table.
I. $\left(S_{0}, S_{1}\right)\left(S_{2}, S_{3}\right)\left(S_{4}, S_{5}, S_{7}\right)\left(S_{0}, S_{2}, S_{3}\right)\left(S_{1}, S_{4}\right)$ $\left(S_{5}, S_{6}, S_{7}\right)$
II. $\left(S_{1}, S_{3}\right)\left(S_{1}, S_{2}\right)\left(S_{3}, S_{4}\right) \times 2\left(S_{2}, S_{5}\right)\left(S_{5}, S_{6}\right) \times 2$ $\left(S_{6}, S_{7}\right)$
III. $\left(S_{0}, S_{1}, S_{3}, S_{5}, S_{6}\right)\left(S_{4}, S_{7}\right)$
$S_{0}=000, S_{1}=001, S_{2}=010, S_{3}=011, S_{4}=111$, $S_{5}=110, S_{6}=100, S_{7}=101$

|  | $Q_{1}^{+} Q_{2}^{+} Q_{3}^{+}$ |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2} Q_{3}$ | $X=0$ | 1 | $X=0$ | $X=1$ |
| 000 | 001 | 011 | 00 | 00 |
| 001 | 001 | 010 | 00 | 00 |
| 010 | 111 | 011 | 10 | 00 |
| 011 | 111 | 011 | 00 | 00 |
| 111 | 110 | 010 | 01 | 00 |
| 110 | 110 | 100 | 00 | 00 |
| 100 | 101 | 100 | 00 | 00 |
| 101 | 110 | 100 | 01 | 00 |

Unit 15 Solutions

### 15.30 (contd)

| $\mathrm{Q}_{1}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 1 | 1 | 0 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 1 | 0 |

$\mathrm{Q}_{1}^{+}$

$\mathrm{J}_{1}=\mathrm{X}^{\prime} \mathrm{Q}_{2}$

$\mathrm{K}_{1}=\mathrm{XQ}_{2} \mathrm{Q}_{3}$

$\mathrm{Z}_{1}=X^{\prime} \mathrm{Q}_{1}^{\prime} \mathrm{Q}_{2} \mathrm{Q}_{3}^{\prime}$

$Z_{2}=X^{\prime} Q_{1} Q_{3}$

| $\mathrm{Q}_{2} \mathrm{Q}_{3}$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 1 | 1 | 0 | 1 |
| 01 | 1 | 0 | 0 | 0 |
| 11 | 1 | 0 | 0 | 1 |
| 10 | 1 | 0 | 0 | 1 |
| $\mathrm{Q}_{3}^{+}$ |  |  |  |  |


$\mathrm{J}_{3}=\mathrm{Q}_{1}{ }^{\prime}+\mathrm{X}^{\prime} \mathrm{Q}_{2}^{\prime}$

$\mathrm{K}_{2}=\mathrm{XQ}_{1} \mathrm{Q}_{3}{ }^{\prime}$
$\mathrm{K}_{3}=\mathrm{Q}_{1}+\mathrm{XQ}_{2}^{\prime}$


15.31 Row reduction of the solution to 14.6 given on FLD p. 724 easily gives 4 states. Renaming them gives:

| Present <br> State | Next State |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 01 | 11 | 10 | $Z$ |  |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{1}$ | $S_{0}$ | 0 |
| $S_{1}$ | $S_{0}$ | $S_{1}$ | $S_{1}$ | $S_{3}$ | 0 |
| $S_{2}$ | $S_{2}$ | $S_{3}$ | $S_{2}$ | $S_{0}$ | 1 |
| $S_{3}$ | $S_{2}$ | $S_{3}$ | $S_{2}$ | $S_{3}$ | 1 |

See p. 146 in this manual for the state table.
I. $\left(S_{0}, S_{1}\right) \times 3\left(S_{2}, S_{3}\right) \times 2\left(S_{0}, S_{2}\right)\left(S_{1}, S_{3}\right)$
II. $\left(S_{0}, S_{1}\right)\left(S_{0}, S_{1}, S_{3}\right)\left(S_{0}, S_{2}, S_{3}\right)\left(S_{2}, S_{3}\right)$
III. $\left(S_{0}, S_{1}\right)\left(S_{2}, S_{3}\right)$
$S_{0}=00, S_{1}=01, S_{2}=10, S_{3}=11$

|  | $Q_{1}^{+} Q_{2}^{+}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 | $Z$ |
| 00 | 00 | 01 | 01 | 00 | 0 |
| 01 | 00 | 01 | 01 | 11 | 0 |
| 10 | 10 | 11 | 10 | 00 | 1 |
| 11 | 10 | 11 | 10 | 11 | 1 |


15.32 See answers to 14.23 for the state table. The four-state table is minimum.
I. $\left(S_{0}, S_{1}\right) \times 3\left(S_{0}, S_{3}\right)\left(S_{1}, S_{2}\right)\left(S_{2}, S_{3}\right) \times 3$
II. $\left(S_{0}, S_{1}\right)\left(S_{0}, S_{1}, S_{2}\right)\left(S_{2}, S_{3}\right)\left(S_{0}, S_{2}, S_{3}\right)$
III. $\left(S_{0}, S_{1}\right)\left(S_{2}, S_{3}\right)$

|  | $Q_{1}^{+} Q_{2}^{+}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | 00 | 01 | 11 | 10 | $Z$ |
| 00 | 01 | 01 | 00 | 00 | 0 |
| 01 | 01 | 01 | 11 | 00 | 0 |
| 11 | 11 | 10 | 11 | 10 | 1 |
| 10 | 11 | 10 | 00 | 10 | 1 |





$\mathrm{D}_{2}=\mathrm{X}_{1}^{\prime} \mathrm{X}_{2}^{\prime}+\mathrm{X}_{1}^{\prime} \mathrm{Q}_{1}^{\prime}+\mathrm{X}_{2}^{\prime} \mathrm{Q}_{1} \mathrm{Q}_{2}$
15.33

$\mathrm{T}_{\mathrm{A}}$

$\mathrm{T}_{\mathrm{B}}$

$B^{+}$

| Present | Next State |  | $Z$ |  |
| :---: | ---: | ---: | :---: | :---: |
| State | $W=0$ | 1 | 0 | 1 |
| 0 | 1 | 3 | 0 | 0 |
| 1 | 3 | 5 | 0 | 0 |
| 2 | 4 | 7 | 1 | 0 |
| 3 | 5 | 0 | 0 | 0 |
| 4 | 7 | 6 | 0 | 0 |
| 5 | 0 | 1 | 0 | 0 |
| 6 | 2 | 4 | 1 | 0 |
| 7 | 6 | 2 | 0 | 0 |

## Unit 15 Solutions

15.33 I. None
(contd) II. $(4,7)^{\checkmark}(6,7) \checkmark(2,4)^{\checkmark}(2,6) \checkmark$
Assignment:
$S_{0}=000, S_{2}=100, S_{4}=111, S_{6}=110, S_{7}=101$
в с


| Present <br> State | Next State |  |  |  |
| :---: | ---: | :---: | :---: | :---: |
| $W=0$ | 1 | Output |  |  |
| $S_{0}$ | $S_{0}$ | $S_{0}$ | 0 | 0 |
| $S_{2}$ | $S_{4}$ | $S_{7}$ | 1 | 0 |
| $S_{4}$ | $S_{7}$ | $S_{6}$ | 0 | 0 |
| $S_{6}$ | $S_{2}$ | $S_{4}$ | 0 | 0 |
| $S_{7}$ | $S_{6}$ | $S_{2}$ | 0 | 0 |


| Present <br> State | Next State <br> $W=0$ |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 |  |  |  |
| 000 | 000 | 000 | 0 | 0 |
| 100 | 111 | 101 | 1 | 0 |
| 111 | 101 | 110 | 0 | 0 |
| 110 | 100 | 111 | 0 | 0 |
| 101 | 110 | 100 | 0 | 0 |

T input equations derived from the transition table using Karnaugh maps:
$T_{\mathrm{A}}=0 ; \quad T_{\mathrm{B}}=W^{\prime} A ; \quad T_{\mathrm{C}}=W B+A B^{\prime} ; \quad Z=W^{\prime} A B^{\prime} C^{\prime}$
15.34

15.35

By inspecting incoming arrows, we get:
$Q_{0}{ }^{+}=D_{0}=X^{\prime} Y Q_{0}+Y^{\prime} Q_{1}+X^{\prime} Y Q_{2}$
$Q_{1}^{+}=D_{1}=X Y Y^{\prime} Q_{0}+X Y Q_{1}+Y^{\prime} Q_{2}$
$Q_{2}^{+}=D_{2}=X Y Q_{0}+X^{\prime} Y^{\prime} Q_{0}+X^{\prime} Y Q_{1}+X Y Q_{2}$
$Z=X^{\prime} Y Q_{1}+X Y Q_{2}+X^{\prime} Y Q_{2}=X^{\prime} Y Q_{1}+Y Q_{2}$
15.37 (a) By inspecting incoming arrows, we get:

$$
\begin{aligned}
& D_{\mathrm{A}}=Q_{\mathrm{A}}^{+}=X \\
& D_{\mathrm{B}}=Q_{\mathrm{B}}^{+}=X^{\prime} Q_{\mathrm{A}} \\
& D_{\mathrm{C}}=Q_{\mathrm{C}}^{+}=X^{\prime} Q_{\mathrm{B}} \\
& D_{\mathrm{D}}=Q_{\mathrm{D}}^{+}=X^{\prime}\left(Q_{\mathrm{C}}+Q_{\mathrm{D}}\right) \\
& Z=X Q_{\mathrm{C}}
\end{aligned}
$$

By inspecting incoming arrows, we get:
$D_{0}=Q_{0}{ }^{+}=X^{\prime} Y^{\prime} Q_{0}+X Y Q_{3}$
$D_{1}=Q_{1}^{+}=X Q_{0}+Y^{\prime} Q_{1}+X Y^{\prime} Q_{3}$
$D_{2}=Q_{2}{ }^{+}=X^{\prime} Y Q_{0}+X^{\prime} Q_{2}+X^{\prime} Y Q_{3}$
$D_{3}=Q_{3}{ }^{+}=Y Q_{1}+X Q_{2}+X^{\prime} Y^{\prime} Q_{3}$
$S=Y Q_{1}+X Q_{2}$
$P=X^{\prime} Y^{\prime} Q_{3}$

### 15.36

|  | Clr, Ld, Cnt | $\begin{aligned} & C l r=Q_{1}{ }^{\prime}+Q_{0}{ }^{\prime}+x \\ & L d=Q_{1} Q_{0} \end{aligned}$ |
| :---: | :---: | :---: |
| $Q_{2} Q_{1} Q_{0}$ | $X=0$ |  |
| 000 | 101101 | $P_{2}=0$ |
| 010 | 101101 | $P_{1}=0$ |
| 010 | 101101 | $P_{0}=1$ |
| 011 | 0-- 11- |  |
| 100 | --- --- |  |
| 101 | --- --- |  |
| 110 | --- --- |  |
| 111 | --- --- |  |

15.37 (b)

|  | $Q_{1}^{+} Q_{0}{ }^{+}$ |  | $Z$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $Q_{1} Q_{0}$ | $X=0$ | 1 | $X=0$ | $X=1$ |
| 00 | 01 | 00 | 0 | 0 |
| 01 | 11 | 00 | 0 | 0 |
| 11 | 10 | 00 | 0 | 1 |
| 10 | 10 | 00 | 0 | 0 |

15.37 (c) For the counter, a better state assignment is $\mathrm{A}=00$, $\mathrm{B}=01, \mathrm{C}=10$ and $\mathrm{D}=11$.

15.37 (d) Another possibility is to duplicate state D and use (contd) 1110 and 1111 as state assignments for the two D's.

|  | $\mathrm{s}_{1} \mathrm{~S}_{0}$ |  | $Z$ |  |
| :---: | ---: | :---: | :---: | :---: |
| $Q_{3} Q_{2} Q_{1} Q_{0}$ | $X=0$ | 1 | $X=0$ | $X=1$ |
| 0000 | 01 | 11 | 0 | 0 |
| 1000 | 01 | 11 | 0 | 0 |
| 1100 | 01 | 11 | 0 | 1 |
| 1110 | 01 | 11 | 0 | 0 |
| 1111 | 01 | 11 | 0 | 0 |

$s_{1}=X, s_{0}=1, Z=X Q_{2} Q_{1}{ }^{\prime}, s_{\text {in }}=1$, $P_{3}=-, P_{2}=-, P_{1}=-, P_{0}=-$
15.38 (b)

|  | $Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  |
| :---: | ---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | 1 |
| 00 | 00 | 01 |
| 01 | 00 | 10 |
| 11 | 00 | 11 |
| 10 | 00 | 11 |


|  | $T_{1} T_{2}$ |  |
| :---: | ---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | 1 |
| 00 | 00 | 01 |
| 01 | 00 | 10 |
| 11 | 00 | 11 |
| 10 | 00 | 11 |

The equations for $T_{1}$ and $T_{2}$ are the same as in Part (a).

15.38 (d) |  | $J_{1} K_{1}, J_{2} K_{2}$ |  |
| :---: | :---: | :---: |
|  | $Q_{1} Q_{2}$ | $X=0$ |
| 00 | $0-, 0-$ | $0-, 1-$ |
| 01 | $0-,-1$ | $1-,-1$ |
| 11 | $-1,0-$ | $-0,1-$ |
| 10 | $-1,-1$ | $-0,-0$ |

The equations for $J_{1}, K_{1}, J_{2}$ and $K_{2}$ are the same as in Part (c).
15.37 (d) For the shift register, the state assignment $\mathrm{A}=0000$, $\mathrm{B}=1000, \mathrm{C}=1100$ and $\mathrm{D}=1110$ makes use of the shift function.

| $Q_{3} Q_{2} Q_{1} Q_{0}$ | $\mathrm{S}_{1} \mathrm{~S}_{0}$ | Z |  |
| :---: | :---: | :---: | :---: |
|  | $X=0 \quad 1$ | $X=0$ | $X=1$ |
| 0000 | 0111 | 0 | 0 |
| 1000 | 0111 | 0 | 0 |
| 1100 | 0111 | 0 | 1 |
| 1110 | 0011 | 0 | 0 |

15.38 (a) $Q_{1}{ }^{+}=X Q_{1}+X Q_{2}=X Q_{1}+X Q_{2}\left(Q_{1}+Q_{1}{ }^{\prime}\right)$

$$
=X Q_{1}+X Q_{2} Q_{1}^{\prime}=\left(X+Q_{1}^{\prime}\right)\left(X^{\prime}+Q_{2}^{\prime}+Q_{1}\right) Q_{1}
$$

$$
+\left(X^{\prime} Q_{1}+X Q_{2} Q_{1}^{\prime}\right) Q_{1}^{\prime}
$$

$$
=\left(X^{\prime} Q_{1}+X Q_{2} Q_{1}^{\prime}\right)^{\prime} Q_{1}+\left(X^{\prime} Q_{1}+X Q_{2} Q_{1}^{\prime}\right) Q_{1}^{\prime}
$$

$$
\text { so } T_{1}=\left(X^{\prime} Q_{1}+X Q_{2} Q_{1}^{\prime}\right)
$$

$$
Q_{2}^{+}=X Q_{1}+X Q_{2}^{\prime}=X Q_{1}\left(Q_{2}+Q_{2}^{\prime}\right)+X Q_{2}^{\prime}
$$

$$
=X Q_{1} Q_{2}+X Q_{2}^{\prime}
$$

$$
\text { so } T_{2}=\left(X Q_{1}\right)^{\prime} Q_{2}+X Q_{2}^{\prime}
$$

$$
=X^{\prime} Q_{2}+Q_{1}^{\prime} Q_{2}+X Q_{2}^{\prime}
$$

15.38 (c) $Q_{1}{ }^{+}=X Q_{1}+X Q_{2}=X Q_{1}+X Q_{2}\left(Q_{1}+Q_{1}{ }^{\prime}\right)$

$$
=X Q_{1}+X Q_{2} Q_{1}^{\prime}
$$

$$
\text { so } J_{1}=X Q_{2}, K_{1}=X^{\prime}
$$

$$
Q_{2}^{+}=X Q_{1}+X Q_{2}^{\prime}=X Q_{1}\left(Q_{2}+Q_{2}^{\prime}\right)+X Q_{2}^{\prime}
$$

$$
=X Q_{1} Q_{2}+X Q_{2}
$$

$$
\text { so } J_{2}=X, K_{2}=\left(X Q_{1}\right)^{\prime}=X^{\prime}+Q_{1}^{\prime}
$$

15.39 (a) $Q_{1}{ }^{+}=J_{1} Q_{1}{ }^{\prime}+K_{1}{ }^{\prime} Q_{1}=Q_{2} Q_{1}{ }^{\prime}+Q_{1}{ }^{\prime} Q_{1}=Q_{2} Q_{1}{ }^{\prime}$ so $T_{1}=Q_{1}+Q_{2} Q_{1}^{\prime}=Q_{1}+Q_{2}$
$Q_{2}{ }^{+}=J_{2} Q_{2}{ }^{\prime}+K_{2}{ }^{\prime} Q_{2}=\left(X+Q_{1}{ }^{\prime}\right) Q_{2}{ }^{\prime}+(1)^{\prime} Q_{2}$
$=\left(X+Q_{1}{ }^{\prime}\right) Q_{2}{ }^{\prime}$
so $T_{2}=Q_{2}+\left(X+Q_{1}{ }^{\prime}\right) Q_{2}{ }^{\prime}$
$=Q_{2}+X+Q_{1}{ }^{\prime}$

Unit 15 Solutions
15.39 (b)

|  | $J_{1} K_{1}, J_{2} K_{2}$ |  |
| :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ |
| 00 | 00,11 | 00,11 |
| 01 | 10,11 | 10,11 |
| 11 | 11,01 | 11,11 |
| 10 | 01,01 | 01,11 |


|  | $Q_{1}{ }^{+} Q_{2}{ }^{+}$ |  |
| :---: | ---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | 1 |
| 00 | 01 | 01 |
| 01 | 10 | 10 |
| 11 | 00 | 00 |
| 10 | 00 | 01 |


|  | $T_{1} T_{2}$ |  |
| :---: | ---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | 1 |
| 00 | 01 | 01 |
| 01 | 11 | 11 |
| 11 | 11 | 11 |
| 10 | 10 | 11 |

The equations for $T_{1}$ and $T_{2}$ are the same as in Part (a).

$$
\begin{aligned}
& 15.39 \text { (c) } Q_{1}{ }^{+}=S_{1}+R_{1}{ }^{\prime} Q_{1}=Q_{2} Q_{1}{ }^{\prime}+Q_{1}{ }^{\prime} Q_{1} \\
& \text { so } S_{1}=Q_{2} Q_{1}{ }^{\prime} \text { and } R_{1}=Q_{1} \\
& Q_{2}{ }^{+}=S_{2}+R_{2}{ }^{\prime} Q_{2}=\left(X+Q_{1}{ }^{\prime}\right) Q_{2}{ }^{\prime}+\left(Q_{2}\right)^{\prime} Q_{2} \\
& \text { so } S_{2}=\left(X+Q_{1}^{\prime}\right) Q_{2}^{\prime} \text { and } R_{2}=Q_{2}
\end{aligned}
$$

15.39 (d) |  | $S_{1} R_{1}, S_{2} R_{2}$ |  |
| :---: | :---: | :---: |
| $Q_{1} Q_{2}$ | $X=0$ | $X=1$ |
| 00 | $0-, 10$ | $0-, 10$ |
| 01 | 10,01 | 10,01 |
| 11 | 01,01 | 01,01 |
| 10 | $01,0-$ | 01,10 |

The equations for $S_{1}, R_{1}, S_{2}$ and $R_{2}$ are the same as in Part (c).

## Unit 16 Problem Solutions

16.1- See Lab Solutions in this manual.
16.14
16.15 See FLD p. 729 for solution.
16.16 See FLD p. 729 for solution.
16.17 (a) The state meanings are given in the following table:

| Name | Meaning |
| :---: | :--- |
| $S_{0}$ | No 1's have occurred |
| $S_{1}$ | One 1 has occurred (an odd number < 2) |
| $S_{2}$ | Two 1's or an even number of 1's $>2$ <br> have occurred |
| $S_{3}$ | An odd number of 1's $>2$ has occurred. |



16.17 (b) |  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{1}$ | $S_{2}$ | 0 |
| $S_{2}$ | $S_{2}$ | $S_{3}$ | 0 |
| $S_{3}$ | $S_{3}$ | $S_{2}$ | 1 |

I: $(1,3)$
II: $(0,1)(1,2)(2,3) 2 x$


|  |  | $a_{i+1} b_{i+1}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $a_{\mathrm{i}} b_{\mathrm{i}}$ | $X=0$ | $X=1$ | $Z$ |
| $S_{0}$ | 00 | 00 | 10 | 0 |
| $S_{1}$ | 10 | 10 | 01 | 0 |
| $S_{2}$ | 01 | 01 | 11 | 0 |
| $S_{3}$ | 11 | 11 | 01 | 1 |



$$
a_{i+1}=x_{i}^{\prime} a_{i}+x_{i} a_{i}^{\prime}
$$




Note: Solution in FLD p. 729 uses state assignment $S_{0}=00$, $S_{1}=01, S_{2}=10, S_{3}=11$.
16.17 (c) Since no 1 's have occurred, $\mathrm{a}_{1}$ and $\mathrm{b}_{1}$ are the same
as $\mathrm{S}_{0}$ or, $a_{1}=0 ; b_{1}=0$;
$\left.\begin{array}{l}a_{2}=x_{1} a_{1}{ }^{\prime}+x_{1}{ }_{1} a_{1}=x_{1} ; \\ b_{2}=b_{1}+x_{1} a_{1}=0\end{array}\right\}$ first cell

## Unit 16 Solutions

16.17 (d)

16.18 (a) The output becomes 1 whenever an even $\# 0$ 's or an even \#1's (greater than 0 ) occurs.


The state meanings are given in the following table:

| Name | Meaning |
| :---: | :--- |
| $S_{0}$ | even \#0's and even \#1's received |
| $S_{1}$ | even \#0's and odd \#1's received |
| $S_{2}$ | odd \#0's and even \#1's received |
| $S_{3}$ | odd \#0's and odd \#1's received |


| Present | Next State |  | Output |  |
| :---: | ---: | ---: | :---: | :---: |
| State | $X=0$ | 1 | $X=0$ | $X=1$ |
| $S_{0}$ | $S_{2}$ | $S_{1}$ | 0 | 0 |
| $S_{1}$ | $S_{3}$ | $S_{0}$ | 0 | 1 |
| $S_{2}$ | $S_{0}$ | $S_{3}$ | 1 | 0 |
| $S_{3}$ | $S_{1}$ | $S_{2}$ | 1 | 1 |

Guidelines: I: --

$$
\text { II: }(1,2) 2 x,(0,3) 2 x
$$

An assignment is

|  | $\mathrm{A}^{+} \mathrm{B}^{+}$ |  | Z |  |
| :---: | :--- | ---: | :---: | ---: |
| AB | $\mathrm{X}=0$ | 1 | $X=0$ | $X=1$ |
| 00 | 11 | 0 | 1 | 0 |
| 01 | 10 | 0 | 0 | 0 |
| 11 | 00 | 10 | 1 | 0 |
| 10 | 01 | 11 | 1 | 1 |


$\mathrm{D}_{\mathrm{A}}=\mathrm{X}^{\prime} \mathrm{A}^{\prime}+\mathrm{XA}$

$D_{B}=B^{\prime}$

$Z=X^{\prime} A+X A^{\prime} B+A B^{\prime}$
16.18 (b)

| A B | $\begin{gathered} \mathrm{J}_{\mathrm{A}} \mathrm{~K}_{\mathrm{A}} \\ \mathrm{X}=0 \quad 1 \end{gathered}$ | A B | $\begin{gathered} \mathrm{J}_{\mathrm{B}} \mathrm{~K}_{\mathrm{B}} \\ \mathrm{X}=0 \quad 1 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| 00 | 1 X 0 X | 00 | 1 X 1 X |
| 01 | 1 X 0 X | 01 | X 1 X 1 |
| 11 | X 1 X 0 | 11 | X 1 X 1 |
| 10 | X 1 X 0 | 10 | 1 X 1 X |

16.18 (b) (contd)

16.18 (c) The state meanings are given in the following table:

| Name | Meaning |
| :---: | :--- |
| $S_{0}$ | reset state |
| $S_{1}$ | even \#0's and even \#1's received |
| $S_{2}$ | odd \#0's and even \#1's received |
| $S_{3}$ | odd \#0's and even \#1's received |
| $S_{4}$ | even \#0's and odd \#1's received |
| $S_{5}$ | even \#0's and odd \#1's received |
| $S_{6}$ | odd \#0's and odd \#1's received |

16.18 (c) (contd)


Guidelines: I: $(0,1) 2 x,(2,3) 2 x,(4,5) 2 x$
II: $(2,5) 2 x,(1,6) 4 x,(3,4)$ An assignment is


| Present <br> State | Next State |  |  |
| :---: | ---: | ---: | :---: |
|  | $\mathrm{X}=0$ | 1 | Z |
| $S_{0}$ | $S_{2}$ | $S_{5}$ | 0 |
| $S_{1}$ | $S_{2}$ | $S_{5}$ | 1 |
| $S_{2}$ | $S_{1}$ | $S_{6}$ | 0 |
| $S_{3}$ | $S_{1}$ | $S_{6}$ | 1 |
| $S_{4}$ | $S_{6}$ | $S_{1}$ | 1 |
| $S_{5}$ | $S_{6}$ | $S_{1}$ | 0 |
| $S_{6}$ | $S_{4}$ | $S_{3}$ | 0 |


|  |  | $A^{+} B^{+} C^{+}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $A B C$ | $X=0 \quad 1$ | Z |  |
| $\mathrm{S}_{0}$ | 000 | 110100 | 0 |  |
| $\mathrm{~S}_{1}$ | 001 | 110100 | 1 |  |
| $\mathrm{~S}_{6}$ | 011 | 101111 | 0 |  |
| -- | 010 | $---\quad---$ | - |  |
| $\mathrm{S}_{5}$ | 100 | 011001 | 0 |  |
| $\mathrm{~S}_{4}$ | 101 | 011001 | 1 |  |
| $\mathrm{~S}_{3}$ | 111 | 001011 | 1 |  |
| $\mathrm{~S}_{2}$ | 110 | 001011 | 0 |  |

Unit 16 Solutions
16.18 (c) (contd)


$T_{C}=A^{\prime} B^{\prime} C+A C^{\prime}$

16.19 (a)


For assignment $S_{0}=00, S_{1}=01, S_{2}=11$ :

|  | $\mathrm{A}^{+} \mathrm{B}^{+}$ |  | Z |  |
| :---: | :--- | ---: | :---: | :---: |
| AB | $\mathrm{X}=0$ | 1 | $X=0$ | $X=1$ |
| 00 | 00 | 01 | 0 | 1 |
| 01 | 00 | 11 | 1 | 0 |
| 11 | 01 | 11 | 0 | 1 |
| 10 | -- | -- | - | - |

16.19 (b)

16.19 (c)

| AB X | $D_{A} D_{B} Z$ |  |  |
| :---: | :--- | :--- | :--- |
| -1 | 1 | 1 | 0 | 0

$D_{\mathrm{A}}=X B$
$D_{\mathrm{B}}=X+A$
$Z=X^{\prime} A^{\prime} B+X B^{\prime}+X A$
16.20 (a)

16.20 (c) Using the assignment $S_{0}=000, S_{1}=001, S_{2}=010, S_{3}=011, S_{4}=100, S_{5}=101$,
$\mathrm{S}_{6}=110, \mathrm{~S}_{7}=111$ :

$$
\begin{aligned}
& D_{0}=\left(X_{2}{ }^{\prime} X_{1} X_{0}+X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(S_{0}+S_{1}+S_{2}+S_{3}+S_{4}+S_{5}+S_{6}\right) \\
& =\left(X_{2}{ }^{\prime} X_{1} X_{0}+X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(Q_{0}{ }^{\prime}+Q_{1}{ }^{\prime}+Q_{2}{ }^{\prime}\right)^{*} \\
& =X_{2}{ }^{\prime} X_{1} X_{0}+X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime *} \\
& D_{1}=\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(S_{0}+S_{1}+S_{3}+S_{5}+S_{4}+S_{6}\right)+\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right) S_{1} \\
& =\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(Q_{1}{ }^{\prime}+Q_{2}{ }^{\prime} Q_{0}+Q_{2} Q_{0}{ }^{\prime}\right)+\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right) Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0} \\
& =\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(Q_{1}{ }^{\prime}+Q_{0}+Q_{2}\right)+\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right) Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}{ }^{*} \\
& D_{2}=\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(S_{2}+S_{4}+S_{6}\right)+\left(X_{2}{ }^{\prime} X_{1}^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(S_{3}+S_{5}\right) \\
& =\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(Q_{2} Q_{0}{ }^{\prime}+Q_{1} Q_{0}{ }^{\prime}\right)+\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(Q_{2}{ }^{\prime} Q_{1} Q_{0}+Q_{2} Q_{1}{ }^{\prime} Q_{0}\right) \\
& =\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(Q_{2} Q_{0}^{\prime}+Q_{1} Q_{0}{ }^{\prime}\right)+\left(X_{2}{ }^{\prime} X_{1}^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(Q_{1} Q_{0}+Q_{2} Q_{0}\right)^{*} \\
& I=S_{5}=Q_{2} Q_{1}{ }^{\prime} Q_{0}=Q_{2} Q_{0}{ }^{*} \quad D=S_{6}=Q_{2} Q_{1} Q_{0}{ }^{\prime}=Q_{2} Q_{1}{ }^{*} \\
& { }^{*} S_{7} \text { never occurs so } 111 \text { is a don't care input combination. }
\end{aligned}
$$


16.21 (b) $D_{0}=X_{2}{ }^{\prime} X_{1} X_{0}+X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime}$
$D_{1}=\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(Q_{0}+Q_{2}+Q_{4}\right)$
$D_{3}=\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right)\left(Q_{1}+Q_{3}\right)$
$D_{2}=\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(Q_{0}+Q_{1}+Q_{3}\right)$
$D_{4}=\left(X_{2} X_{1}+X_{2} X_{0}\right)\left(Q_{2}+Q_{4}\right)$
$I=\left(X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{2}{ }^{\prime} X_{0}{ }^{\prime}\right) Q_{3}$
$D=\left(X_{2} X_{0}+X_{2} X_{1}\right) Q_{4}$

## Unit 16 Solutions

16.21 (c) Using the assignment $\mathrm{S}_{0}=000, \mathrm{~S}_{1}=001, \mathrm{~S}_{2}=010, \mathrm{~S}_{3}=011, \mathrm{~S}_{4}=100$ :

$$
\begin{aligned}
D_{2} & =X_{2} X_{0} Q_{1} Q_{0}{ }^{\prime}+X_{2} X_{1} Q_{1} Q_{0}^{\prime}+X_{2} X_{0} Q_{2}+X_{2} X_{1} Q_{2} \text { or } \\
& =X_{2} X_{0} Q_{2}^{\prime} Q_{1}^{\prime}+X_{2} X_{1} Q_{2}^{\prime} Q_{1}^{\prime}+X_{2} X_{0} Q_{0}+X_{1} X_{0}^{\prime} Q_{0}+X_{2} X_{1}^{\prime} Q_{0} \\
D_{1} & =X_{2} X_{0} Q_{2} Q_{1}^{\prime}+X_{2} X_{1} Q_{2} Q_{1}^{\prime}+X_{2} X_{1} Q_{0}+X_{1} X_{0} Q_{0}+X_{2}^{\prime} X_{0}{ }^{\prime} Q_{0} \\
D_{0} & =X_{2}^{\prime} X_{1}^{\prime}+X_{2} X_{0}^{\prime} \\
I & =X_{2}^{\prime} X_{1}^{\prime} Q_{1} Q_{0}+X_{2}{ }^{\prime} X_{0}{ }^{\prime} Q_{1} Q_{0} \quad D=X_{2} X_{0} Q_{2}+X_{2} X_{1} Q_{2}
\end{aligned}
$$

16.22 (a)


$$
16.22 \text { (b) } \begin{aligned}
& D_{0}=0 \\
& D_{1}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}+Q_{4}+Q_{5}+Q_{6}\right) \\
& D_{2}=\left(X^{\prime} Y^{\prime}+X Y\right) Q_{1} \\
& D_{3}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{2}+Q_{3}\right) \\
& D_{6}=\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{0}+Q_{1}+Q_{2}+Q_{3}\right) \\
& D_{5}=\left(X^{\prime} Y+X Y^{\prime}\right) Q_{6} \\
& D_{4}=\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{4}+Q_{5}\right) \\
& Z_{0}=Q_{1}+Q_{3}+Q_{5} \\
& Z_{1}=Q_{2}+Q_{3}+Q_{6} \\
& Z_{2}=Q_{4}+Q_{5}+Q_{6}
\end{aligned}
$$

16.22 (c) Using the assignment $\mathrm{S}_{0}=000, \mathrm{~S}_{1}=001, \mathrm{~S}_{2}=010, \mathrm{~S}_{3}=011, \mathrm{~S}_{4}=100, \mathrm{~S}_{5}=101, \mathrm{~S}_{6}=110$ :

$$
\begin{aligned}
D_{0} & =\left(X^{\prime} Y^{\prime}+X Y\right)\left(S_{0}+S_{4}+S_{5}+S_{6}+S_{2}+S_{3}\right)+\left(X^{\prime} Y+X Y^{\prime}\right) S_{6}{ }^{\prime} \\
& =\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}^{\prime}+Q_{2}^{\prime} Q_{1}+Q_{2} Q_{1}^{\prime}\right)+\left(X^{\prime} Y+X Y^{\prime}\right) Q_{2} Q_{1} Q_{0}^{\prime}{ }^{\prime} \\
& =\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}^{\prime}+Q_{1}+Q_{2}\right)+\left(X^{\prime} Y+X Y^{\prime}\right) Q_{2} Q_{1}^{*} \\
D_{1} & =\left(X^{\prime} Y^{\prime}+X Y\right)\left(S_{1}+S_{2}+S_{3}\right)+\left(X^{\prime} Y+X Y^{\prime}\right)\left(S_{0}+S_{1}+S_{2}+S_{3}\right) \\
& =\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{2}^{\prime} Q_{1}+Q_{2}^{\prime} Q_{0}\right)+\left(X^{\prime} Y+X Y^{\prime}\right) Q_{2}^{\prime} \\
D_{2} & =\left(X^{\prime} Y+X Y^{\prime}\right)\left(S_{0}+S_{1}+S_{2}+S_{3}+S_{4}+S_{5}+S_{6}\right) \\
& =\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{0}^{\prime}+Q_{1}^{\prime}+Q_{2}\right)^{*}=\left(X^{\prime} Y+X Y^{\prime}\right)^{*} \\
Z_{0} & =S_{1}+S_{3}+S_{5}=Q_{1} Q_{0}+Q_{2}^{\prime} Q_{0}=Q_{0}{ }^{*} \\
Z_{1} & =S_{2}+S_{3}+S_{6}=Q_{2}^{\prime} Q_{1}+Q_{1} Q_{0}^{\prime}=Q_{1}^{*} * \\
Z_{2} & =S_{4}+S_{5}+S_{6}=Q_{2} Q_{1}^{\prime}+Q_{2} Q_{0}^{\prime}=Q_{2}^{*}
\end{aligned}
$$

* S7 never occurs so 111 is a don't care input combination.

16.23 (b) $D_{0}=0$
$D_{1}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}+Q_{5}+Q_{6}\right)$
$D_{2}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{1}+Q_{2}\right)$
$D_{6}=\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{0}+Q_{1}+Q_{2}\right)$
$D_{5}=\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{6}+Q_{5}\right)$
$Z_{0}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}+Q_{2}+Q_{5}\right)+Q_{6}$
$Z_{1}=\left(X^{\prime} Y+X Y^{\prime}\right) Q_{0}+Q_{1}+Q_{2}$
$Z_{2}=\left(X^{\prime} Y+X Y^{\prime}\right)$
16.23 (c) Using the assignment $\mathrm{S}_{0}=000, \mathrm{~S}_{1}=001, \mathrm{~S}_{2}=101$,
$S_{6}=111, S_{5}=011$
$D_{2}=Q_{1}{ }^{\prime} Q_{0}+X Y^{\prime} Q_{1}{ }^{\prime}+X^{\prime} Y Q_{1}{ }^{\prime}$ or
$=Q_{1}{ }^{\prime} Q_{0}+X Y^{\prime} Q_{0}{ }^{\prime}+X^{\prime} Y Q_{1}{ }^{\prime}$ or
$=Q_{1}{ }^{\prime} Q_{0}+X Y^{\prime} Q_{1}{ }^{\prime}+X^{\prime} Y Q_{0}{ }^{\prime}$ or
$=Q_{1}{ }^{\prime} Q_{0}+X Y^{\prime} Q_{0}{ }^{\prime}+X^{\prime} Y Q_{0}{ }^{\prime}$
$D_{1}=X^{\prime} Y+X Y^{\prime}$
$D_{0}=1$
$Z_{2}=X^{\prime} Y+X Y^{\prime}$
$Z_{1}=Q_{1}{ }^{\prime} Q_{0}+X Y^{\prime} Q_{1}{ }^{\prime}+X^{\prime} Y Q_{1}{ }^{\prime}$ or
$=Q_{1}{ }^{\prime} Q_{0}+X Y^{\prime} Q_{0}{ }^{\prime}+X^{\prime} Y Q_{1}{ }^{\prime}$ or
$=Q_{1} Q_{0}+X Y^{\prime} Q_{1}^{\prime}+X^{\prime} Y Q_{0}{ }^{\prime}$ or
$=Q_{1} Q_{0}+X Y^{\prime} Q_{0}{ }^{\prime}+X^{\prime} Y Q_{0}{ }^{\prime}$
$Z_{0}=X^{\prime} Y^{\prime} Q_{0}{ }^{\prime}+X Y Q_{0}{ }^{\prime}+X^{\prime} Y^{\prime} Q_{2}+X Y Q_{2}$
$+Q_{2} Q_{1}+X^{\prime} Y^{\prime} Q_{1}+X Y Q_{1}$
16.24 (b) $D_{0}=0$
$D_{1}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}+Q_{4}+Q_{5}+Q_{6}+Q_{7}\right)$
$D_{2}=\left(X^{\prime} Y^{\prime}+X Y\right) Q_{1}$
$D_{3}=\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{2}+Q_{3}\right)$
$D_{7}=\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{0}+Q_{1}+Q_{2}+Q_{3}\right)$
$D_{6}=\left(X^{\prime} Y+X Y^{\prime}\right) Q_{7}$
$D_{5}=\left(X^{\prime} Y+X Y^{\prime}\right) Q_{6}$
$D_{4}=\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{4}+Q_{5}\right)$
$Z_{0}=Q_{1}+Q_{3}+Q_{5}+Q_{7}$
$Z_{1}=Q_{2}+Q_{3}+Q_{6}+Q_{7}$
$Z_{2}=Q_{4}+Q_{5}+Q_{6}+Q_{7}$
16.25 (a)

16.24 (a)

16.24 (c) Using the assignment $\mathrm{S}_{0}=000, \mathrm{~S}_{1}=001, \mathrm{~S}_{2}=010$,

$$
\begin{aligned}
\mathrm{S}_{3}= & 011, \mathrm{~S}_{4}=100, \mathrm{~S}_{5}=101, \mathrm{~S}_{6}=110, \mathrm{~S}_{7}=111: \\
D_{0}= & \left(X^{\prime} Y^{\prime}+X Y\right)\left(S_{0}+S_{4}+S_{5}+S_{6}+S_{7}+S_{2}+\right. \\
& \left.S_{3}\right)+\left(X^{\prime} Y+X Y^{\prime}\right)\left(S_{0}+S_{1}+S_{2}+S_{3}+S_{6}\right) \\
= & \left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}^{\prime}+Q_{1}+Q_{2}\right)+ \\
& \left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{2}^{\prime}+Q_{1} Q_{0}{ }^{\prime}\right) \\
D_{1}= & \left(X^{\prime} Y^{\prime}+X Y\right)\left(S_{1}+S_{2}+S_{3}\right)+ \\
& \left(X^{\prime} Y+X Y^{\prime}\right)\left(S_{0}+S_{1}+S_{2}+S_{3}+S_{7}\right) \\
= & \left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{2}^{\prime} Q_{1}+Q_{2}^{\prime} Q_{0}\right)+ \\
& \left(X^{\prime} Y+X Y\right)\left(Q_{2}^{\prime}+Q_{1} Q_{0}\right) \\
D_{2}= & \left(X^{\prime} Y+X Y^{\prime}\right) \\
& \\
Z_{0}= & S_{1}+S_{3}+S_{5}+S_{7}=Q_{0} \\
Z_{1}= & S_{2}+S_{3}+S_{6}+S_{7}=Q_{1} \\
Z_{2}= & S_{4}+S_{5}+S_{6}+S_{7}=Q_{2}
\end{aligned}
$$

16.25 (b) $D_{0}=0$

$$
\begin{aligned}
D_{1}= & \left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{0}+Q_{5}+Q_{6}+Q_{7}\right) \\
D_{2}= & \left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{1}+Q_{2}\right) \\
D_{7}= & \left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{0}+Q_{1}+Q_{2}\right) \\
D_{6}= & \left(X^{\prime} Y+X Y^{\prime}\right) Q_{7} \\
D_{5}= & \left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{5}+Q_{6}\right) \\
Z_{0}= & Q_{0}+\left(X^{\prime} Y^{\prime}+X Y\right)\left(Q_{2}+Q_{5}\right) \\
& +\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{1}+Q_{6}\right) \\
Z_{1}= & Q_{1}+Q_{2}+\left(X^{\prime} Y+X Y^{\prime}\right)\left(Q_{0}+Q_{7}\right) \\
Z_{2}= & \left(X^{\prime} Y+X Y^{\prime}\right)
\end{aligned}
$$

## Unit 16 Solutions

16.25 (c) Using the assignment $\mathrm{S}_{0}=000, \mathrm{~S}_{1}=001, \mathrm{~S}_{2}=100$,
$S_{5}=011, S_{6}=010, S_{7}=111$ :
$D_{2}=X^{\prime} Y Q_{1}{ }^{\prime}+X Y^{\prime} Q_{1}{ }^{\prime}+Q_{1}{ }^{\prime} Q_{0}$
$D_{1}=X^{\prime} Y+X Y^{\prime}$
$D_{0}=Q_{1}{ }^{\prime}+X^{\prime} Y^{\prime}+X Y+Q_{2}{ }^{\prime}$
$Z_{2}=X^{\prime} Y+X Y^{\prime}$
$Z_{1}=X^{\prime} Y Q_{1}{ }^{\prime}+X Y^{\prime} Q_{1}{ }^{\prime}+Q_{1}{ }^{\prime} Q_{0}+X^{\prime} Y Q_{2}+X Y^{\prime} Q_{2}$
$Z_{0}=Q_{0}{ }^{\prime}+X^{\prime} Y Q_{1}{ }^{\prime}+X Y^{\prime} Q_{1}{ }^{\prime}+X^{\prime} Y^{\prime} Q_{1}+X Y Q_{1}$ $+Q_{2} Q_{1}{ }^{\prime}$
16.26 (b)

16.26 (c) With the state assignment $S_{0}=00, S_{1}=01, S_{2}=10, S_{3}=11$, we have:

16.27 (a)


Outputs: LC, $L B, L A, R A, R B, R C$
16.27 (b) First, assign $L C=Q_{1}, L B=Q_{2}, L A=Q_{3}, R A=Q_{4}, R B=Q_{5}, R C=Q_{6}$. So $S_{0}=000000, S_{1}=001000, S_{2}=011000$, etc.

This state machine has too many state variables to use Karnaugh maps. Instead, we will write down equations for each flip-flop by inspection.

First consider $Q_{1} . Q_{1}=1$ in states $S_{3}$ or $S_{7}$ only.

- $\quad S_{7}$ is reached whenever $H=1$ and we are not already in $S_{7}: H\left(Q_{1} Q_{2} Q_{3} Q_{4} Q_{5} Q_{6}\right)^{\prime}$. But $S_{7}$ is the only state in which both $Q_{3}=1$ and $Q_{4}=1$, so assuming we are always in a valid state, we can use $H\left(Q_{3} Q_{4}\right)^{\prime}=H Q_{3}^{\prime}+H Q_{4}^{\prime}$. Note: Any combination of one left light and one right light will also work, i.e. $H Q_{1}^{\prime}+H Q_{5}^{\prime}$.
- $S_{3}$ is reached whenever we are in $S_{2}$ and $L=1$ while $H=0: L H^{\prime} Q_{1}^{\prime} Q_{2} Q_{3} Q_{4}^{\prime} Q_{5}^{\prime} Q_{6}^{\prime}$. But $Q_{3}=1$ whenever $Q_{2}=1$, and $Q_{4}=Q_{5}=Q_{6}=0$ whenever $Q_{1}=0$. So we can use $L H^{\prime} Q_{1}^{\prime} Q_{2}$.
- $\quad$ So $D_{1}=L H^{\prime} Q_{1}^{\prime} Q_{2}+H Q_{3}^{\prime}+H Q_{4}^{\prime}=L Q_{1}^{\prime} Q_{2}+H Q_{3}^{\prime}+H Q_{4}^{\prime}\left(\right.$ using $\left.X+X^{\prime} Y=X+Y\right)$ Similarly $Q_{2}=1$ in states $S_{3}, S_{2}$, and $S_{7}$ only.
- $\quad S_{3}$ and $S_{2}$ are reached whenever we are in $S_{2}$ or $S_{1}$ and $L=1$ while $H=0$.

$$
L H^{\prime} Q_{1}^{\prime} Q_{2} Q_{3} Q_{4}^{\prime} Q_{5}^{\prime} Q_{6}^{\prime}+L H^{\prime} Q_{1}^{\prime} Q_{2}^{\prime} Q_{3} Q_{4}^{\prime} Q_{5}^{\prime} Q_{6}^{\prime}=L H^{\prime} Q_{1}^{\prime} Q_{3} Q_{4}^{\prime} Q_{5}^{\prime} Q_{6}^{\prime}
$$

But again, $Q_{4}=Q_{5}=Q_{6}=0$ whenever $Q_{1}=0$, so $D_{2}=L Q_{1} '^{\prime} Q_{3}+H Q_{3}^{\prime}+H Q_{4}^{\prime}$
We can also get by inspection: $D_{3}=L Q_{1}^{\prime} Q_{4}^{\prime}+H Q_{3}^{\prime}+H Q_{4}^{\prime} ; \quad D_{4}=R Q_{3}^{\prime} Q_{6}{ }^{\prime}+H Q_{3}^{\prime}+H Q_{4}^{\prime}$;
$D_{5}=R Q_{4} Q_{6}^{\prime}+H Q_{3}^{\prime}+H Q_{4}^{\prime} ; \quad D_{6}=R Q_{5} Q_{6}^{\prime}+H Q_{3}^{\prime}+H Q_{4}^{\prime}$
16.27 (c)

| State | $L R H=000$ | 001 | 010 | 011 | 100 | 101 | 110 | 111 | LC LB LA RA RB RC |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{0}$ | $S_{0}$ | $S_{7}$ | $S_{4}$ | $S_{7}$ | $S_{1}$ | $S_{7}$ | - | - | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $S_{1}$ | $S_{0}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | $S_{2}$ | $S_{7}$ | - | - | 0 | 0 | 1 | 0 | 0 | 0 |  |
| $S_{2}$ | $S_{0}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | $S_{3}$ | $S_{7}$ | - | - | 0 | 1 | 1 | 0 | 0 | 0 |  |
| $S_{3}$ | $S_{0}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | - | - | 1 | 1 | 1 | 0 | 0 | 0 |  |
| $S_{4}$ | $S_{0}$ | $S_{7}$ | $S_{5}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | - | - | 0 | 0 | 0 | 1 | 0 | 0 |  |
| $S_{5}$ | $S_{0}$ | $S_{7}$ | $S_{6}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | - | - | 0 | 0 | 0 | 1 | 1 | 0 |  |
| $S_{6}$ | $S_{0}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | $S_{0}$ | $S_{7}$ | - | - | 0 | 0 | 0 | 1 | 1 | 1 |  |
| $S_{7}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{0}$ | - | - | 1 | 1 | 1 | 1 | 1 | 1 |  |

I. $\left(S_{0}, S_{1}, S_{2}, S_{3}, S_{4}, S_{5}, S_{6}\right)$ for $S_{7}$ in $L R H=001,011,101$

$$
\begin{aligned}
& \left(S_{1}, S_{2}, S_{3}, S_{6}, S_{7}\right) \text { for } S_{0} \text { in } L R H=010 \\
& \left(S_{3}, S_{4}, S_{5}, S_{6}, S_{7}\right) \text { for } S_{0} \text { in } L R H=100
\end{aligned}
$$

II. Every state matches $S_{0}$ and $S_{7}$. But $S_{0}$ and $S_{7}$ match the best, so $\left(S_{0}, S_{7}\right) \times$ (many times)
III. $\left(S_{1}, S_{2}, S_{3}, S_{7}\right)\left(S_{4}, S_{5}, S_{6}, S_{7}\right)$ etc.

From LogicAid:
$D_{1}=H Q_{2}+R Q_{1} Q_{2} Q_{3}^{\prime}+H Q_{3}+L Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}+H Q_{1}^{\prime}+R Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}^{\prime}$

$D_{2}=R H^{\prime} Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}^{\prime}+R H^{\prime} Q_{1} Q_{2}+L H^{\prime} Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}^{\prime}$
$D_{3}=L H^{\prime} Q_{1}^{\prime} Q_{2} Q_{3}^{\prime}+L H^{\prime} Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}+R H^{\prime} Q_{1} Q_{2}$
$L \stackrel{C}{C}=Q_{1} Q_{2}^{\prime} ; \quad L B=Q_{1} Q_{2}^{\prime}+Q_{2}^{\prime} Q_{3} ; \quad L A=Q_{1} Q_{2}^{\prime}+Q_{2}^{\prime} Q_{3}+Q_{1}^{\prime} Q_{2} Q_{3}^{\prime}$
$R C=Q_{1} Q_{2}^{\prime} Q_{3}^{\prime}+Q_{1}^{\prime} Q_{2} Q_{3} ; R B=Q_{1} Q_{2}^{\prime} Q_{3}^{\prime}+Q_{2} Q_{3} ; \quad R A=Q_{1} Q_{3}^{\prime}+Q_{2} Q_{3}$
Other minimum solutions can be found for $D_{2}$ and $D_{3}$ with this assignment.


Note: This state graph assumes that only one of the buttons $S T, P L, R E$, and $F F$ can be pressed at any given time. The graph is incompletely specified and must be augmented before using LogicAid. For example, the arc from REW to PLAY should be labeled PLST' $F F^{\prime}$.


$$
\begin{aligned}
D_{1}= & S T^{\prime} F F P S Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}+S T^{\prime} R E P L Q_{1}^{\prime} Q_{2}^{\prime} Q_{3} \\
& +S T^{\prime} M Q_{1} \\
D_{2}= & S T^{\prime} F F Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}^{\prime}+S T^{\prime} R E Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}^{\prime} \\
& +S T^{\prime} R E^{\prime} P L^{\prime} Q_{2} Q_{3}+S T^{\prime} F F^{\prime} P L^{\prime} Q_{2} Q_{3}^{\prime} \\
D_{3}= & S T^{\prime} R E^{\prime} F F Q_{1}^{\prime} Q_{2}^{\prime} Q_{3}^{\prime}+S T^{\prime} R E^{\prime} F F^{\prime} Q_{3} \\
& +S T^{\prime} F F^{\prime} P L Q_{2} Q_{3}^{\prime}+S T^{\prime} R E^{\prime} Q_{2} Q_{3} \\
& +S T^{\prime} M^{\prime} Q_{1}+S T^{\prime} Q_{1} Q_{3}+S T^{\prime} R E^{\prime} P L Q_{1}^{\prime} Q_{2}^{\prime} \\
P= & Q_{1}^{\prime} Q_{2}^{\prime} Q_{3} ; \\
R= & Q_{2} Q_{3}^{\prime}+Q_{1} Q_{3}^{\prime} ; \\
F= & Q_{2} Q_{3}+Q_{1} Q_{3}
\end{aligned}
$$

16.29 (a)

16.29 (b)

| Present <br> State | Next State <br> $\mathrm{X}=0$ |  | 1 |
| :---: | :---: | :---: | :---: | Z


|  |  | $A^{+} B^{+} C^{+}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $A B C$ | $X=0 \quad 1$ | Z |  |
| $\mathrm{S}_{0}$ | 000 | 000001 | 0 |  |
| $\mathrm{~S}_{1}$ | 001 | 010011 | 0 |  |
| $\mathrm{~S}_{2}$ | 010 | 000101 | 0 |  |
| $\mathrm{~S}_{3}$ | 011 | 100011 | 0 |  |
| $\mathrm{~S}_{5}$ | 100 | 000110 | 0 |  |
| $\mathrm{~S}_{4}$ | 101 | 010110 | 0 |  |
| $\mathrm{~S}_{6}$ | 110 | 110110 | 1 |  |
| -- | 111 | $---\quad--$ | - |  |


$\mathrm{a}_{\mathrm{i}+1}=\mathrm{x}_{\mathrm{i}}{ }^{\prime} \mathrm{b}_{\mathrm{i}} \mathrm{c}_{\mathrm{i}}+\mathrm{a}_{\mathrm{i}} \mathrm{b}_{\mathrm{i}}+\mathrm{x}_{\mathrm{i}} \mathrm{b}_{\mathrm{i}} \mathrm{c}_{\mathrm{i}}{ }^{\prime}+\mathrm{x}_{\mathrm{i}} \mathrm{a}_{\mathrm{i}}$

$\mathrm{b}_{\mathrm{i}+1}=\mathrm{b}_{\mathrm{i}} \mathrm{c}_{\mathrm{i}}+\mathrm{a}_{\mathrm{i}} \mathrm{b}_{\mathrm{i}}+\mathrm{x}_{\mathrm{i}} \mathrm{c}_{\mathrm{i}}+\mathrm{x}_{\mathrm{i}} \mathrm{a}_{\mathrm{i}}$

$\mathrm{c}_{\mathrm{i}+1}=\mathrm{x}_{\mathrm{i}} \mathrm{a}_{\mathrm{i}}{ }^{\prime}$
16.29 (c)

16.30 (a)


## Unit 16 Solutions

16.30 (b)

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $x_{\mathrm{i}}=0$ | $x_{\mathrm{i}}=1$ | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 0 |
| $S_{1}$ | $S_{2}$ | $S_{3}$ | 1 |
| $S_{2}$ | $S_{2}$ | $S_{1}$ | 1 |
| $S_{3}$ | $S_{3}$ | $S_{3}$ | 0 |



$\mathrm{a}_{\mathrm{i}+1}=\left(\mathrm{x}_{\mathrm{i}}+\mathrm{a}_{\mathrm{i}}\right)\left(\mathrm{x}_{\mathrm{i}}{ }^{\prime}+\mathrm{b}_{\mathrm{i}}{ }^{\prime}\right)$

$b_{i+1}=\left(x_{i}+b_{i}\right)\left(x_{i}+a_{i}^{\prime}\right)$

16.30 (c) $a_{1}=b_{1}=0$

$$
a_{2}=\left(x_{1}+0\right)\left(x_{1}^{\prime}+1\right)=x_{1}
$$

$$
b_{2}=\left(x_{1}+1\right)\left(x_{1}+0\right)=x_{1}
$$

16.30 (d)


## Unit 17 Problem Solutions

17.1 See FLD p. 731 for solution. 17.2 See FLD p. 732 for solution.
17.3
(a, b)


See FLD p. 732-733 for solutions.
17.4 See FLD p. 733-734 for solution.
17.5 See FLD p. 734 for solution.
17.6 See FLD p. 734-735 for solutions.
(a, b)

17.7 (a) See FLD p. 736 for solution.
17.7 (b)

17.8 See FLD p. 738 for solution.

## Unit 17 Solutions

17.11 A rising edge triggered D-CE flip flop with asynchronous clear and preset.
17.12


```
17.10 library IEEE;
    use IEEE.STD_LOGIC_1164.ALL;
    use IEEE.STD_LOGIC_ARITH.ALL;
    use IEEE.STD_LOGIC_UNSIGNED.ALL;
    -- D-G Latch
    entity dglatch is
        port (d, g : in bit;
        q:out bit);
    end dglatch;
    architecture Behavioral of dglatch is
        begin
        process(g, d)
        begin
        if g='1' then q <= d; end if;
        end process;
end Behavioral;
-- D flip flop using D-G latches
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity dff is
    port (d, clk : in bit;
        q : out bit);
end dff;
architecture Behavioral of dff is
component dglatch is
    port (d, g: in bit;
        q : out bit);
end component;
signal p, clkn : bit;
    begin
    clkn <= not clk;
    dg1 : dglatch port map(d, clkn, p);
    dg2 : dglatch port map(p, clk, q);
end Behavioral;
```

```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity myreg is
    port(en, ld, clk : in std_logic;
        d : in std_logic_vector(7 downto 0);
        q : out std_logic_vector(7 downto 0));
end myreg;
architecture Behavioral of myreg is
signal qint : std_logic_vector(7 downto 0):="00000000";
    begin
    q <= qint when en ='1' else "ZZZZZZZZ";
    process(clk)
            begin
            if clk' event and clk='1' then
                if Id='1' then qint <= d; end if;
            end if;
    end process;
end Behavioral;
```

```
17.13 library IEEE;
    use IEEE.STD_LOGIC_1164.ALL;
    use IEEE.STD_LOGIC_ARITH.ALL;
    use IEEE.STD_LOGIC_UNSIGNED.ALL;
    entity encoder is
    port (y0, y1, y2, y3 : in bit;
        a, b, c : out bit);
end encoder;
architecture Behavioral of encoder is
    begin
    process(y0, y1, y2, y3)
        begin
        if y3='1' then a <= '1'; b <= '1'; c <= '1';
            -- y3 has highest priority
        elsif y2='1' then
            a <= '1'; b <= '0'; c <= '1';
        elsif y1='1' then
            a <= '0'; b <= '1'; c <= '1';
        elsif y0='1' then
            a <= '0'; b <= '0'; c <= '1';
        else a <= '0'; b <= '0'; c <= '0'; end if;
    end process;
end Behavioral;
17.15 library IEEE;
    use IEEE.STD_LOGIC_1164.ALL;
    use IEEE.STD_LOGIC_ARITH.ALL;
    use IEEE.STD_LOGIC_UNSIGNED.ALL;
    entity super is
    port (a: in std_logic_vector(2 downto 0);
        d : in std_logic_vector(5 downto 0);
        rsi, Isi, clk : in std_logic;
        q : out std_logic_vector(5 downto 0));
    end super;
    architecture Behavioral of super is
    signal qint: std_logic_vector(5 downto 0);
    begin
    q <= qint;
    process(clk)
        begin
        if clk' event and clk='1' then
            case a is
            when "111"=> qint <= d;
            when "110"=> qint <= qint-1;
            when "101"=> qint <= qint+1;
            when "100"=> qint <= "111111";
            when "011"=> qint <= "000000";
            when "010"=> qint <= rsi&qint(5 downto 1);
            when "001"=> qint <= qint(4 downto 0)&|si;
            when others=> NULL;
            end case;
        end if;
    end process;
end Behavioral;
```

Unit 17 Solutions

```
17.17 (a)
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity Mealy_XOR is
    Port (CLK, clr, x : in std_logic;
        z : out std_logic);
end Mealy_XOR;
architecture df1 of Mealy_XOR is
signal q, d: std_logic;
begin
\(\mathrm{z}<=\mathrm{x}\) XOR q after 10ns;
d <= x ;
process (CLK, clr)
    begin
        if \(\mathrm{clr}=\) ' 0 ' then
            q <= '0' after 10ns;
        elsif CLK'event and CLK = ' 1 '
                then q <= d after 10ns;
        end if;
    end process;
end df1;
```

17.17 (c)

```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity Moore_XOR is
    Port (CLK, clr, X : in std_logic;
        Z : out std_logic);
end Moore_XOR;
architecture df1 of Moore_XOR is
signal Q1, Q2, D1, D2: std_logic;
begin
Z <= Q1 XOR Q2 after 10ns;
D1 <= X;
D2 <= Q1;
process (CLK, clr)
        begin
            if clr = '0' then
            Q1 <= '0' after 10ns;
            Q2 <= '0' after 10ns;
            elsif CLK'event and CLK = '1'
            then Q1 <= D1 after 10ns;
                    Q2 <= D2 after 10ns;
            end if;
    end process;
end df1;
```


17.17 (e) The Mealy model output is valid before the positive clock edge while the corresponding Moore model output (contd) becomes valid after the clock edge. Also, the Mealy output is not valid after the clock edge until the input has changed to its next value. The Meal model does not have an output corresponding to the Moore output prior to the first clock edge.

17.18 (a) )Z0 = Q0 Q1'Q3' or Q1'Q2'Q3'
$\mathrm{Z} 1=\mathrm{Q} 0 \mathrm{Q} 1$
$\mathrm{Z} 2=\mathrm{Q} 0{ }^{\prime} \mathrm{Q} 1 \mathrm{Q} 2$ ' or Q0'Q2'Q3'
Z3 = Q1 Q2
Z4 = Q1'Q2 Q3' or Q0'Q1'Q3'
$\mathrm{Z} 5=\mathrm{Q} 2 \mathrm{Q} 3$
Z6 = Q0'Q2'Q3 or Q0'Q1'Q2'
$\mathrm{Z} 7=\mathrm{Q} 0 \mathrm{Q} 3$
17.18 (b) D0 = Q1'Q2'
$\mathrm{D} 1=\mathrm{Q} 2 \mathrm{Q}^{\prime}$
D2 = Q0'Q3'
D3 = Q0'Q1'
17.18 (c) $\mathrm{CE} 0=\mathrm{Q} 2 ', \mathrm{D} 0=\mathrm{Q} 1$ ' or $\mathrm{CE} 0=\mathrm{Q} 1 ', \mathrm{D} 0=\mathrm{Q}^{\prime}$
$\mathrm{CE} 1=\mathrm{Q} 3 ', \mathrm{D} 1=\mathrm{Q} 2$ ' or $\mathrm{CE} 1=\mathrm{Q} 2 ', \mathrm{D} 1=\mathrm{Q} 3^{\prime}$
$\mathrm{CE} 2=\mathrm{Q} 3 ', \mathrm{D} 2=\mathrm{Q} 0^{\prime}$ or $\mathrm{CE} 2=\mathrm{Q} 0^{\prime}, \mathrm{D} 2=\mathrm{Q} 3^{\prime}$
$\mathrm{CE} 3=\mathrm{Q} 1 ', \mathrm{D} 3=\mathrm{Q} 0$ ' or $\mathrm{CE} 3=\mathrm{Q} 0 ', \mathrm{D} 3=\mathrm{Q} 1^{\prime}$
17.18 (d) stt_trnstn: process(CLK,CIrN) (contd)

[^0]17.18 (d) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
entity mod8_counter is
port (CLK, CIrN : in std_logic;
Z0, Z1, Z2, Z3, Z4, Z5, Z̄6, Z7 : out std_logic);
end mod8_counter;
architecture bhvr of mod8_counter is
signal Q, Q_plus: std_logic_vector(0 to 3);
begin
cmb_Igc: process(Q)
begin
Z0 <= '0'; Z1 <= '0'; Z2 <= '0'; Z3 <= '0';
Z4 <= '0'; Z5 <= '0'; Z6 <= '0'; Z7 <= '0';
case Q is
when "1000" =>
Z0 <= ' 1 ';
Q_plus <= "1100";
when "1100" =>
Z1 <= '1';
Q_plus <= "0100";
when "0100" =>
Z2 <= '1';
Q_plus <= "0110";
when "0110" =>
Z3 <= '1';
Q_plus <= "0010";
when "0010" =>
Z4 <= '1';
Q_plus <= "0011";
when "0011" => Z5 <= '1';
Q_plus <= "0001";
when "0001" =>
Z6 <= ' 1 ';
Q_plus <= "1001";
when "1001" =>
Z7 <= '1'; Q_plus <= "1000";
when others =>
Q_plus <= "XXXX";
end case;
end process cmb_lgc;

## Unit 17 Solutions

17.18 (e) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
entity mod8_counter is port (CLK, ClrN : in std_logic;
Z0, Z1, Z2, Z3, Z4, Z5, Z6, Z7 : out std_logic);
end mod8_counter;
architecture df1 of mod8_counter is
signal Q, D : std_logic_vector(0 to 3);
begin
cmb_lgc: process(Q)
begin
$\mathrm{Z} 0<=\mathrm{Q}(0)$ and not $\mathrm{Q}(1)$ and not $\mathrm{Q}(3)$;
$\mathrm{Z} 1<=\mathrm{Q}(0)$ and $\mathrm{Q}(1)$;
$\mathrm{Z} 2<=\operatorname{not} \mathrm{Q}(0)$ and $\mathrm{Q}(1)$ and not $\mathrm{Q}(2)$;
$\mathrm{Z} 3<=\mathrm{Q}(1)$ and $\mathrm{Q}(2)$
$\mathrm{Z} 4<=$ not $\mathrm{Q}(1)$ and $\mathrm{Q}(2)$ and not $\mathrm{Q}(3)$;
$\mathrm{Z} 5<=\mathrm{Q}(2)$ and $\mathrm{Q}(3)$;
$\mathrm{Z} 6<=$ not $\mathrm{Q}(0)$ and not $\mathrm{Q}(2)$ and $\mathrm{Q}(3)$;
$\mathrm{Z7}<=\mathrm{Q}(0)$ and $\mathrm{Q}(3)$
$D(0)<=$ not $Q(1)$ and not $Q(2)$;
$D(1)<=$ not $Q(2)$ and not $Q(3)$;
$D(2)<=$ not $Q(0)$ and not $Q(3)$;
$\mathrm{D}(3)<=$ not $\mathrm{Q}(0)$ and not $\mathrm{Q}(1)$;
end process cmb_lgc;
stt_trnstn: process(CLK,ClrN)
begin
if $\mathrm{ClrN}=$ = 0 ' then
Q <= "1000";
elsif Rising_Edge (CLK) then
Q <= D;
end if;
end process stt_trnstn;
end df1;

```
17.18 (f) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
entity mod8_counter is
    port (CLK, ClrN : in std_logic;
    Z0, Z1, Z2, Z3, Z4, Z5, Z6, Z7 : out std_logic);
end mod8_counter;
architecture df2 of mod8_counter is
    signal Q, CE, D : std_logic_vector(0 to 3);
begin
    cmb_lgc: process(Q)
    begin
    Z0<= Q(0) and not Q(1) and not Q(3);
    Z1<= Q(0) and Q(1);
    Z2 <= not Q(0) and Q(1) and not Q(2);
    Z3 <= Q(1) and Q(2);
    Z4 <= not Q(1) and Q(2) and not Q(3);
    Z5 <= Q(2) and Q(3);
    Z6 <= not Q(0) and not Q(2) and Q(3);
    Z7 <= Q(0) and Q(3);
    CE(0) <= not Q(2); D(0) <= not Q(1);
    CE(1) <= not Q(3); D(1) <= not Q(2);
    CE(2) <= not Q(0); D(2) <= not Q(3);
    CE(3)<= not Q(1); D(3) <= not Q(0);
end process cmb_lgc;
    stt_trnstn: process(CLK,ClrN)
    begin
        if ClrN = '0' then
        Q <= "1000";
        elsif Rising_Edge (CLK) then
                if CE(0) = '1' then Q(0) <= D(0); end if;
                if CE(1) = '1' then Q (1) <= D(1); end if;
                if CE(2)= '1' then Q(2)<= D(2); end if;
                if CE (3) = '1' then Q (3) <= D(3); end if;
        end if;
    end process stt_trnstn;
```

end df2;



## Unit 17 Solutions

17.19 (e) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
entity mod8_counter2 is port (CLK, ClrN : in std_logic;
Z0, Z1, Z2, Z3, Z4, Z5, Z6, Z7 : out std_logic);
end mod8_counter2;
architecture df1 of mod8_counter2 is
signal Q, D : std_logic_vector(0 to 3);
begin
cmb_lgc: process(Q)
begin
$\mathrm{Z} 0<=\mathrm{Q}(0)$ and not $\mathrm{Q}(1)$ and not $\mathrm{Q}(3)$;
$\mathrm{Z} 1<=\mathrm{Q}(1)$ and not $\mathrm{Q}(2)$;
$\mathrm{Z} 2<=\mathrm{Q}(0)$ and $\mathrm{Q}(1)$ and $\mathrm{Q}(2)$;
$\mathrm{Z} 3<=$ not $\mathrm{Q}(0)$ and $\mathrm{Q}(1)$;
$\mathrm{Z} 4<=$ not $\mathrm{Q}(1)$ and $\mathrm{Q}(2)$ and not $\mathrm{Q}(3)$;
$\mathrm{Z} 5<=$ not $\mathrm{Q}(0)$ and $\mathrm{Q}(3)$;
$\mathrm{Z} 6<=\mathrm{Q}(0)$ and $\mathrm{Q}(2)$ and $\mathrm{Q}(3)$;
Z 7 <= not $\mathrm{Q}(2)$ and $\mathrm{Q}(3)$;
$\mathrm{D}(0)<=\mathrm{Q}(3)$ or not $\mathrm{Q}(2)$;
$D(1)<=Q(0)$ and not $Q(3)$;
$D(2)<=$ not $Q(0)$ or $Q(1)$;
$\mathrm{D}(3)<=$ not $\mathrm{Q}(1)$ and $\mathrm{Q}(2)$;
end process cmb_lgc;
stt_trnstn: process(CLK,ClrN)
begin
if $\mathrm{ClrN}=\mathrm{O} 0$ ' then Q <= "1000"; elsif Rising_Edge (CLK) then Q <= D;
end if;
end process stt_trnstn;
end df1;
17.19 (f) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
entity mod8_counter2 is
port (CLK, ClrN : in std_logic;
Z0, Z1, Z2, Z3, Z4, Z5, Z6, Z7 : out std_logic); end mod8_counter2;
architecture df2 of mod8_counter2 is signal Q, CE, D : std_logic_vector(0 to 3); begin
cmb_lgc: process(Q)
begin
$\mathrm{Z} 0<=\mathrm{Q}(0)$ and not $\mathrm{Q}(1)$ and not $\mathrm{Q}(3)$;
$\mathrm{Z} 1<=\mathrm{Q}(1)$ and not $\mathrm{Q}(2)$;
$\mathrm{Z} 2<=\mathrm{Q}(0)$ and $\mathrm{Q}(1)$ and $\mathrm{Q}(2)$;
$\mathrm{Z} 3<=$ not $\mathrm{Q}(0)$ and $\mathrm{Q}(1)$;
$\mathrm{Z} 4<=$ not $\mathrm{Q}(1)$ and $\mathrm{Q}(2)$ and not $\mathrm{Q}(3)$;
$\mathrm{Z} 5<=$ not $\mathrm{Q}(0)$ and $\mathrm{Q}(3)$;
$\mathrm{Z} 6<=\mathrm{Q}(0)$ and $\mathrm{Q}(2)$ and $\mathrm{Q}(3)$;
Z 7 <= not $\mathrm{Q}(2)$ and $\mathrm{Q}(3)$;
$\mathrm{CE}(0)<=\mathrm{Q}(2)$; $\mathrm{D}(0)<=\mathrm{Q}(3)$;
$\mathrm{CE}(1)<=\operatorname{not} \mathrm{Q}(3) ; \mathrm{D}(1)<=\mathrm{Q}(0)$;
CE(2) <= Q (0); D(2) <= Q(1);
$\mathrm{CE}(3)<=\operatorname{not} \mathrm{Q}(1) ; \mathrm{D}(3)<=\mathrm{Q}(2)$;
end process cmb_lgc;
stt_trnstn: process(CLK,ClrN)
begin
if $\mathrm{ClrN}=$ ' 0 ' then
Q <= "1000";
elsif Rising_Edge (CLK) then if $\mathrm{CE}(0)=$ ' 1 ' then $\mathrm{Q}(0)<=\mathrm{D}(0)$; end if; if $\mathrm{CE}(1)=$ ' 1 ' then $\mathrm{Q}(1)<=\mathrm{D}(1)$; end if; if $C E(2)=$ ' 1 ' then $\mathrm{Q}(2)<=\mathrm{D}(2)$; end if; if $\mathrm{CE}(3)=$ ' 1 ' then $\mathrm{Q}(3)<=\mathrm{D}(3)$; end if; end if;
end process stt_trnstn;
end df2;
17.19 (e)



| 17.21 (a) | $\begin{aligned} & \text { so = si AND a' } \\ & \text { b = si AND a } \end{aligned}$ |
| :---: | :---: |
|  |  |
| 17.21(c) | library IEEE; <br> use IEEE.STD_LOGIC_1164.ALL; <br> entity pr_sel_1bit is <br> port (a, si : in std_logic; <br> b, so : out std_logic); <br> end pr_sel_1bit; <br> architecture prdf of pr_sel_1bit is begin <br> so <= si and not a; $\mathrm{b}<=$ si and a ; <br> end prdf; <br> library IEEE; <br> use IEEE.STD_LOGIC_1164.ALL; <br> -- Code for the 4 bit priority selector entity pr_sel_4bit is port( x : in std_logic_vector(3 downto 0); isel : in std_logic; y : out std_logic_vector(3 downto 0); osel : out std_logic); <br> end pr_sel_4bit; |
|  | ```architecture Pr_Struc of pr_sel_4bit is component pr_sel_1bit port (a, si : in std_logic; b, so : out std_logic); end component; signal sel : std_logic_vector(3 downto 0); begin pr_sel_1bit_3 :pr_sel_1bit port map (x(3), sel(3), y(3), sel(2)); pr_sel_1bit_2 :pr_sel_1bit port map (x(2), sel(2), y(2), sel(1)); pr_sel_1bit_1 :pr_sel_1bit port map (x(1), sel(1), y(1),sel(0)); pr_sel_1bit_0 :pr_sel_1bit port map (x(0), sel(0), y(0), osel); sel(3) <= isel; end Pr_Struc;``` |

17.23


```
--The state assignment is as follows (q0q1q2q3)-
--S0-1000; S1-0100; S2 - 0010; S3 - 0001
--VHDL code using equations derived by inspection from state graph
entity sm1 is
    port (x, clk : in bit;
        z : out bit);
end sm1;
architecture equations of sm1 is
signal q0 : bit := '1';
signal q1, q2, q3 : bit:='0';
    begin
    process(clk)
        begin
        if clk'event and clk='1' then
            q0 <= (x and q0) or (not x and q1) or (not x and q3);
            q1 <= (not x and q0) or (x and q3);
                q2 <= (x and q2) or (x and q1);
                q3 <= not x and q2;
                    end if;
    end process;
    z <= (not x and q1) or (x and q3) or q2;
end equations;
```

17.24 There are three problems with this code.

1) The sensitivity list for the process contains the signal select. It should be sel. (Select is a VHDL reserved word).
2) When sel is true, there are two assignments to muxsel in the process and only the second one has any effect. Hence, if sel is true for two successive executions of the process, muxsel will be incremented to 2.
3) Since muxsel is not changed until the process terminates, the selection uses the old value of muxsel not the new value.

|  | Next State |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | $X=0 \quad X=1$ |  |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 10 | 00 |
| $S_{1}$ | $S_{1}$ | $S_{2}$ | 01 | 01 |
| $S_{2}$ | $S_{2}$ | $S_{3}$ | 01 | 01 |
| $S_{3}$ | $S_{0}$ | $S_{0}$ | 00 | 10 |

### 17.26

|  | Next State |  |  |
| :---: | :---: | :---: | :---: |
| State | $X=0$ | $X=1$ | Output |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | 1 |
| $S_{1}$ | $S_{3}$ | $S_{2}$ | 0 |
| $S_{2}$ | $S_{1}$ | $S_{0}$ | 0 |
| $S_{3}$ | $S_{0}$ | $S_{1}$ | 0 |

17.27

|  | Next State |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | :--- |
| State | $X_{1} X_{2}=00$ | 01 | 10 | 11 | $Z$ |
| $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{0}$ | 0 |
| $S_{1}$ | $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{1}$ | 0 |
| $S_{2}$ | $S_{0}$ | $S_{1}$ | $S_{2}$ | $S_{2}$ | 1 |

Unit 17 Solutions

17.28 (a) \begin{tabular}{c|c|cc|c|}

\hline | Present |
| :---: |
| State | \& \multicolumn{3}{|c|}{| Next State |
| :---: |
| xin 0 |} \& xin $=1$


 zout 

\hline$S_{1}$ \& $S_{2}$ \& $S_{10}$ \& 0 <br>
\hline$S_{2}$ \& $S_{2}$ \& $S_{3}$ \& 0 <br>
\hline$S_{3}$ \& $S_{4}$ \& $S_{6}$ \& 1 <br>
\hline$S_{4}$ \& $S_{7}$ \& $S_{8}$ \& 0 <br>
\hline$S_{5}$ \& $S_{9}$ \& $S_{10}$ \& 1 <br>
\hline$S_{6}$ \& $S_{9}$ \& $S_{10}$ \& 0 <br>
\hline$S_{7}$ \& $S_{2}$ \& $S_{3}$ \& 0 <br>
\hline$S_{8}$ \& $S_{4}$ \& $S_{5}$ \& 1 <br>
\hline$S_{9}$ \& $S_{7}$ \& $S_{8}$ \& 0 <br>
\hline$S_{10}$ \& $S_{9}$ \& $S_{10}$ \& 0 <br>
\hline
\end{tabular}

17.28 (c) \begin{tabular}{|c|cc|c|}

\hline | Present |
| :---: |
| State | \& \multicolumn{3}{|c|}{| Next State |
| :---: |
| xin |} <br>

\hline$S_{1}$ \& $S_{2}$ \& $S_{6}$ \& 0 <br>
\hline$S_{2}$ \& $S_{2}$ \& $S_{3}$ \& 0 <br>
\hline$S_{3}$ \& $S_{4}$ \& $S_{6}$ \& 1 <br>
\hline$S_{4}$ \& $S_{2}$ \& $S_{8}$ \& 0 <br>
\hline$S_{6}$ \& $S_{4}$ \& $S_{6}$ \& 0 <br>
\hline$S_{8}$ \& $S_{4}$ \& $S_{3}$ \& 1 <br>
\hline
\end{tabular}

17.28 (d) The output is 1 for an input sequence ending in either 01 or 1011
17.28 (b)


$$
\mathrm{z}=(0) 010110011
$$

17.29 df1, df2, and df3 are the same. They have two flip-flops, y 0 and $\mathrm{y} 1 ; \mathrm{y} 1$ has input xin and y 0 has input the complement of y 0 . The output z is y 0 AND (xin XOR y 1 or equivalent AND-OR logic.

17.29 df4 is the same except that $z$ is "registered" in a flip-flop.
(contd)

17.29 (b) The output for df4 only changes on positive clock edges and is delayed with respect to the output for df1, df2 and df3. See the simulation waveforms below..
Simulation for df1, df2 and df3.


Simulation for df4.

17.30


```
library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity mask_8 is
    port (X : in std_logic_vector(7 downto 0);
        Store, Set, Clk : in std_logic;
        Z : out std_logic_vector(7 downto 0));
end mask_8;
architecture Behavioral of mask_8 is
signal M : std_logic_vector(7 downto 0);
    begin
        process(Set, Clk)
        begin
        if Set='1' then M <= "11111111";
        elsif Clk'event and Clk='1' then
                if Store='1' then M<=X; end if;
            end if;
        end process;
        Z <= M and X;
end Behavioral;
```

17.31 library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity Seq_143 is
port (Clk, X : in std_logic;
Z : out std_logic);
end Seq_143;
architecture Moore of Seq_143 is
signal State : integer := 0;
signal NextState : integer range 0 to 3 ;
begin
process(State, X)
begin
case State is
when 0 => Z <= '0';
if $\mathrm{X}=$ ' 0 ' then NextState $<=0$;
else NextState $<=1$; end if;
when 1 => Z <= ' 0 ';
if $\mathrm{X}=\mathrm{O}$ ' then NextState $<=2$;
else NextState <= 1; end if;
when 2 => $Z$ <= ' 0 ';
if $X=$ ' 0 ' then NextState $<=0$;
else NextState <=3; end if;
when 3 => Z <= '1';
if $\mathrm{X}=$ ' 0 ' then NextState $<=2$;
else NextState <= 1; end if;
end case;
end process;
process(Clk)
begin
if Clk'event and Clk='1' then
State <= NextState; end if;
end process;
end Moore;

## Unit 18 Problem Solutions

18.3 See FLD p. 736 for circuit. Notice that the $Q$ output of the flip-flop is $b_{\mathrm{in}}$, while the D input is $b_{\text {out }}$.

18.4 See FLD p. 737. AND-ing with $x_{\mathrm{i}}$ is like $M / A d$ if $x_{\mathrm{i}}$ is 1 . Shifting is like moving from AND gates involving $x_{1}$ to those involving $x_{2}$, or from $x_{2}$ to $x_{3}$.
18.5 See FLD p. 737. Compare to divider state graph of FLD Figure 18-11.
18.6 See FLD p. 737.
18.7 (a) Overflow occurs only on division by 0 , so $V=y_{0}^{\prime} y_{1}^{\prime} y_{2}^{\prime} y_{3}^{\prime} y_{4}^{\prime}=\left(y_{0}+y_{1}+y_{2}+y_{3}+y_{4}\right)^{\prime}$
18.7 (b) See FLD p. 738.

- (d)
18.8 See FLD p. 738.


Notes: 1) The value in the carry FF does not matter for the first addition. 2) Only a half-adder is needed since all the additions are of two bits.

Next State Table: $\mathrm{S}_{0}=00, \mathrm{~S}_{1}=01, \mathrm{~S}_{2}=11$

|  | St Z |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 00 | 01 | 11 | 10 |
| 00 | 00 | 00 | 01 | 01 |
| 01 | 11 | 11 | 11 | 11 |
| 11 | 11 | 00 | 00 | 11 |
| 10 | -- | -- | -- | -- |

$D_{0}=\left(S t+Q_{0}\right)\left(Q_{1}^{\prime}+Z^{\prime}\right)$
$Q_{1}{ }^{\prime} Q_{0}+Z^{\prime} Q_{0}+S t Q_{1}{ }^{\prime}$

Output Table: L Sh Dec C

|  | St Z |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 00 | 01 | 11 | 10 |
| 00 | 0000 | 0000 | 1000 | 1000 |
| 01 | 0111 | 0111 | 0111 | 0111 |
| 11 | 0110 | 0000 | 0000 | 0110 |
| 10 | --- | --- | --- | ---- |


| Sh $\left.=\left(\operatorname{St}+Q_{0}\right)^{\prime}\right)^{\prime}$ |
| :--- |
| $\left.C=\left(Q_{1}+Q_{1}{ }^{\prime}\right)^{\prime}+Z^{\prime}\right) Q_{0}$ |

## Unit 18 Solutions

18.10


Notes: 1) The carry FF must contain 0 for the first addition. 2) Only a half-adder is needed since all the additions are of two bits; the first addition will produce a 0 carry.
Next State Table: $\mathrm{S}_{0}=00, \mathrm{~S}_{1}=01, \mathrm{~S}_{2}=11, \mathrm{~S}_{3}=$ 10

|  | St Z |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 00 | 01 | 11 | 10 |
| 00 | 00 | 00 | 01 | 01 |
| 01 | 11 | 11 | 11 | 11 |
| 11 | 10 | 10 | 10 | 10 |
| 10 | 10 | 00 | 00 | 10 |


| $D_{1}=\left(Q_{0}+Z^{\prime}\right)\left(Q_{0}+Q_{1}\right)$ |
| :--- |
| $D_{0}=\left(Q_{0}+S t\right) Q_{1}{ }^{\prime}$ |

Output Table: L Sh Dec C

|  St Z    <br> $\mathrm{Q}_{1} \mathrm{Q}_{0}$ 00 01 11 10 <br> 00 0000 0000 1000 1000 <br> 01 0110 0110 0110 0110 <br> 11 0111 0111 0111 0111 <br> 10 0110 0000 0000 0110 |
| :--- |
| Sh=(St'+Q1+Q$=$Sec $=\left(Q_{0}\right)^{\prime}$ <br> $C=\left(Z_{1}^{\prime}\right)\left(Q_{0}+Q_{1}\right)$ <br> $C$ |

18.11


Notes: 1) The carry FF must contain 0 for the first addition. 2) A full-adder is needed since the second addition may be of three bits. 3)The next state and output equations are the same as in 18.10 except $\mathrm{C}=\mathrm{Q}_{0}$.
18.12 (a) Inputs and outputs are given in decimal in the table. Inputs 10 through 14 are assumed to never occur.

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | A, 0 | B, 9 | B, 8 | B, 7 | B, 6 | B, 5 | B, 4 | B, 3 | B, 2 | B, 1 | A, 15 |
| B | B, 9 | B, 8 | B, 7 | B, 6 | B, 5 | B, 4 | B, 3 | B, 2 | B, 1 | B, 0 | A, 15 |

18.12 (b) Use the state assignment $\mathrm{Q}=0$ for state A and $\mathrm{Q}=1$ for state B . There are 159 minimum sum-of-product equations for $\mathrm{Q}^{+}$; one solution is
$Q^{+}=X_{3}{ }^{\prime} X_{0}+X_{3}{ }^{\prime} X_{2}+X_{3}{ }^{\prime} X_{1}+X_{3} X_{2}{ }^{\prime}+Q X_{2}{ }^{\prime}$.
The minimum sum-of-product equations for the outputs are
$Z_{3}=X_{3}{ }^{\prime} X_{2}{ }^{\prime} X_{1}{ }^{\prime} X_{0}+Q X_{2}{ }^{\prime} X_{1} X_{0}{ }^{\prime}+Q X_{3}{ }^{\prime} X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{3} X_{2}$ or
$=X_{3}{ }^{\prime} X_{2}{ }^{\prime} X_{1}{ }^{\prime} X_{0}+Q X_{2}{ }^{\prime} X_{1} X_{0}{ }^{\prime}+Q X_{3}{ }^{\prime} X_{2}{ }^{\prime} X_{1}{ }^{\prime}+X_{3} X_{1}$
$Z_{2}=X_{2}{ }^{\prime} X_{1} X_{0}+X_{2} X_{1}{ }^{\prime}+Q^{\prime} X_{2} X_{0}{ }^{\prime}+Q X_{2}{ }^{\prime} X_{1}+X_{3} X_{2}$ or
$=X_{2}{ }^{\prime} X_{1} X_{0}+X_{2} X_{1}^{\prime}+Q^{\prime} X_{2} X_{0}^{\prime}+Q X_{2}{ }^{\prime} X_{1}+X_{3} X_{1}$
$Z_{1}=X_{1} X_{0}+Q^{\prime} X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime}+Q^{\prime} X_{3} X_{0}{ }^{\prime}+Q X_{1}$
$Z_{0}=Q^{\prime} X_{0}+Q X_{0}{ }^{\prime}+X_{3} X_{2}$ or
$=Q^{\prime} X_{0}+Q X_{0}{ }^{\prime}+X_{3} X_{1}$
18.13 (a) Inputs and outputs are given in decimal in the table. Inputs 1, 2, 13, 14, and 15 are assumed to never occur.

|  | 0 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | A, 0 | A, 3 | B, 12 | B, 11 | B, 10 | B, 9 | B, 8 | B, 7 | B, 6 | B, 5 | B, 4 |
| B | A, 0 | B, 12 | B, 11 | B, 10 | B, 9 | B, 8 | B, 7 | B, 6 | B, 5 | B, 4 | B, 3 |

18.13 (b) Use the state assignment $\mathrm{Q}=0$ for state A and $\mathrm{Q}=1$ for state B . The minimum sum-of-product equations for $\mathrm{Q}^{+}$and the outputs are
$Q^{+}=X_{3}+X_{2}+Q X_{0}$ or
$=X_{3}+X_{2}+Q X_{1}$
$Z_{3}=X_{3}{ }^{\prime} X_{2}+Q^{\prime} X_{3} X_{2}{ }^{\prime} X_{1}{ }^{\prime} X_{0}{ }^{\prime}+Q X_{3}{ }^{\prime} X_{1}$ or
$=X_{3}{ }^{\prime} X_{2}+Q^{\prime} X_{3} X_{2}{ }^{\prime} X_{1}{ }^{\prime} X_{0}{ }^{\prime}+Q X_{3}{ }^{\prime} X_{0}$
$Z_{2}=Q^{\prime} X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime}+Q X_{3} X_{2}{ }^{\prime}+Q X_{2}{ }^{\prime} X_{0}+X_{3} X_{0}+X_{3} X_{1}$ or
$=Q^{\prime} X_{2} X_{1}{ }^{\prime} X_{0}{ }^{\prime}+Q X_{3} X_{2}{ }^{\prime}+Q X_{2}{ }^{\prime} X_{1}+X_{3} X_{0}+X_{3} X_{1}$
$Z_{1}=X_{1}{ }^{\prime} X_{0}+Q^{\prime} X_{1} X_{0}{ }^{\prime}+Q X_{2} X_{1}{ }^{\prime}+Q X_{3} X_{1}{ }^{\prime}+X_{3}{ }^{\prime} X_{2}{ }^{\prime} X_{1}$ or
$=X_{1}{ }^{\prime} X_{0}+Q^{\prime} X_{1} X_{0}{ }^{\prime}+Q X_{2} X_{1}{ }^{\prime}+Q X_{3} X_{1}{ }^{\prime}+X_{3}{ }^{\prime} X_{2}{ }^{\prime} X_{0}$
$Z_{0}=Q^{\prime} X_{0}+Q X_{2} X_{0}{ }^{\prime}+Q X_{3} X_{0}{ }^{\prime}$
18.14 The ONE ADDER is similar to a serial adder, except that there is only one input. This means that the carry will be added to $X$. Thus, if the carry flipflop is initially set to 1,1 will be added to the input. The signal I can be used to preset the carry flip-flop to 1 .

Let $S_{0}$ represent Carry $=0$, and let $S_{1}$ represent
Carry $=1$. The state graph is as follows:


Unit 18 Solutions
18.14 (contd)

18.15 (a)

18.15 (b)

18.15 (c)

| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | add |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 | 0 | 1 | 1 |  |  |  |  |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | shift |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | shift |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | add |
|  | 1 | 0 | 1 | 1 |  |  |  |  |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | shift |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |  |

Unit 18 Solutions
18.15 (d)

| Present State | Next State | Ad Sh Load Done |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | StM:00 011011 | 00 | 01 | 10 | 11 |
| $S_{0}$ | $\begin{array}{lllll}S_{0} & S_{0} & S_{1} & S_{1}\end{array}$ | 0000 | 0000 | 0010 | 0010 |
| $S_{1}$ | $\begin{array}{lllll}S_{3} & S_{2} & S_{3} & S_{2}\end{array}$ | 0100 | 1000 | 0100 | 1000 |
| $S_{2}$ | $\begin{array}{lllll}S_{3} & S_{3} & S_{3} & S_{3}\end{array}$ | 0100 | 0100 | 0100 | 0100 |
| $S_{3}$ | $\begin{array}{lllll}S_{5} & S_{4} & S_{5} & S_{4}\end{array}$ | 0100 | 1000 | 0100 | 1000 |
| $S_{4}$ | $\begin{array}{lllll}S_{5} & S_{5} & S_{5} & S_{5}\end{array}$ | 0100 | 0100 | 0100 | 0100 |
| $S_{5}$ | $\begin{array}{lllll}S_{7} & S_{6} & S_{7} & S_{6}\end{array}$ | 0100 | 1000 | 0100 | 1000 |
| $S_{6}$ | $\begin{array}{lllll}S_{7} & S_{7} & S_{7} & S_{7}\end{array}$ | 0100 | 0100 | 0100 | 0100 |
| $S_{7}$ | $S_{0} S_{0} S_{0} S_{0}$ | 0001 | 0001 | 0001 | 0001 |

I. $\left(S_{0}, S_{7}\right)\left(S_{1}, S_{2}\right)\left(S_{3}, S_{4}\right)\left(S_{5}, S_{6}\right)$
II. $\left(S_{0}, S_{1}\right)\left(S_{2}, S_{3}\right)\left(S_{4}, S_{5}\right)\left(S_{6}, S_{7}\right)$
III. $\left(S_{1}, S_{3}, S_{5}\right)\left(S_{2}, S_{4}, S_{6}\right)$ etc.

(Other assignments are possible.)

For this assignment, from LogicAid:
$J_{\mathrm{A}}=S t B^{\prime} C^{\prime}+M C ; \quad K_{\mathrm{A}}=M^{\prime}+B+C ; \quad J_{\mathrm{B}}=A^{\prime} C ; \quad K_{\mathrm{B}}=A^{\prime} C^{\prime} ; \quad J_{\mathrm{C}}=A B^{\prime} ; \quad K_{\mathrm{C}}=A^{\prime} B ;$
$A d=M A B^{\prime} C^{\prime}+M A^{\prime} C ; S h=M^{\prime} A+M^{\prime} C+A B+A C ; \quad$ Load $=S t A^{\prime} B^{\prime} C^{\prime} ; \quad$ Done $=A^{\prime} B C^{\prime}$


## Unit 18 Solutions

18.16 (a)
product

18.16 (b) See solution to 18.15 (b).
18.16 (d) Graph is same as 18.15 , so from LogicAid, using the same state assignment:
$D_{\mathrm{A}}=S t A^{\prime} B^{\prime} C^{\prime}+M A B^{\prime} C^{\prime}+M A^{\prime} C$
$D_{\mathrm{B}}=A^{\prime} C+A B$
$D_{\mathrm{C}}=A B^{\prime}+B^{\prime} C+A C$
Ad, Sh, Ld, Done: See solution to 18.15 (d)

| $S t$ | $M$ | $A$ | $B$ | $C$ | $D_{\mathrm{A}}$ | $D_{\mathrm{B}}$ | $D_{\mathrm{C}}$ | Ad | Sh | Ld Done |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| - | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| - | 1 | 0 | - | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| - | - | 0 | - | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| - | - | 1 | 1 | - | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| - | - | 1 | 0 | - | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| - | - | - | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| - | - | 1 | - | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| - | 0 | 1 | - | - | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| - | 0 | - | - | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| - | - | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

18.16 (c)

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 11100$ shift 0000000111 add | 1 | 0 | 1 | 0 | 0 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |$\quad$ shift 0001001000011 add 10100


| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| shift |  |  |  |  |  |  |  |  | 0011111000


18.16 (d) (contd)

18.17 (b)

| State | Counter | $X$ | St $M$ |  |  |  |  | K Ad Sh |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{0}$ | 00 | 000000111 | 1 | 1 | 0 | 1 | 0 |  |
| $S_{1}$ | 00 | 011001111 | 0 | 1 | 0 | 0 | 1 |  |
| $S_{2}$ | 01 | 001100111 | 0 | 1 | 0 | 1 | 0 |  |
| $S_{1}$ | 01 | 100101111 | 0 | 1 | 0 | 0 | 1 |  |
| $S_{2}$ | 10 | 010010111 | 0 | 1 | 1 | 1 | 0 |  |
| $S_{1}$ | 10 | 101011111 | 0 | 1 | 1 | 0 | 1 |  |
| $S_{0}$ | 00 | 010101111 | 0 | 1 | 0 | 0 | 0 |  |

18.18 (a)


## Unit 18 Solutions

18.18 (b)

18.18 (d)

| $\left.\begin{array}{lllllll} 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right\rvert\,$ | shift C $=0$ |
| :---: | :---: |
| $\begin{array}{lllllll\|l\|} \hline 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & & & & & \end{array}$ | sub. $\mathrm{C}=1$ |
| $\begin{array}{llllllll}0 & 1 & 0 & 0 & 1 & 1 & 1\end{array}$ | shift $\mathrm{C}=0$ |
| $\begin{array}{rrrrrrrr} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ & 1 & 1 & & & & & \\ \hline \end{array}$ | 1 |
| 0 0 1 0 1 1 1 1 | shift $\mathrm{C}=0$ |
| $\begin{array}{lllll\|lll} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & & & & & \end{array}$ | ift C = 0 |
| $\begin{array}{llll\|llll} \hline 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ & 1 & 1 & & & & & \end{array}$ | sub. $C=1$ |
| $\begin{array}{llllllll}0 & 1 & 0 & 1 & 1 & 1 & 0 & 1\end{array}$ | shift $\mathrm{C}=0$ |
| $\begin{array}{lll\|lllll} 1 & 0 & 1 \\ & 1 & 1 & & & & & \\ \hline \end{array}$ | sub. $C=1$ |
| $\underbrace{0}_{\text {remainder }} 1 \begin{array}{cc} 1 & 0 \\ 1 & 1 \end{array} 0 \begin{array}{lll} 1 & 1 \\ \text { quotient } \end{array}$ | $\mathrm{C}=0$ |


18.19 (c)

18.19 (d)

| $\begin{array}{rllllll} 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & & \\ \hline \end{array}$ | shift $\mathrm{C}=0$ |
| :---: | :---: |
| $\begin{array}{llllll\|l\|} \hline 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ & 1 & 1 & 0 & 1 & & \\ \hline \end{array}$ | sub. $C=1$ |
| 0 1 0 0 1 1 1 <br> 1 1 0 1    | shift $\mathrm{C}=0$ |
| $\begin{array}{lllll\|ll} \hline 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ & 1 & 1 & 0 & 1 & & \\ \hline \end{array}$ | sub. $\mathrm{C}=1$ |
|  |  |

18.20 (a)

18.20 (b) $D_{0}=S t^{\prime} Q_{0}+K Q_{1}+K Q_{2} ; \quad D_{1}=S t Q_{0}+K^{\prime} B^{\prime} Q_{1}+K^{\prime} B Q_{2} ; \quad D_{2}=K^{\prime} B Q_{1}+K^{\prime} B^{\prime} Q_{2} ; \quad R=S t Q_{0}$ $S h=K^{\prime} B^{\prime} Q_{1}+K^{\prime} B Q_{1}+K^{\prime} B Q_{2}+K^{\prime} B^{\prime} Q_{2}=K^{\prime} Q_{1}+K^{\prime} Q_{2} ; \quad X=K Q_{1}+K^{\prime} B Q_{1}+K^{\prime} B Q_{2}=K Q_{1}+B Q_{1}+K^{\prime} B Q_{2}$
18.21 (a)

18.21 (b)

18.21 (c) $D_{1}=S t^{\prime} Q_{1}+K Q_{2}+K Q_{3}$



$D_{3}=K^{\prime} S_{0} Q_{2}+K^{\prime} Q_{3}$
Note: The signal marked by $\downarrow$ is the shift register serial output $S_{\text {o }}$ not a state.
18.22 (a)


## Unit 18 Solutions

18.22 (b)


| Present <br> State | $S t K$ |  |  |  |  | Sh |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |  |  |
| $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{1}$ | 0 | 0 | 1 | 1 |  |  |
| $S_{1}$ | - | - | $S_{2}$ | $S_{1}$ | - | - | 1 | 1 |  |  |
| $S_{2}$ | $S_{0}$ | $S_{0}$ | $S_{2}$ | $S_{2}$ | 0 | 0 | 0 | 0 |  |  |

18.22 (c) I. $\left(S_{0}, S_{2}\right) \times 2\left(S_{1}, S_{2}\right)\left(S_{0}, S_{1}\right)$
II. $\left(S_{0}, S_{2}\right) \times 2\left(S_{1}, S_{2}\right)\left(S_{0}, S_{1}\right) \times 2$

From Karnaugh maps:
$D_{0}=Q_{0}^{+}=S t Q_{0}+K Q_{0}^{\prime} Q_{1}$
$D_{1}=Q_{1}^{+}=S t ; \quad S h=S t Q_{0}^{\prime}$

| 1 | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ |
| :--- | :--- | :--- |

Alternative: $Q_{0}^{+}=S t Q_{0}+S t K Q_{1}$

| $S t$ | $K$ | $Q_{0}$ | $Q_{1}$ | $D_{0}$ | $D_{1}$ | Sh |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | 1 | - | 1 | 0 | 0 |
| - | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | - | - | - | 0 | 1 | 0 |
| 1 | - | 0 | - | 0 | 0 | 1 |

18.23 (a)

18.23 (b)


| State | Meaning |
| :---: | :--- |
| $S_{0}$ | Reset |
| $S_{1}$ | Find AND of $A$ \& $B$ |
| $S_{2}$ | Find XOR of $A$ \& $B$ |

18.23 (c) $Q_{0}^{+}=S t^{\prime} Q_{0}+K Q_{1}+K Q_{2} ; \quad Q_{1}^{+}=S t C Q_{0}+K^{\prime} Q_{1} ;$
$Q_{2}^{+}=S t C^{\prime} Q_{0}+K^{\prime} Q_{2}$;
$S h=S t C Q_{0}+S t C^{\prime} Q_{0}+K^{\prime} Q_{1}+K Q_{1}+K^{\prime} Q_{2}+K Q_{2}$
$D=S t C Q_{0}+K^{\prime} Q_{1}+K Q_{1}$

| St | $C$ | $K$ | $Q_{0}$ | $Q_{1}$ | $Q_{2}$ | $Q_{1}^{+}$ | $Q_{2}^{+}$ | $Q_{3}^{+}$ | Sh | $D$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | - | - | 1 | - | - | 1 | 0 | 0 | 0 | 0 |
| - | - | 1 | - | 1 | - | 1 | 0 | 0 | 1 | 1 |
| - | - | 1 | - | - | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | - | 1 | - | - | 0 | 1 | 0 | 1 | 1 |
| - | - | 0 | - | 1 | - | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | - | 1 | - | - | 0 | 0 | 1 | 1 | 0 |
| - | - | 0 | - | - | 1 | 0 | 0 | 1 | 1 | 0 |

18.24 (a)

18.24 (c) $J_{\mathrm{A}}=B ; \quad K_{\mathrm{A}}=B ; \quad J_{\mathrm{B}}=S t+A ; \quad K_{\mathrm{B}}=1$;
$S h=S t+A+B ; \quad M \stackrel{B}{=} C_{1}^{\prime} C_{2}+X_{0}^{\prime} C_{1} C_{2}$
18.23 (d) Change $C^{\prime}$ to $D^{\prime}$ in 18.22 (d) $S I=D^{\prime} a b+D a b^{\prime}+D a^{\prime} b$
18.24 (b)


Note: M can be determined independently of the state of the system, so it is not included in the state graph.

18.25 (c)

|  | $S t K$ |  |  |  | ABLd |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| State | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
| $S_{0}$ | $S_{0}$ | $S_{0}$ | $S_{1}$ | $S_{1}$ | 000 | 000 | 001 | 001 |
| $S_{1}$ | $S_{1}$ | $S_{2}$ | - | - | 010 | 001 | - | - |
| $S_{2}$ | $S_{2}$ | $S_{0}$ | - | - | 100 | 000 | - | - |

$D_{1}=K Q_{2}+K^{\prime} Q_{1} ; \quad D_{2}=S t+K^{\prime} Q_{2} ; \quad A=K^{\prime} Q_{1} ;$
$B=K^{\prime} Q_{2} ; \quad L d=S t+K Q_{2}$

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18.26

18.27 (b) $J=S T$; $K=$ ZER1 ZER2;

Done $=$ ZER1 ZER2 Q; CLR =STQ';
$L D 2=S T Q^{\prime} ; L D 1=S T Q^{\prime}+Z E R 1$ ZER2' $Q$;
$C T 1=$ ZER1' $Q ; C T 2=$ ZER1 ZER2' $Q$
18.27 (c) $\left(N_{1}+1\right) N_{2}$ cycles
18.28 (a) Initial PU,PL: 00000000

1st Add Lower half PU, PL: 00001011 1st Add Upper half PU, PL: 00001011 2nd Add Lower half PU, PL: 00000110 2nd Add Upper half PU, PL: 00010110 3rd Add Lower half PU, PL: 00010001 3rd Add Upper half PU, PL: 00100001 4th Add Lower half PU, PL: 00101100 4th Add Upper half PU, PL: 00101100 5th Add Lower half PU, PL: 00100111 5th Add Upper half PU, PL: 00110111
18.28 (b)

18.28 (d) Assume two $\mathrm{FFs} \mathrm{Q}_{1} \mathrm{Q}_{0}$ and the following encoding:
$\mathrm{S}_{0}=00, \mathrm{~S}_{1}=01, \mathrm{~S}_{2}=11$, and $\mathrm{S}_{3}=10$. Then, $D_{0}=S\left(S_{0}\right)+S_{2}+\left(B Z^{\prime}\right) S_{1}$
$=S Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}+Q_{1} Q_{0}+(B Z ') Q_{1}{ }^{\prime} Q_{0}$
$=S Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}+Q_{1} Q_{0}+\left(B Z^{\prime}\right) Q_{0}$
$D_{1}=\left(B Z^{\prime}\right) S_{1}+(B Z) S_{1}+S\left(S_{3}\right)$
$=S_{1}+S\left(S_{3}\right)$
$=Q_{1}{ }^{\prime} Q_{0}+S Q_{1} Q_{0}{ }^{\prime}$
$C P=L A=L B=C C=S_{0}=Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}$
$L P L=E A=D B=\left(B Z^{\prime}\right) S_{1}=\left(B Z^{\prime}\right) Q_{1}{ }^{\prime} Q_{0}$
$L P U=M S=S_{2}=Q_{1} Q_{0}$
$D=S_{3}=Q_{1} Q_{0}{ }^{\prime}$
18.29 (a) When the multiplier is negative, the $B$ counter can be incremented to zero. Two control inputs are assumed: DB (decrement B) and IB (Increment B). Also, when the multiplier is negative, the multiplicand must be subtracted to produce the product. This can be done by adding an ExclusiveOR array after the AND array; when IA $=0$, the output of the Exclusive-OR array is equal to its input and when IA = 1, it inverts its input. To produce a two's complement subtract, the carry FF must be set; SC (set carry) has been added to the carry FF. When the product is negative, $\left(\mathrm{A}_{3} \oplus\right.$ $\left.\mathrm{B}_{3}\right)=1$, IA must be 1 to extend the sign bit when adding the carry to the upper half of the product.
18.29 (b) Answer is the same for both parts of Part (b).

Initial PU,PL: 00000000
1st Add Lower half PU, PL: 00001011
1st Add Upper half PU, PL: 11111011
2nd Add Lower half PU, PL: 11110110
2nd Add Upper half PU, PL: 11110110
3rd Add Lower half PU, PL: 11110001
3rd Add Upper half PU, PL: 11110001
4th Add Lower half PU, PL: 11111100
4th Add Upper half PU, PL: 11101100
5th Add Lower half PU, PL: 11100111
5th Add Upper half PU, PL: 11100111

Note: The PU and PL registers are connected the same way as in Problem 18.28.
18.29 (c)

18.29 (d) Label the 5 FF outputs $S_{0}, S_{1}, S_{2}, S_{3}$ and $S_{4}$.
$D_{0}=S^{\prime} S_{0}+S^{\prime} S_{4}, D_{1}=S\left(S_{0}\right), D_{2}=S_{1}+S_{3}$
$D_{3}=\left(B Z^{\prime}\right) S_{2}, D_{4}=(B Z) S_{2}+S\left(S_{4}\right)$
$C P=L A=L B=S_{0}$
$C C=B_{3}{ }^{\prime}\left(S_{1}+S_{3}\right), S C=B_{3}\left(S_{1}+S_{3}\right)$
$L P L=E A=\left(B Z^{\prime}\right) S_{2}$
$D B=B_{3}{ }^{\prime} S_{3}, I B=B_{3} S_{3}$
$I A=B_{3}\left(B Z^{\prime}\right) S_{2}+\left(A_{3} \oplus B_{3}\right) S_{3}$
$L P U=M S=S_{3}$
$D=S_{4}$

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18.29 (e) Assume three FFs $\mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{Q}_{0}$ and the following encoding: $\mathrm{S}_{0}=000, \mathrm{~S}_{1}=001, \mathrm{~S}_{2}=011, \mathrm{~S}_{3}=010$ and $\mathrm{S}_{4}=100$. Then,

$$
\begin{aligned}
& D_{0}=S\left(S_{0}\right)+S_{1}+S_{3}=S Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}+Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}+Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}=S Q_{1}{ }^{\prime}+Q_{1}{ }^{\prime} Q_{0}+Q_{1} Q_{0}{ }^{\prime} \text { or } \\
& =S Q_{0}{ }^{\prime}+Q_{1}{ }^{\prime} Q_{0}+Q_{1} Q_{0}{ }^{\prime} \\
& D_{1}=S_{1}+S_{3}+\left(B Z^{\prime}\right) S_{2}=Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}+Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}+\left(B Z^{\prime}\right) Q_{2}{ }^{\prime} Q_{1} Q_{0}=Q_{1}{ }^{\prime} Q_{0}+Q_{1} Q_{0}{ }^{\prime}+\left(B Z^{\prime}\right) Q_{1} \text { or } \\
& =Q_{1}^{\prime} Q_{0}+Q_{1} Q_{0}^{\prime}+\left(B Z^{\prime}\right) Q_{0} \\
& D_{2}=(B Z) S_{2}+S\left(S_{4}\right)=(B Z) Q_{2}{ }^{\prime} Q_{1} Q_{0}+S\left(Q_{2} Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}\right)=(B Z) Q_{1} Q_{0}+S\left(Q_{2}\right) \\
& C P=L A=L B=S_{0}=Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}=Q_{1}{ }^{\prime} Q_{0}{ }^{\prime} \\
& C C=B_{3}{ }^{\prime}\left(Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}+Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}\right)=B_{3}{ }^{\prime}\left(Q_{1}{ }^{\prime} Q_{0}+Q_{1} Q_{0}{ }^{\prime}\right) ; S C=B_{3}\left(Q_{2}{ }^{\prime} Q_{1}{ }^{\prime} Q_{0}+Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}\right)=B_{3}\left(Q_{1}{ }^{\prime} Q_{0}+Q_{1} Q_{0}{ }^{\prime}\right) \\
& L P L=E A=\left(B Z^{\prime}\right) Q_{2}{ }^{\prime} Q_{1} Q_{0}=\left(B Z^{\prime}\right) Q_{1} Q_{0} \\
& D B=B_{3}{ }^{\prime} Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}=B_{3}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime} ; I B=B_{3} Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}=B_{3} Q_{1} Q_{0}{ }^{\prime} \\
& I A=B_{3}(B Z ') Q_{2}^{\prime} Q_{1} Q_{0}+\left(A_{3} \oplus B_{3}\right) Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}=B_{3}\left(B Z^{\prime}\right) Q_{1} Q_{0}+\left(A_{3} \oplus B_{3}\right) Q_{1} Q_{0}{ }^{\prime} \\
& L P U=M S=Q_{2}{ }^{\prime} Q_{1} Q_{0}{ }^{\prime}=Q_{1} Q_{0}{ }^{\prime} \\
& D=Q_{2} Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}=Q_{2}
\end{aligned}
$$

Note: These equations simplify because 101, 110 and 111 are don't-care state combinations.
18.30 (a)

18.30 (b) $D=E Z E R O^{\prime} Q+S t Q^{\prime} ;$ Done $=E Z E R O$ Q;

CLR = StQ';
$L O A D=S t Q^{\prime}+I Z E R O$ EZERO' $Q$
$D O W N=I Z E R O^{\prime} E Z E R O^{\prime} Q$
$U P=I Z E R O E Z E R O^{\prime} Q$
18.30 (d) The quotient counter reaches 1111 , and $U P=1$ again.
18.30 (c) $N_{1}+\left(N_{1} / N_{2}\right)$ cycles (round down)
18.30 (e) The quotient will count upward forever, and Done will never be 1 .
18.31


When the done signal comes on, square root is in the 4-bit counter


## Unit 19 Problem Solutions

19.1 See FLD p. 739 for solution.
19.3 See FLD p. 739 for solution.
19.5 See FLD p. 740 for solution.
19.7 See FLD p. 741 for solution.
19.9 See FLD p. 741-742 for solution.
19.11

19.12 (b)

19.2 See FLD p. 739 for solution.
19.4 See FLD p. 740 for solution.
19.6 See FLD p. 741 for solution.
19.8 See FLD p. 741 for solution.
19.10 See FLD p. 742 for solution.
19.12 (a)

19.13

19.14 (a)

19.14 (b) Let $S_{0}, S_{1}, S_{2}$ and $S_{3}$ be the four FF outputs, then $D_{0}=x^{\prime}\left(S_{0}+S_{1}+S_{2}+S_{3}\right)$,
$D_{1}=x S_{0}$,
$D_{2}=x S_{1}$, and
$D_{3}=x\left(S_{2}+S_{3}\right)$.
$Z=S_{3}$
19.14 (d) Using the simplification identity twice, $D_{1}=x\left(Q_{0}+Q_{1}\right)$ and $D_{0}=x\left(Q_{0}{ }^{\prime}+Q_{1}\right)$.
19.14 (c) Using state assignment $\mathrm{S}_{0}=00, \mathrm{~S}_{1}=01, \mathrm{~S}_{2}=10$ and $S_{3}=11$, and denoting state variables $Q_{1}$ and $\mathrm{Q}_{0}, \mathrm{D}_{1}$ is the OR of $\mathrm{D}_{2}$ and $\mathrm{D}_{3}$ from Part b) so $D_{1}=x\left(S_{1}+S_{2}+S_{3}\right)$
$=x\left(Q_{1}{ }^{\prime} Q_{0}+Q_{1} Q_{0}+Q_{1} Q_{0}{ }^{\prime}\right)$.
Similarly, $D_{0}$ is the OR of $D_{1}$ and $D_{3}$ from Part b) so
$D_{0}=x\left(S_{0}+S_{2}+S_{3}\right)$
$=x\left(Q_{1}{ }^{\prime} Q_{0}{ }^{\prime}+Q_{1} Q_{0}{ }^{\prime}+Q_{1} Q_{0}\right)$.
$Z=Q_{1} Q_{0}$
19.15 (a)

19.15 (c) Label the three FF outputs $\mathrm{S}_{1}, \mathrm{~S}_{2}$ and $\mathrm{S}_{3}$.
$D_{1}=S t^{\prime} S_{1}+K S_{2}+K S_{3} ; \quad D_{2}=K^{\prime} S_{0} S_{2}+S t S_{1}$
$D_{3}=K^{\prime} S_{0} S_{2}+K^{\prime} S_{3} ; \quad C l r=S t S_{1}$
$S h=K^{\prime} S_{2}+K S_{0}{ }^{\prime} S_{2}+K^{\prime} S_{3}=K^{\prime} S_{2}+S_{0}{ }^{\prime} S_{2}+K^{\prime} S_{3}$
$E r=K S_{0} S_{2} ; \quad S I=K ' S_{0} S_{2}+K ' S_{0} S_{3}$
19.15 (b) Next State Table for $\mathrm{Q}_{1}{ }^{+} \mathrm{Q}_{0}{ }^{+}: \mathrm{S}_{1}=00, \mathrm{~S}_{2}=01$ and $S_{3}=11$.

|  | St K S |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 000 | 001 | 011 | 010 |
| 00 | 00 | 00 | 00 | 00 |
| 01 | 01 | 11 | 00 | 00 |
| 11 | 11 | 11 | 00 | 00 |
| 10 | -- | -- | -- | -- |


|  | St K S |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 100 | 101 | 111 | 110 |
| 00 | 01 | 01 | 01 | 01 |
| 01 | 01 | 11 | 00 | 00 |
| 11 | 11 | 11 | 00 | 00 |
| 10 | -- | -- | -- | -- |

$D_{1}=K^{\prime} Q_{1}+K^{\prime} S_{0} Q_{0}, D_{0}=K^{\prime} Q_{0}+S t Q_{0}{ }^{\prime}$
Output Table for Clr Sh Er SI

|  | St K S |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 000 | 001 | 011 | 010 |
| 00 | 0000 | 0000 | 0000 | 0000 |
| 01 | 0100 | 0101 | 0010 | 0100 |
| 11 | 0101 | 0100 | 0000 | 0000 |
| 10 | -- | -- | -- | -- |


|  | St K S $_{0}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Q}_{1} \mathrm{Q}_{0}$ | 100 | 101 | 111 | 110 |
| 00 | 1000 | 1000 | 1000 | 1000 |
| 01 | 0100 | 0101 | 0010 | 0100 |
| 11 | 0101 | 0100 | 0000 | 0000 |
| 10 | -- | -- | -- | -- |

$E r=K S_{0} Q_{1}{ }^{\prime} Q_{0} ; S I=K ' S_{0}{ }^{\prime} Q_{1}+K S_{0} Q_{1}{ }^{\prime} Q_{0}$
19.15 (d) Label the two FF outputs $\mathrm{Q}_{1}, \mathrm{Q}_{0}$ and the decoder outputs $S_{1}=00, S_{2}=01$ and $S_{3}=11^{*}$, then
$D_{0}=K^{\prime} S_{0}{ }_{0} S_{2}+S t S_{1}+K^{\prime} S_{0} S_{2}+K^{\prime} S_{3}$
$=K^{\prime} S_{2}+S t S_{1}+K^{\prime} S_{3}$
$D_{1}=K^{\prime} S_{0} S_{2}+K^{\prime} S_{3} ; \quad C l r=S t S_{1}$
$S h=K^{\prime} S_{2}+S_{0}{ }^{\prime} S_{2}+K^{\prime} S_{3}$
$E r=K S_{0} S_{2} ; \quad S I=K ' S_{0} S_{2}+K ' S_{0} S_{3}$ *Note: Other solutions are possible for different encodings.

Unit 19 Solutions
19.16

19.17

19.18


| State | $Q_{0}$ | $Q_{1}$ | $S t$ | $M$ | $K$ | $Q_{0}^{+}$ | $Q_{1}^{+}$ | Ad | Sh |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{0}$ | 0 | 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_{0}$ | 0 | 0 | 1 | - | - | 0 | 1 | 0 | 0 | 1 | 0 |
| $S_{1}$ | 0 | 1 | - | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| $S_{1}$ | 0 | 1 | - | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| $S_{1}$ | 0 | 1 | - | 1 | - | 1 | 1 | 1 | 0 | 0 | 0 |
| $S_{2}$ | 1 | 1 | - | - | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| $S_{2}$ | 1 | 1 | - | - | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| $S_{3}$ | 1 | 0 | - | - | - | 0 | 0 | 0 | 0 | 0 | 1 |

19.19 (a)

19.19 (b)

$$
\begin{aligned}
A^{+}= & A^{\prime} B X_{2}+A^{\prime} B^{\prime} X_{2}\left(X_{1}^{\prime}+X_{3}\right)+\{A B\} \\
& =B X_{2}+A^{\prime} X_{2}\left(X_{1}^{\prime}+X_{3}\right) \\
B^{+}= & A^{\prime} B^{\prime}\left(X_{2}^{\prime \prime}+X_{1} X_{3}^{\prime}\right)+A B^{\prime} X_{1}^{\prime}+A^{\prime} B X_{2}^{\prime}+\{A B\} \\
& =A X_{1}^{\prime}+A^{\prime} B^{\prime} X_{1} X_{3}^{\prime}+A^{\prime} X_{2}^{\prime} \\
Z_{1}= & A+B+X_{2} ; \quad Z_{2}=A^{\prime} B^{\prime} X_{2}^{\prime} ; \quad Z_{3}=A^{\prime} B^{\prime} X_{1}^{\prime} X_{2}
\end{aligned}
$$

In the preceding equations, curly brackets ( $\}$ ) indicate a don't care term.
19.19 (c) PLA table obtained by tracing link paths:

| State | $A B$ | $X_{1} X_{2} X_{3}$ | $A^{+} B^{+}$ | $Z_{1} Z_{2} Z_{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| $S_{0}$ | 00 | $-0-$ | 01 | 010 |
|  | 00 | $01-$ | 10 | 101 |
|  | 00 | 110 | 01 | 100 |
|  | 00 | 111 | 10 | 100 |
| $S_{1}$ | 01 | $-0-$ | 01 | 100 |
|  | 01 | $-1-$ | 10 | 100 |
| $S_{2}$ | 10 | $0--$ | 01 | 100 |
|  | 10 | $1--$ | 00 | 100 |


19.19 (d) $2^{5} \times 5$ ROM

| $A B$ | $X_{1} X_{2} X_{3}$ | $A^{+} B^{+}$ | $Z_{1} Z_{2} Z_{3}$ |
| :---: | :---: | :---: | :---: |
| 00 | 000 | 01 | 010 |
| 00 | 001 | 01 | 010 |
| 00 | 010 | 10 | 101 |
| 00 | 011 | 10 | 101 |
| 00 | 100 | 01 | 010 |

19.20 (b) Let $S_{0}, S_{1}, S_{2}$ and $S_{3}$ be the four FF outputs, then $D_{0}=s^{\prime}\left(S_{0}+S_{3}\right) ; D_{1}=s S_{0}+(T C)^{\prime} S_{2}$
$D_{2}=S_{1} ; \quad D_{3}=(T C) S_{2}$
$L D N=S_{1}+S_{2}$ or $L D N=S_{1}+S_{2}+S_{3}$
$C E=S_{1}$ or $C E=S_{1}+S_{3}$
$z_{1}=S_{1} ; \quad z_{2}=S_{2}$
$P_{3}=0 ; P_{2}=0 ; P_{1}=1 ; P_{0}=1$
19.20 (c) Using state assignment $S_{0}=00, S_{1}=01, S_{2}=11$,
$S_{3}=10$, and denoting the state variables as $Q_{1} Q_{0}$,
$D_{1}$ is the OR of $D_{2}$ and $D_{3}$ from Part (b) so
$D_{1}=S_{1}+(T C) S_{2}=Q_{1} Q_{0}+(T C) Q_{1} Q_{0}$ $=Q_{1}{ }^{\prime} Q_{0}+(T C) Q_{0}$.
Similarly, $\mathrm{D}_{0}$ is the OR of $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ from Part (b)
so $D_{0}=s S_{0}+(T C) S_{2}+S_{1}$

$$
\begin{aligned}
& =s Q_{1}{ }^{\prime} Q_{0}^{\prime}+(T C)^{\prime} Q_{1} Q_{0}+Q_{1}{ }^{\prime} Q_{0} \\
& =s Q_{1}^{\prime}+(T C)^{\prime} Q_{0}+Q_{1}^{\prime} Q_{0} .
\end{aligned}
$$

The outputs are
$L D N=S_{1}+S_{2}=Q_{0} ; C E=S_{1}=Q_{1}{ }^{\prime} Q_{0}$ $z_{1}=Q_{1}{ }^{\prime} Q_{0}$; and $z_{2}=Q_{1} Q_{0}$
19.21 (a) Initial PU,PL: 00000000

1st Add Lower half PU, PL: 00001011
1st Add Upper half PU, PL: 00001011
2nd Add Lower half PU, PL: 00000110
2nd Add Upper half PU, PL: 00010110
3rd Add Lower half PU, PL: 00010001
3rd Add Upper half PU, PL: 00100001
4th Add Lower half PU, PL: 00101100
4th Add Upper half PU, PL: 00101100
5th Add Lower half PU, PL: 00100111
5th Add Upper half PU, PL: 00110111

## Unit 19 Solutions


19.21 (c) Label the 4 FF outputs $\mathrm{S}_{0}, \mathrm{~S}_{1}, \mathrm{~S}_{2}$ and $\mathrm{S}_{3}$. $D_{0}=S^{\prime} S_{0}+S^{\prime} S_{3}$
$D_{1}=S\left(S_{0}\right)+S_{2}$
$D_{2}=(B Z ') S_{1}$
$D_{3}=(B Z) S_{1}+S\left(S_{3}\right)$
$C P=L A=L B=C C=S_{0}$
$L P L=E A=D B=\left(B Z^{\prime}\right) S_{1}$
$L P U=M S=S_{2}$
$D=S_{3}$
19.22


* Although SB is 1 in state $S_{2}$, shifting does not occur until the next clock.
19.21 (d) Assume two $F F s Q_{1} Q_{0}$ and the following encoding:
$\mathrm{S}_{0}=00, \mathrm{~S}_{1}=01, \mathrm{~S}_{2}=11$ and $\mathrm{S}_{3}=10$. (The
decoder outputs are labeled $\mathrm{S}_{0}, \mathrm{~S}_{1}, \mathrm{~S}_{2}$ and $\mathrm{S}_{3}$.)
Then,
$D_{0}=S\left(S_{0}\right)+S_{2}+\left(B Z^{\prime}\right) S_{1}$
$D_{1}=\left(B Z Z^{\prime}\right) S_{1}+(B Z) S_{1}+S\left(S_{3}\right)=S_{1}+S\left(S_{3}\right)$
The output equations are the same as in Part (c).
19.23 (a)

19.24 (a)

19.24 (b) See answer to 16.26 (c) on page 192.

Unit 19 Solutions


19.27


Unit 19 Solutions

## Unit 20 Problem Solutions

20.1 See FLD p. 743 for solution.
20.2 See FLD p. 743-744 for solution.
20.3

Replace line 12 with:
signal State, Nextstate: integer range 0 to 5 ;
Replace lines 27-33 with:
when $1|2| 3 \mid 4$ =>
if $\mathrm{M}=$ ' 1 ' then Ad <='1';
Nextstate <= State;
else $\mathrm{Sh}<=$ '1'; Nextstate <= State +1 ; end if;
when 5 => Done <= ' 1 '; Nextstate <= 0 ;
20.4 See FLD p. 744-745 for solution.
20.5

See FLD p. 745 for solution.
20.6

See FLD p. 746 for solution.
Replace lines 39-41 with:
if Load = '1' then ACC <= "00000" \& MPlier; end if;
if $\mathrm{Ad}=$ ' 1 ' then $\mathrm{ACC}(8$ downto 4$)<=$ addout; $\operatorname{ACC}(0)<=$ ' 0 '; end if;
20.7 Replace line 14 with:
signal Counter: integer range 0 to 4;
signal State, NextState: integer range 0 to 3;
After line 22, add:
K<='1' when Counter=3 else ' 0 ';
Replace lines 33-36 with:
when 2 =>
if $\mathrm{C}=$ ' 1 ' then $\mathrm{Su}<=$ ' 1 '; NextState <= 2 ;
elsif K = '1' then Sh <='1'; NextState <= 3;
else Sh <= '1'; NextState <= 2; end if;
when 3 =>
Replace line 47 with:
if Sh = ' 1 ' then Dividend $<=$ Dividend ( 7 downto 0 ) \& ' 0 ';
Counter <= Counter + 1; end if;
20.8

Entity and architecture for DiceGame goes here.
(contd)
entity GameTest is
end GameTest;
architecture dicetest of GameTest is
component DiceGame
port (CLK, Rb, Reset : in bit;
Sum: in integer range 2 to 12 ;
Roll, Win, Lose: out bit);
end component;
signal rb, reset, clk, roll, win, lose: bit;
signal sum: integer range 2 to 12 ;
type arr is $\operatorname{array}(0$ to 11$)$ of integer;
constant Sumarray:arr := (7,11,2,4,7,5,6,7,6,8,9,6);
begin
CLK <= not CLK after 20 ns ;
Dice: Dicegame port map(rb,reset,clk,sum,roll,win,lose);

## Contintued next column

20.8


```
process
    begin
    for i in 0 to 11 loop
```

        Rb <= '1'; -- push roll button
        wait until roll = '1';
            wait until clk'event and clk = ' 1 ';
            \(\mathrm{Rb}<=\) '0'; -- release roll button
            wait until roll <= '0';
            sum <= Sumarray(i);
            -- read roll of dice from array
            wait until clk'event and clk = ' 1 ';
            wait until clk'event and clk = ' 1 ';
            if win = ' 1 ' or lose = ' 1 ' then reset <= ' 1 ';
            end if;
            wait until clk'event and clk = ' 1 ';
            reset <= '0';
    end loop;
                            wait; -- test completed, do not execute process
                                    again
                                    end process;
    end dicetest;

## Unit 20 Solutions

20.9 Replace lines 6-11 with:

Port (Dividend_in: in std_logic_vector(4 downto 0);
Divisor: in std_logic_vector(4 downto 0);
St, Clk: in std_logic;
Quotient: out std_logic_vector(4 downto 0);
Remainder: out std_logic_vector(4 downto 0);
Replace lines 14-17 with:
signal State, NextState: integer range 0 to 6 ;
signal C, Load, Su, Sh, V: std_logic;
signal Subout : std_logic_vector (5 downto 0);
signal Dividend: std_logic_vector (9 downto 0);
Replace lines 19-23 with:
Subout <= '0'\&Dividend(9 downto 5) - Divisor;
C <= not Subout (5);
Remainder <= Dividend (9 downto 5);
V <= ' 1 ' when Divisor = " 00000 " else ' 0 ';
Quotient <= Dividend (4 downto 0);
State_Graph: process (State, St, C, V)
Replace line 25 with:
Load <= '0'; Sh <= '0'; Su <= '0';
Replace lines 28-33 with:
if ( $\mathrm{St}=\mathrm{I} \mathrm{I}^{\prime}$ ) then
if ( $\mathrm{V}={ }^{\prime} \mathrm{O}^{\prime}$ ) then Load <='1'; NextState $<=1$;
else Nextstate <=0; end if;
else Nextstate <=0; end if;
when 1 => Sh <='1'; NextState <= 2;
when 2|3|4|5 =>

Replace line 36 with:
when 6 =>

Replace lines 45-47 with:
if Load = '1' then Dividend <= "00000" \& Dividend_in; end if;
if $\mathrm{Su}=$ ' 1 ' then Dividend(9 downto 5) <= Subout (4 downto 0); Dividend(0) <= ' 1 '; end if;
if Sh = '1' then Dividend <= Dividend (8 downto 0 ) \& ' 0 '; end if;

```
20.10 (a) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity mult20_10 is
    Port (CLK, S: in std_logic;
        Mplier, Mcand : in std_logic_vector(3 downto 0);
        Product : out std_logic_vector(7 downto 0);
        D : out std_logic);
end mult20_10;
architecture Behavioral of mult20_10 is
    signal State, NextState: integer range 0 to 4;
    signal A, B: std_logic_vector (3 downto 0); -- Multiplicand \& Multiplier
    signall PU, PL: std_logic_vector (3 downto 0); -- Product registers
    signal muxout, andarray: std_logic_vector (3 downto 0);
    signal addout: std_logic_vector (4 downto 0);
    signal BZ, LA, CP, DB, LPU, LPL, EA, MS, CC, C: std_logic;
    begin
        \(B Z<=~ ' 1\) ' when \(B=\) " 0000 " else ' 0 ';
        muxout <= PU when MS = ' 1 ' else PL;
        andarray <= A when EA = '1' else "0000";
        addout <= ('O' \& muxout) + ('O' \& andarray) + ("0000" \& C); -- adder output is
        Product \(<=\) PU \& PL; -- 5 bits including carry
        process (S, State, BZ)
        begin
        CP <= '0'; LA <= '0'; DB <= '0'; MS <= '0'; CC <= '0';
            EA <= ' 0 '; LPU <= ' 0 '; LPL <= ' 0 '; D <= ' 0 '; \(\quad\)-- control signals are ' 0 ' by default
        case State is
        when \(0=>\)
            CP <= ' 1 '; LA <= '1'; CC <= ' 1 ';
            if \(S=\) ' 1 ' then NextState \(<=1\); else NextState \(<=0\); end if;
        when \(1=>\)
            if \(B Z=\) ' 1 ' then NextState <= 3 ; else LPL <= ' 1 '; EA <= ' 1 '; DB <= ' 1 '; NextState <= 2 ; end if;
        when 2 =>
            LPU <= '1'; MS <= '1'; NextState <= 1;
        when 3 =>
            D <= '1';
            if \(S=\) ' 1 ' then NextState \(<=3\); else NextState <= 0 ; end if;
        end case;
    end process;
        process (CLK)
        begin
        if CLK'event and CLK = '1' then -- update registers on rising edge of clk
            if LA = '1' then \(B<=\) Mplier; \(A<=\) Mcand; end if; -- load multiplier \& multiplicand
            if \(C P=\) ' 1 ' then PU <= "0000"; PL <= "0000"; end if; -- clear product registers
            if \(\mathrm{DB}=\) ' 1 ' then \(\mathrm{B}<=\mathrm{B}-1\); end if; -- decrement multiplier
            if \(\mathrm{LPL}=1\) ' then \(\mathrm{PL}<=\) addout( 3 downto 0 ); end if;
            if LPU = '1' then PU <= addout(3 downto 0); end if;
            if CC = '1' then C <= '0'; else C <= addout(4); end if -- load carry flip-flop
            State <= NextState;
        end if;
    end process;
end Behavioral;
```

Unit 20 Solutions

```
20.10 (b) library IEEE;
    use IEEE.STD_LOGIC_1164.ALL;
    use IEEE.STD_LOGIC_ARITH.ALL;
    use IEEE.STD_LOGIC_UNSIGNED.ALL;
    entity test20_10 is
    end test20_10;
    architecture test1 of test20_10 is
    component mult20_10
    port (Clk: in std_logic;
        S: in std_logic;
        Mplier, Mcand : in std_logic_vector(3 downto 0);
        Product : out std_logic_vector(7 downto 0);
        D: out std_logic);
    end component;
    constant N: integer := 4;
    type arr is array(1 to N) of std_logic_vector(3 downto 0);
    constant Mcandarr: arr := ("1011", "1011", "1111", "0000");
    constant Mplierarr: arr := ("0101", "0000", "1111", "1111");
    signal CLK: std_logic :='0';
    signal S, D: std_logic;
    signal Mplier, Mcand: std_logic_vector(3 downto 0);
    signal Product: std_logic_vector(7 downto 0);
    begin
        mult1: mult20_10 port map(CLK, S, Mplier, Mcand, Product, D);
        CLK <= not CLK after 10 ns; -- clock has }20\mathrm{ ns period
        process
        begin
        for i in 1 to N loop
        Mcand <= Mcandarr(i);
        Mplier <= Mplierarr(i);
        S <= '1';
        wait until CLK = '1' and CLK'event;
        S <= '0';
        wait until D = '1';
        wait until CLK = '1' and CLK'event;
        end loop;
    end process;
end test1;
```

```
20.11 (a) library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_ARITH.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;
entity mult20_11 is
    Port (CLK, S: in std_logic;
        Mplier, Mcand : in std_logic_vector(3 downto 0);
        Product : out std_logic_vector(7 downto 0);
        D : out std_logic);
end mult20_11;
architecture Behavioral of mult20_11 is
    signal State, NextState: integer range 0 to 4;
    signal B: std_logic_vector (3 downto 0); -- Multiplier counter
    signal A: std_logic_vector (3 downto 0); -- Multiplicand register
    signal PU: std_logic_vector (3 downto 0); -- Upper half of product register
    signal PL: std_logic_vector (3 downto 0); -- Lower half of product register
    signal andarray: std_logic_vector (3 downto 0);
    signal addout: std_logic_vector (4 downto 0);
    signal muxout: std_logic_vector (3 downto 0);
    signal BZ, LA, CP, DB, IB, IA, LPU, LPL, EA, MS, CC, SC, C: std_logic;
    alias B3: std_logic is B(3);
    alias A3: std_logic is A(3);
    begin
        BZ <= '1' when B = "0000" else '0';
        muxout <= PU when MS = '1' else PL;
        andarray <= A when EA = ' }1\mathrm{ ' and IA = ' }0\mathrm{ ' else
            not A when EA = '1' and IA = '1' else
            "1111" when EA = '0' and IA = '1' else "0000";
        addout <= ('0' & muxout) + ('0' & andarray) + ("0000" & C); -- adder output is 5 bits
        Product <= PU & PL; -- including carry
        process (S, State, BZ)
        begin
            CP <= '0'; LA <= '0'; DB <= '0'; IB <= '0'; MS <= '0'; CC <= '0'; -- control signals are '0'
            SC <= '0'; EA <= '0'; IA <= '0'; LPU <= '0'; LPL <= '0'; D <= '0'; -- by default
            case State is
            when 0 =>
            CP <= '1'; LA <= '1';
            if S = '1' then NextState <= 1;
            else NextState <= 0; end if;
            when 1 =>
            NextState <= 2;
            if B3 = '1' then SC <= ' }1\mathrm{ ';
            else CC <= '1'; end if;
        when 2 =>
                            if BZ = '1' then NextState <= 4;
            else NextState <= 3; LPL <= '1'; EA <= '1'; end if;
            if BZ = ' }0\mathrm{ ' and B3 = '1' then IA <= '1'; end if;
            when 3 =>
            LPU <= '1'; MS <= '1'; NextState <= 2;
            if B3 = '0' then CC <= '1'; DB <= '1';
                    else SC <= '1'; IB <= ' }1\mathrm{ '; end if;
            if (A3 xor B3) = '1' then IA <= '1'; end if;
            when 4 =>
            D <= '1';
            if S = '1' then NextState <= 4;
            else NextState <= 0; end if;
        end case;
    end process;
```

Unit 20 Solutions

```
20.11 (a) process (CLK)
(contd) begin
    if CLK'event and CLK = '1' then -- update registers on rising edge of clk
        if LA = '1' then B <= Mplier;
            A <= Mcand; end if; -- load multiplier \& multiplicand
    if \(\mathrm{CP}=\) ' 1 ' then \(\mathrm{PU}<=\) " 0000 ";
        PL <= "0000"; end if; -- clear product registers
    if \(D B=\) ' 1 ' then \(B<=B-1\); end if; \(\quad-\) decrement multiplier
    if \(I B=\) ' 1 ' then \(B<=B+1\); end if; \(\quad-\) increment multiplier
    if \(\mathrm{LPL}=\) ' 1 ' then \(\mathrm{PL}<=\) addout(3 downto 0 ); end if;
    if LPU = ' 1 ' then \(\mathrm{PU}<=\) addout(3 downto 0 ); end if;
    if \(C C=\) ' 1 ' then \(C<=\) ' 0 '; elsif \(S C=\) ' 1 ' then \(C<=\) ' 1 ';
        else \(C<=\) addout(4); end if; -- load carry flip-flop
    State <= NextState;
    end if;
    end process;
end Behavioral;
```

20.11 (b) --Same as 20-10(b) except
constant N : integer :=6;
type arr is array( 1 to N ) of std_logic_vector(3 downto 0 );
constant Mcandarr: arr := ("0111", "0000", "0111", "1000", "0111", "1000");
constant Mplierarr: arr := ("0111", "0111", "0000", "0111", "1000", "1000");


[^0]:    begin if $\mathrm{ClrN}=$ ' 0 ' then Q <= "1000"; elsif Rising_Edge (CLK) then Q <= Q_plus; end if; end process stt_trnstn;
    end bhvr;

